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# Performance Identification for REITs by Using Draw Measures

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We use existing drawdown measures as well as modifications and extensions to gain insight into pronounced periods of gains and losses among global real estate companies. While there is no indication on heavier loss periods for companies that had experienced higher drawups in the previous market upturn, it appears that companies which fell the most during the recent crisis experienced larger upward phases in the following period.

### **Keywords**

Maximum Drawdown; Conditional Drawdown; Drawup; Real Estate Stocks

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## 1. Introduction

The use of drawdown statistics dates back to the early 1990s, with Grossman and Zhou (1993) who define the drawdown term as the value obtained in relation to the previous all-time high. Going forward, the drawdown measure made its way into the financial markets where it is a commonly mentioned term in portfolio advertisements and analyses. A considerable amount of research that concern the measure has emerged as well<sup>1</sup>, with recent literature that still enhances the knowledge in that area.

Reveiz and Leon (2008) use a risk metric, the maximum drawdown, in an effort to derive mean-variance curves of superior efficiency in a portfolio context, and Berkelaar and Kouwenberg (2010) integrate drawdown measures in an asset-liability framework. Elie and Touzi (2008) employ drawdown metrics for lifetime consumption and investment. Some studies related to asset-liability and pension fund space are concerned with relative measures as well. When it comes to relative measurement, studies in the field of performance analysis and those that make comparisons with benchmarks are interesting as well in the drawdown context; drawdown-based performance measures and ranking relations with the Sharpe ratio (Sharpe 1994) are studied by Schuhmacher and Eiling (2011), for example.

Apart from pure portfolio optimization and composition contexts, others including Vecer (2006 and 2007), James and Yang (2010) and Carr et al. (2011) provide innovations that concern drawdown insurance, hedging and trading. The latter take into account the upside phases of the respective series as well, as we do when considering drawup measures as counterparts to downside metrics. The evaluation of drawup phases is found as well in de Melo Mendes and Ratton Brandi (2004), Rebonato and Gaspari (2006), and Pospisil et al. (2009), among others.

As the measure is still commonly used in practice and academic research, several disadvantages and admittedly poor statistical properties of the measure, like the fact that they are not coherent, have led to related innovations and enhancements, and many of which are found in the studies mentioned above<sup>2</sup>. We will review the classical drawdown measure and present several modifications and extensions in Section 2 and apply the presented tools onto real estate companies from a global sample in Section 3, followed by concluding remarks in Section 4.

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<sup>1</sup> Incorporating the drawdown statistic in a portfolio or investment context is not only carried out by Grossman and Zhou (1993), a variety of studies emerged very quickly after that, see Cvitanic and Karatzas (1995),

<sup>2</sup> See Artzner et al. (1999) and Delbaen (2000 and 2002) for discussions on coherent risk measures.

## 2. Drawdown Measures: Classical, Extensions and Ratios

Generally, the absolute drawdown is defined as:

$$AbDD(W(t)) = \max\{W(j), j = 0, 1, \dots, t\} - W(t)$$

where  $(W(t))$  is the cumulative return in  $[0, t] = R(1) + \dots + R(t)$  for any returns of a fund or benchmark  $R(i)$  in any period  $i$ . Therefore, the average drawdown is defined as:

$$ADD(T) = \{AbDD(1) + \dots + AbDD(T)\} / T.$$

Similarly, the absolute and average drawups can be accordingly computed:

$$AbDU(W(t)) = -\min\{W(j), j = 0, 1, \dots, t\} - W(t)$$

$$ADU(T) = \{AbDU(1) + \dots + AbDU(T)\} / T$$

From the measures above, it is easy to compute the maximum drawdown and the maximum drawup:

$$MDD(T) = \max(ADD(W(t)), t = 1 \dots T)$$

$$MDU(T) = \max(ADU(W(t)), t = 1 \dots T)$$

By measuring the strength of upward periods against downward movements in the observation periods, we compute the maximum drawup /down ratio (max draw ratio) and the average drawup /down ratio (average draw ratio):

$$MDR(T) = MDU(T) / MDD(T)$$

$$ADR(T) = ADU(T) / ADD(T)$$

All of the above definitions are straightforward with respect to the commonly known definition of drawdowns. The drawup metric was previously used in several studies in different fashions, and Ortobelli et al. (2010) proposed the use of the ratio for investment management.

The drawdown measures lack several properties that have led to enhancements and extensions which are expected to be more in line with risk management and measurement, and better suited for them<sup>3</sup>. A very similar measure to the expected tail loss (ETL) or the conditional value at risk (CVaR) for continuous distributions is provided by defining the conditional drawdown (CDD)<sup>4</sup>:

$$CDD_{1-\alpha}(W) = E\{AbDd(W) | AbDD(W) > AbDD_{1-\alpha}(W)\}$$

<sup>3</sup> See Rachev et al. (2008) for discussions on risk, uncertainty and performance measures. The conditional value at risk (CVaR) corresponds to the average value at risk (AVaR), see Pflug and Romisch (2007) for example.

<sup>4</sup> See Chekhlov et al. (2003 and 2005) for detailed introductions with regards to the CVaR measure.

As can be seen from the definition above, the CDD at risk is the counterpart of the CVaR, as it measures the expected drawdown for the  $\alpha$  % of the worst drawdowns. Admittedly, the CDD was also termed “conditional drawdown at risk, CDaR” in the past, which is sort of a misnomer, as it is not the drawdown which is at risk, but wealth. Accordingly, the CDD would be more in line with what the measure represents, and used in the following.

