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Re-Default Risk of Modified Mortgages

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During the recent housing recession and financial crisis, mortgage modification has been heavily promoted by the U.S. government as a way to stabilize the housing and the national banking systems