For the sake of comparison and completeness, consider the ETL or CVaR<sup>5</sup>,

$$ETL_{1-\alpha}(R_a) = E(\max(-R_a, 0) | -R_a > VaR_{1-\alpha}(R_a))$$

where  $ETL_{1-\alpha}(R_p)$  is the expected tail loss with tail probability  $\alpha$  for asset returns  $R_a$  and VaR denotes the value at risk. In accordance with common usage for risk measures, such as VaR, are values of 1% or 5% for  $\alpha$ , which correspond to confidence levels of 99% and 95%, respectively. For any confidence level, the ETL is higher than the VaR as it measures the expected losses in the case of a tail event rather than measuring the loss not to be exceeded with the respective confidence. In terms of risk measurement, the choice of an appropriate measure is another way to overcome erroneous estimations, as for example, the VaR at a 95% confidence of a normal distribution may be the same as the corresponding measure for a stable distribution or a  $t$  distribution, but the ETLs or CVaRs at 95% may largely differ.

Given the measures of CVaR and CDD above, we have the definition for risk measures that are informative on how high the losses are in case they exceed the VaR at a certain confidence level (CVaR) or the severity of the drawdown when it is larger than that obtained at a certain confidence level.

Having specified the CVaR and CDD, we introduce a risk measure that makes perfect sense when employed for analyses that aim at both risk measurement and intertemporal or path dependent analyses. Like for the CDD, our measure aims to link the desirable properties of a risk measure with the intuitiveness of a drawdown measure. The adjustment hereby is the definition of the expected drawdown in relation to a pre-specified threshold value, namely the conditional threshold drawdown (CTDD):

$$CTDD(W) = E(AbDD(W) | AbDD(W) > AbDD_T)$$

With the CTDD, we have introduced a measure that defines the expected drawdown for those that are larger than the pre-specified threshold value. In complementing the CDD, the CTDD is informative on the loss to be incurred

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<sup>5</sup> See Rockafellar and Uryasev (2000 and 2002), Sortino and Satchell (2001), Acerbi and Tasche (2002) and Tasche (2002) among others concerning VaR and CVaR / ETL and ES (expected shortfall).

should the drawdown exceed the threshold, rather than for the relatively worst drawdowns within all drawdowns. This enables investment management with risk thresholds for time periods and not only successive time points as otherwise provided by the ETL or CVaR.

With regards to the estimation of the statistics, one may rely on historical data, as well as simulations. While commonly used Monte Carlo simulations may serve for the calculation of the CVaR, simulating drawdown measures requires the calculation of paths or trees. This may be achieved by estimating the time-series properties for drawdown periods and the whole time span analysed, followed by path generating with simulations by using the parameters found in the previous step. While moving from historical to simulation data imposes the work-load of obtaining simulated paths, the calculation of such could further improve the forecasting properties.<sup>6</sup>

As for the measures shown in Section 2, we can define the respective upside counterparts as well, namely, the conditional tail gain (CTG) for an upward counterpart to the CVaR, and conditional drawup (CDU) and conditional threshold drawup (CTDU) for the intertemporal measures:

$$\begin{aligned}CTG_{1-\alpha}(R_a) &= E(\max(R_a, 0) | R_a > Gain_{1-\alpha}(R_a)) \\CDU_{1-\alpha}(W) &= E(AbDU(W) | AbDU(W) > AbDU_{1-\alpha}(W)) \\CTU(W) &= E(AbDU(W) | AbDU(W) > AbDU_T)\end{aligned}$$

By applying the previous interpretations, the CTG is informative on the average gains should the return be larger than that obtained at the given confidence interval. Accordingly, the CDU is the expected drawup obtained for the case when a drawup is larger than that not to be exceeded at a given confidence level, and the CTDU is the drawup to be realized when a drawup is greater than the pre-specified threshold value. Naturally, as the statistics above are all counterparts to the downside measures, the mathematical properties are the same and the structure of the measures is preserved.

While the drawups and drawdowns, and the related extensions presented, are already informative on a stand-alone basis, their use in ratios may be beneficial for investment management and analyses. The use of performance ratios has been previously well-researched, and many studies have combined both risk and reward measures and other statistics with drawdown-related elements.

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<sup>6</sup> Problems with the backward-looking nature of risk measures however are not limited to the drawdown metrics, as with Zimmermann et al. (2003) and Bhansali (2005) who discuss the risk measure deficiency in unexpected breakdowns. Although the argument is valid, there is nevertheless the necessity to properly define the measures in the first step, and especially in any forecasting framework.

From the previously defined measures, we can already build a large variety of ratios, some of which are discussed in the following, namely, the conditional tail ratio (CTR), the conditional draw ratio (CDR) and the conditional threshold draw ratio (CTDR):

$$CTR_{1-\alpha}(R_a) = \frac{CTG_{1-\alpha}(R_a)}{CTR_{1-\alpha}(R_a)} = \frac{E(\max(R_a, 0) | R_a > Gain_{1-\alpha}(R_a))}{E(\max(-R_a, 0) | -R_a > VaR_{1-\alpha}(R_a))}$$

$$CDR_{1-\alpha}(W) = \frac{CDU_{1-\alpha}(W)}{CDD_{1-\alpha}(W)} = \frac{E(AbDU(W) | AbDU(W) > AbDU_{1-\alpha}(W))}{E(AbDD(W) | AbDD(W) > AbDD_{1-\alpha}(W))}$$

$$CTDR_{1-\alpha}(W) = \frac{CTDU_{1-\alpha}(W)}{CTDD_{1-\alpha}(W)} = \frac{E(AbDU(W) | AbDU(W) > AbDU_T)}{E(AbDD(W) | AbDD(W) > AbDD_T)}$$

Depending on the aim of the analyses, the mixing of numerators and denominators is possible, as well as substituting for the displayed measures. For the sake of brevity, we continue to use the definitions above in the following. While performance and reward-to-risk ratios like the CTR exist in numerous modifications (see Farinelli et al. (2009) for example), ratios that include draw measures did not receive attention on a broad basis. However, several interesting studies which have been mentioned in the introduction section, recommend employing those in a framework designed to identify both upside potentials and downside risk on a statistical basis and in an intuitive way.

The CDR and CTDR, for example, are elegant ways to combine probabilistic measurement of possible drawup and drawdown phases into a relational context. Interpretation is straightforward, as one obtains measures that are informative not only on the length and strength of extended positive and/or negative periods, but how they relate to each other.

### 3. Empirical Analysis

We apply the presented and discussed measures in the context of the global real estate investment trust (REIT) market, which is especially suited for such an analysis when the nature of cyclicity in the markets is taken into consideration, as well as due to the fact that the tremendous rise and sharp fall of the sector in the last 10 years makes for a challenging surrounding for risk and drawdown measures.

By using daily data from October 2002 to October 2011, we have 2365 observations for 250 real estate companies that comprise the GPR 250 Index at the sample's end. The beginning of the study was selected as the trough of the market before the next upturn, which ended in February 2007.

Of course, this means that we omit some companies, but this number is very small with only 2 falling out of the study period due to bankruptcy, according

to information by the GPR. Out of the 250, 166 companies have a history that completely spans the sample size (therefore, mergers and spin-offs are included in the sample which have a complete history) and where used for the analysis<sup>7</sup>. We use the GPR 250 Index itself for the analysis, omitted companies where only the ones that started after 2002 when crucial market periods are identified and stocks are compared with the index. All data were obtained from the GPR and Bloomberg.

Figure 1 gives an overview on the companies and the index. One can clearly identify the pronounced surge in the index in the years before the subprime crisis and the credit crunch, with the downswing being sharp and enduring, followed by a recovery period. We identify the market peak for the index on the 23rd of February 2007 and the trough on the 9th of March 2009. This makes it possible to not only analyse the sample as a whole, but divide the sample into subparts and see how the statistics compare with each other. Furthermore, one has a complete up-and-down, and a complete down-and-up phase to analyse.

By comparing the measures between the two different phases, we can see whether the stocks which had the largest surge in the upswing, are the ones that were hurt the most or more resistant in the downfall following the peak. On the other hand, it is possible to check whether the stocks which experienced the largest declines are the ones that are quickly regaining in the following upswing, or whether they stay low. If this is the case, this should show up in both the respective drawup and drawdown measures, as well as in the ratios thereof.

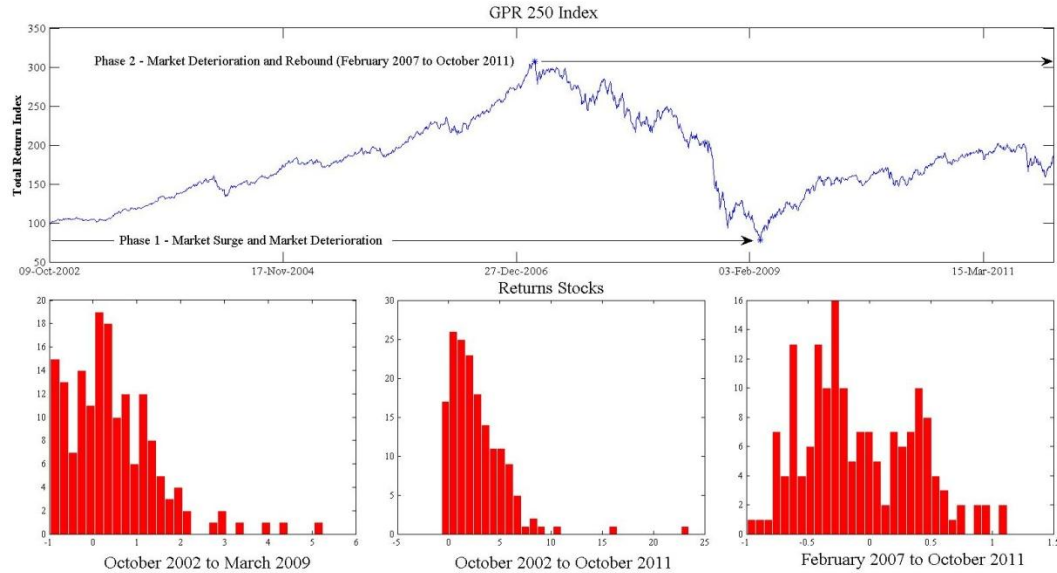
Of course, the peaks and troughs of the single stocks may be before or after those in the index, but tranching the sample at the market turning points is straightforward in the context of analysing how stocks performed in the light of broad market movements. However, to identify how the drawup and drawdown periods of stocks with respect to their own turning points evolved, we will later add the analysis by using company-specific turning points.

We begin the analysis by comparing the maximum, average, conditional, and conditional threshold drawdowns as well as the corresponding drawup measures in two phases, i.e. October 2002 to March 2009, and February 2007 to October 2009. Note that the downward phase from February 2007 to March 2009 is contained in both sub-samples, and with no strong assumptions, one can expect that the most severe and pronounced drawdowns for both sub-samples mainly take place during that time span.

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<sup>7</sup> For some of the drawup analyses below, we had to omit General Growth Properties Inc. and Inmobiliaria Colonial SA from the figures, as their large jumps in price led to extremely high values in drawup measures. The figures therefore would have been unusable as the large values make a large scale necessary and the differences between other companies would not have been visible any more.

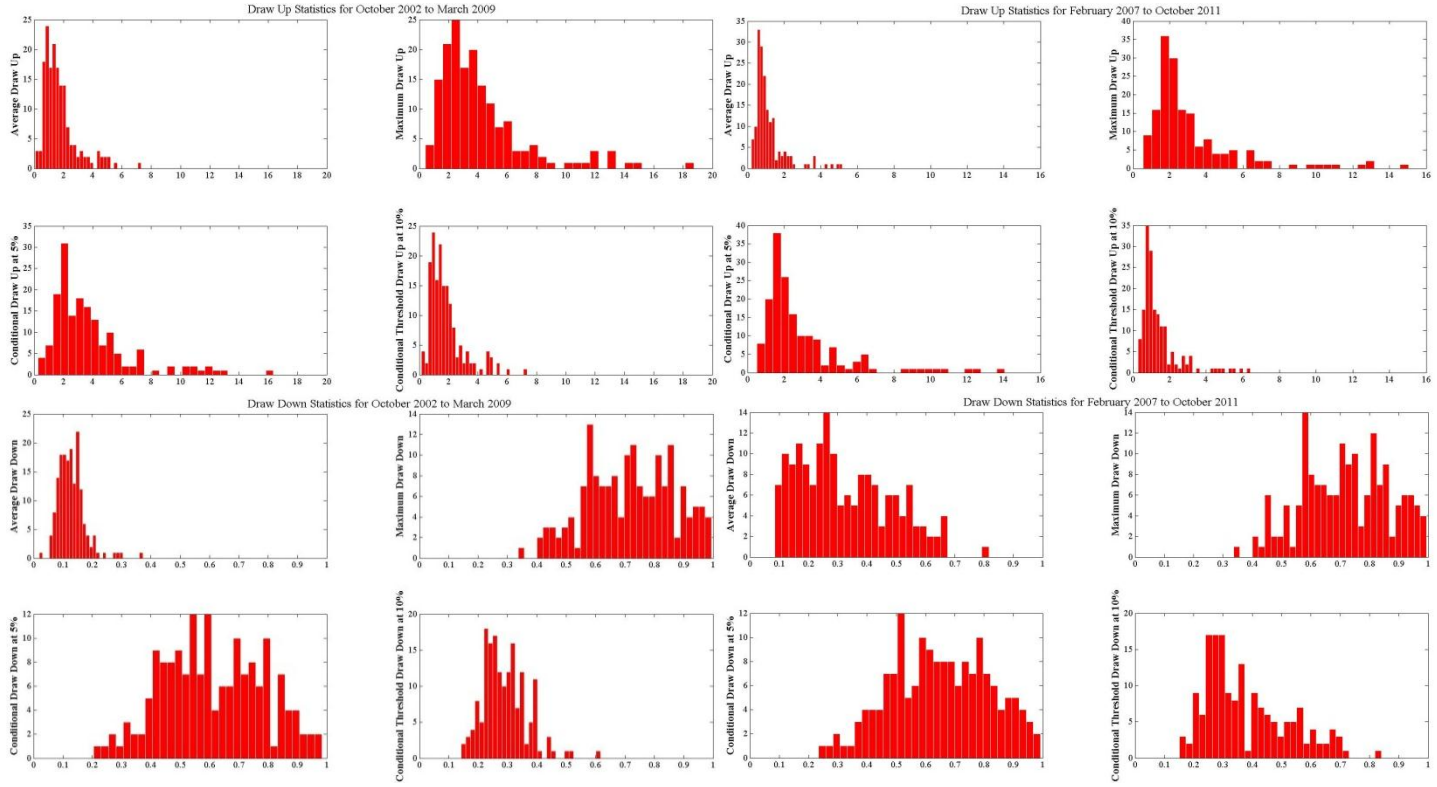
**Figure 1** The GPR 250 Index and the Returns of the Constituents



*Notes:* The upper plot shows the GPR 250 Index for the full sample from the beginning of October 2002 to the end of October 2011. The market peaked on the 23rd of March 2007 and had its trough on the 9th of March 2009. In the lower plot, the distributions of returns for the full sample (middle) and the two chosen phases (left and right) for the constituents with a full history within the specified time are depicted. Please note that while the index itself had total losses during Phase 1 (-22.04%) and Phase 2 (-40.62%), the return over the full sample is +82.33%, as the phases are overlapping and the market decline following the subprime meltdown is included in both sub-samples.



**Figure 2 Drawdown and Drawup Statistics**



It is obvious from Figure 2 that there is large variation in the measures which is due to the fact that 1) the stocks are from a global sample, and countries and regions differ, and 2) the respective stocks may have peaks and troughs that do not coincide with the market turning points. Clearly, the drawups are more pronounced in the first phase, as the run up in the real estate equity markets was much larger than the recent recovery following the crisis – which may or not be over and the phase is still on-going. The largest drawdowns, however, are merely taking place in the market downturn between February 2007 and March 2009, as expected. Therefore, the maximum drawdown distribution, for example, is almost the same for both phases as the February 2007 to March 2009 period is included in both samples – as the second part of Phase 1 and the first part of Phase 2.

Having analysed the separate statistics for both upward and downward measuring, we are interested in obtaining insight into the relation of the respective numbers. By calculating the ratios as described above, we can see for both subsamples, how and whether the amplitudes compare with their counterparts on the other side.

As the identification of market phases may induce a sub-sample truncation that masks the characteristics of the companies themselves, we analyse the stocks by using both the market turning points and the respective turning points of the companies. The results do not differ much, and so we only show<sup>8</sup> the analysis by using the turning points of the companies themselves in Figures 3 and 4.

We can see from Figure 3 that the distribution of the draw ratios is as dispersed as the respective measures that constitute it, with the ratios being considerably smaller for the second phase. This is the first indication that a clear relation of amplitudes in the upward and downward phases may not exist. To obtain more insight into the relation of the performance amplitudes of companies, we plot the respective drawup and drawdown measures against each other for both phases in Figure 4.

While one cannot see a relation for Phase 1, meaning that companies with the strongest performance phases in the market surge before February 2007 have not necessarily lost the most in the following market turmoil in terms of ongoing loss periods, it appears that there is a positive relation in Phase 2. While there still is dispersion among the measures, companies that had experienced the largest drawdowns appear to have accordingly recovered with a slightly positive relation seen in the scatter plots.

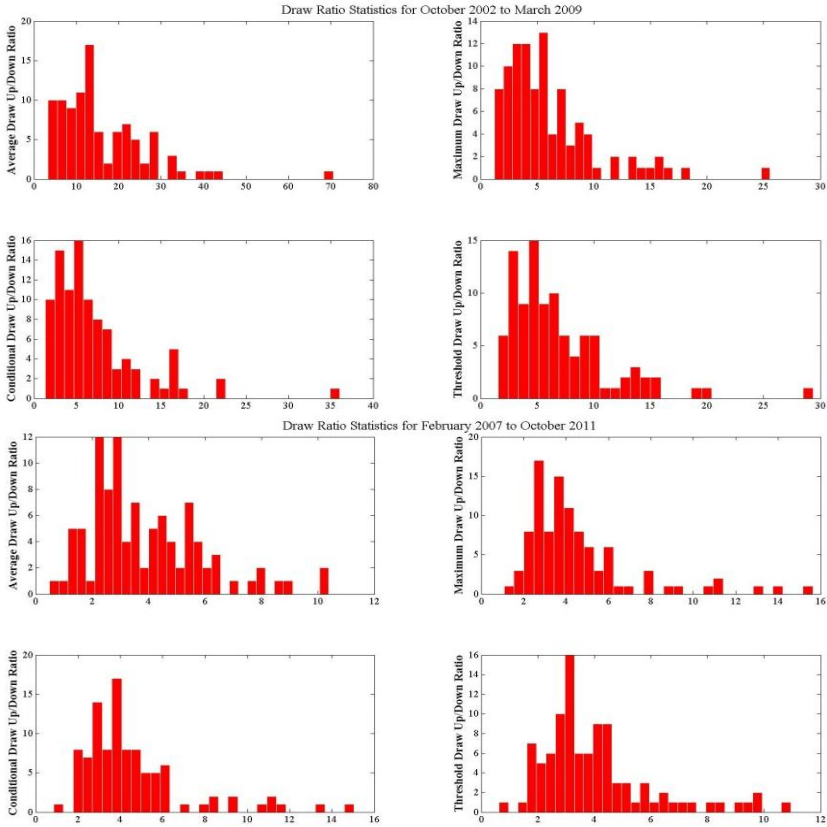
We consider the fact that there is an obvious similarity in the second phase as highly interesting, because the stocks differ so much with respect to their

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<sup>8</sup> All other results are available from the authors upon request

drawdown and drawup measures, and no relation was seen for Phase 1. The question on what to conclude from this difference between Phases 1 and 2 might be answered when considering the fact that there was much uncertainty in the market on whether the peak was reached when the markets started to decline, and the possible severity of a starting downturn. This could be an explanation for the continued phases of gains that were not mirrored by proportionate loss periods in the crisis. On the other hand, observation of stocks that experienced the strongest periods of gains for those that fell the most, may be an indication of the perception that they were overly sold with respect to their fundamental values.

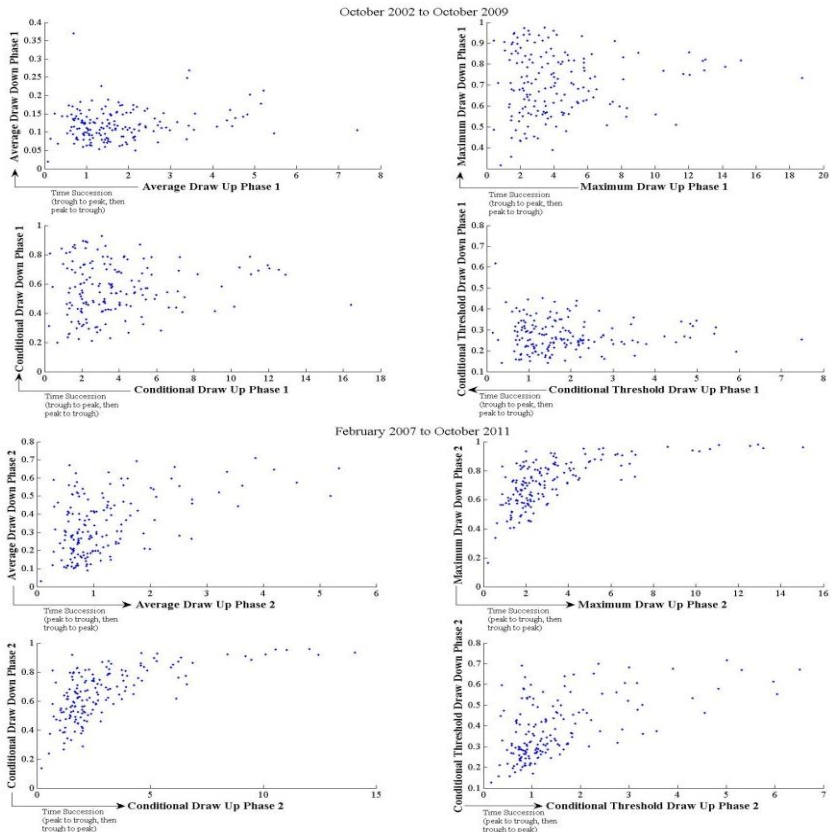
**Figure 3 Draw Ratio Statistics, Respective Phases**



In order to obtain a more robust view of the relationships in both phases and rule out that the observations made were random effects, we divide the sample on a regional basis. While the Africa (6) and South-America (5) stocks are too few in number, enough stocks are included in the index to form sub-samples for Asia (70), North America (114) and Europe (55). Figures 5 and 6 show the comparisons for the regional samples. While the Asian sample for both phases

does not show a clear relation, the results from above resemble those in the Europe and North-America plots: stocks with the most pronounced downturns experience the largest gains in upturn periods thereafter.

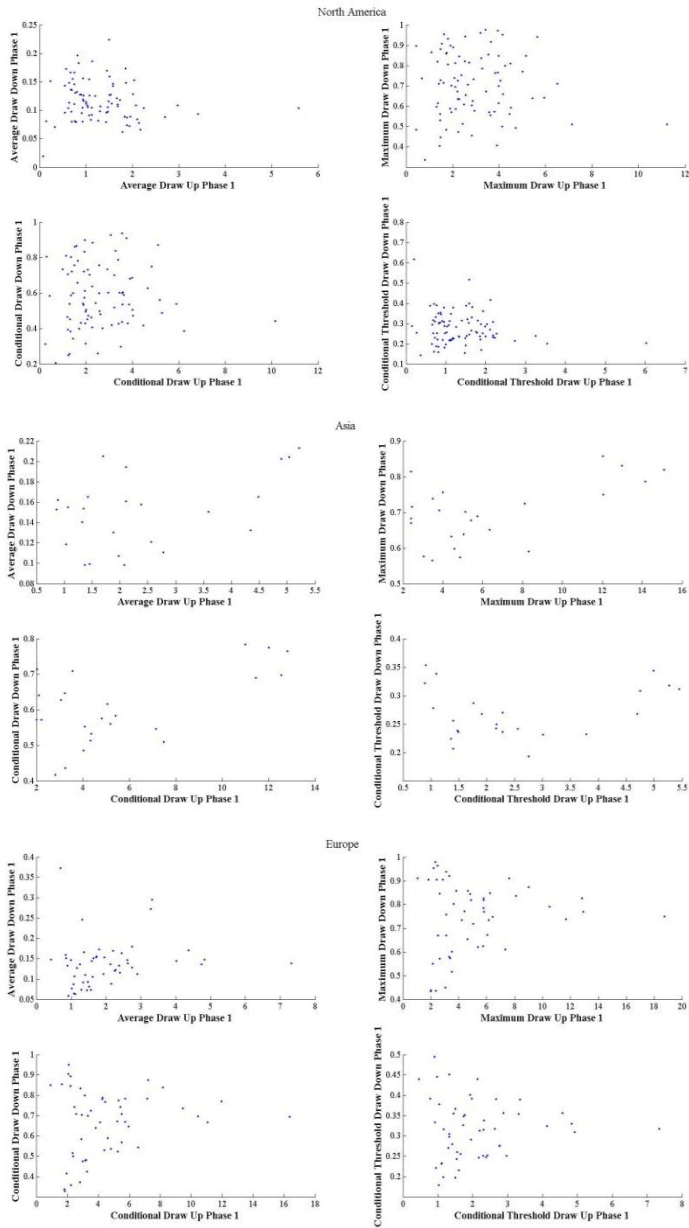
**Figure 4 Comparison of Successive Up- and Down Periods, Respective Phases**



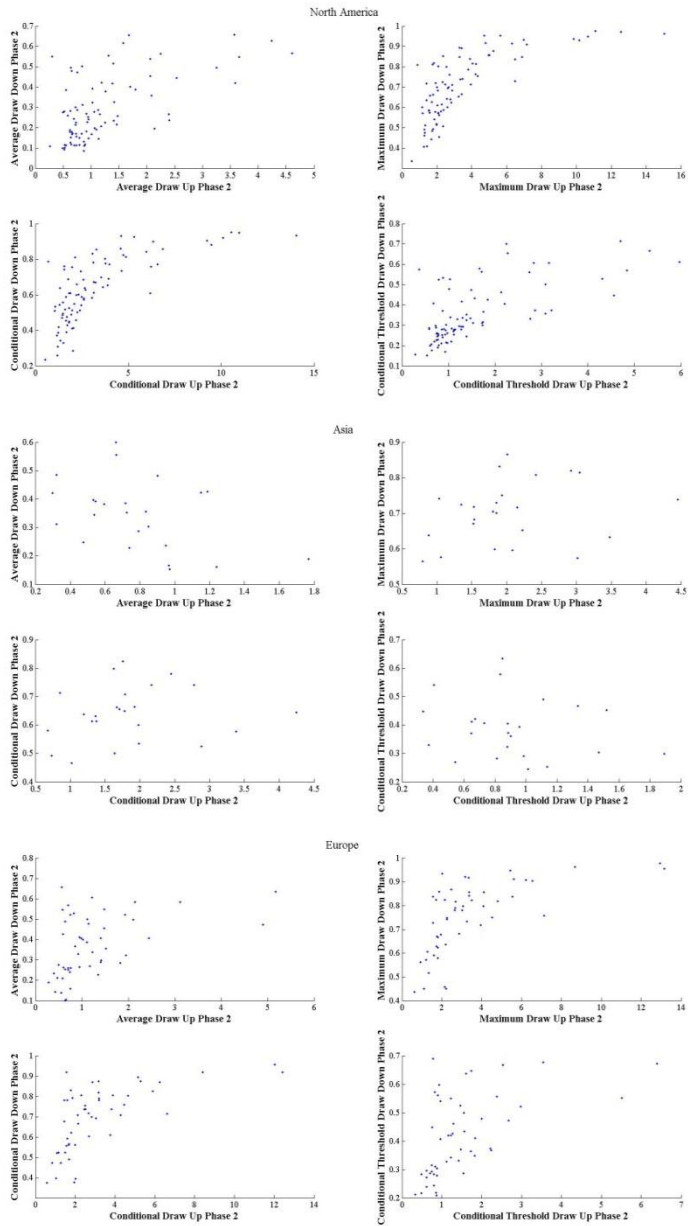
The result for the Asian sample might be attributed to the fact that Japanese stocks comprise most of this regional portfolio of stocks, and Japan (again) experienced a financial market movement as opposed to other developed countries.

To check whether the differences of the drawdown relations in Phases 1 and 2 are significant, we ran the Kolmogorov-Smirnov test to compare the distributions of draw ratios of the two phases. Table 1 presents the results both for the analysis by using the market peak and the respective peaks of the stocks. All test results further indicate that the ratios are not the same in the two different phases.

**Figure 5 Comparison of Successive Up- and Down Periods, Phase 1**



**Figure 6 Comparison of Successive Up- and Down Periods, Phase 2**



**Table 1** Kolmogorov-Smirnov Test on Draw-Ratio Distributions

	Average Draw Ratio	Maximum Draw Ratio	Conditional Draw Ratio	Condit. Threshold Draw Ratio
<i>Analysis by using the GPR 250 market peak</i>				
Test statistic	0.7262	0.2679	0.2917	0.4286
Probability	(5.1086e-40***)	(8.2102e-06***)	(8.2066e-07***)	(3.2445e-14***)
<i>Analysis by using the respective peaks of stocks</i>				
Test statistic	0.7530	0.2831	0.3614	0.4398
Probability	(1.6927e-42***)	(2.2542e-06***)	(4.0508e-10***)	(8.9530e-15***)

**Notes:** This table reports the test statistics of the two-sample Kolmogorov-Smirnov test with a null hypothesis in which the distributions of the respective draw ratios are the same; Probability identifies p-values; \*\*\*, \*\*, and \* denote significance at the 1%, 5%, and 10% levels, respectively.

However, while the distribution test indicates a general divergence with regards to the ratios, from the previous analyses, we still only have a visual indication on whether there is a meaningful relation between the upward phase following the downturn and the lacking of this relation in the downturn after the run-up to the peak. Accordingly, we ran cross-sectional regressions for the drawdown measures of Phase 1 based on its drawup measures and for the drawup measures of Phase 2 based on its drawdown measures (with the latter being largely identical to the drawdown measures of Phase 1 as mentioned above).

Table 2 presents the results of the cross-sectional regressions. As expected, there is a significant and strong positive relationship between the drawdown and drawup measures for Phase 2. Surprisingly, in 3 of the 4 measures for Phase 1, the coefficient of the corresponding measure in the period before is significant as well, although the coefficient is merely zero. This might be an effect of some strongly influential outliers, and we ran the cross-sectional regressions again, thereby excluding the ten strongest observations on both sides for the upside and downside measures. We continued to use  $166-4 \cdot 10 = 126$  observations for the “outlier-free” sample. The results are striking: while the significance and strength for Phase 2 remains the same, none of the 4 measures in Phase 1 show statistical or economic significance. With regards to the previous indications on the relation of the measures in Phase 2 but none in Phase 1, the cross-sectional results are additional evidence for observations made by visual inspection.

From the visual and statistical inspections, we can finally conclude that stocks with stronger drawdowns after the peak have stronger drawups in the following recovery period, while the stocks that previously experienced the largest drawups before the peak apparently do not correspondingly fall afterwards.

#### **4. Conclusion**

We have employed both existing as well as modified measures of drawups and drawdowns to identify pronounced periods of gains and losses in real estate companies in a global sample. While there is no relation between the measures in the phase characterized by the run-up to the peak before the crisis, there appears to be a positive relation between drawup and drawdown measures in the second phase with the recovery following the downturn. This result holds true for phases induced by market peaks and troughs, as well as when considering the respective turning points of the stocks.



**Table 2** Cross-Sectional Regressions of Respective Up- and Down-Periods

	Average Draw Ratio	Maximum Draw Ratio	Conditional Draw Ratio	Condit. Threshold Draw Ratio
<i>Analysis by using the GPR 250 market peak</i>				
Coefficient Phase 1	<b>0.0105</b>	<b>0.0049</b>	<b>0.0095</b>	0.0025
Probability	<b>(3.5191e-05***)</b>	<b>(0.0511*)</b>	<b>(0.0034***)</b>	(0.5345)
Coefficient Phase 2	<b>3.6791</b>	<b>15.4705</b>	<b>12.9611</b>	<b>5.8982</b>
Probability	<b>(2.8433e-06***)</b>	<b>(5.8487e-08***)</b>	<b>(4.9142e-08***)</b>	<b>(8.0531e-08***)</b>
<i>Analysis by using the respective peaks of the stocks</i>				
Coefficient Phase 1	<b>0.0129</b>	<b>0.0052</b>	<b>0.0092</b>	0.0046
Probability	<b>(3.3563e-06***)</b>	<b>(0.0454**)</b>	<b>(0.0077***)</b>	(0.3032)
Coefficient Phase 2	<b>3.7305</b>	<b>14.6138</b>	<b>12.5210</b>	<b>6.1455</b>
Probability	<b>(1.3926e-06***)</b>	<b>(1.0799e-07***)</b>	<b>(7.3818e-08***)</b>	<b>(5.0393e-08***)</b>
<i>Analysis by using the GPR 250 market peak / Excluding top and bottom 10 for both measures</i>				
Coefficient Phase 1	0.0020	-0.0054	0.0022	-0.0036
Probability	(0.6231)	(0.3301)	(0.7549)	(0.5396)
Coefficient Phase 2	<b>1.0077</b>	<b>6.8814</b>	<b>5.3694</b>	<b>2.1927</b>
Probability	<b>(0.0041***)</b>	<b>(1.9795e-11***)</b>	<b>(1.5394e-09***)</b>	<b>(2.8912e-05***)</b>

*(Continued...)*

*(Table 2 Continued)*

	Average Draw Ratio	Maximum Draw Ratio	Conditional Draw Ratio	Condit. Threshold Draw Ratio
<i>Analysis by using the respective peaks of the stocks / Excluding top and bottom 10 for both measures</i>				
Coefficient Phase 1	0.0023	-0.0046	0.0058	-0.0066
Probability	(0.5401)	(0.4169)	(0.4354)	(0.3185)
Coefficient Phase 2	<b>1.0608</b>	<b>6.8466</b>	<b>5.1234</b>	<b>2.1743</b>
Probability	<b>(0.0035***)</b>	<b>(1.5262e-11***)</b>	<b>(2.0002e-09***)</b>	<b>(6.2401e-05***)</b>

**Notes:** This table reports the coefficients of the cross-sectional regressions of the respective measures on the preceding measure that is its opposite, i.e. of drawdown on drawup (phase 1) and of drawup on drawdown (phase 2); Probability identifies p-values; \*\*\*, \*\*, and \* denote significance at the 1%, 5%, and 10% levels, respectively.

The fact that a positive relation is present only for the recovery following the downturn during the recent crisis may be due to the fact that there was either an overly strong sell-off in some real estate stocks that is gradually being corrected, or there was more confidence that the trough is reached, than there was on whether the peak is over, which leads to more stringent relations. By dividing the sample with respect to the various regions, this solidified the findings of the aggregate sample.

Furthermore, the fact that there is large dispersion among the drawups, drawdowns and ratio measures in different market phases has strong implications for risk and portfolio managers, as the diversity provides both chances and threats depending on asset allocation and diversification. Portfolio applications therefore may be promising given the diversity of the company metrics. However, short-term decision making and long-term oriented measures provide challenges in addition to the task of path dependent calculations and non-linear optimization problems.

Interestingly, the proposed extended measures, like conditional and conditional threshold drawdowns, do not add much more insight, which shows that the use of the intuitive measure of maximum drawdowns may serve well for longer horizons. However, possible short-term applications may show improved usefulness of such a measure.

We consider the approach of using the various measures useful in the context of large samples and on a comparison or portfolio basis, as the intuitiveness of the metrics is best taken advantage of on a comparable basis in the presence of a heavily diverse sample. This makes the identification of differences and similarities in the markets, like the relation of drawdowns and drawups in Phase 2, possible.

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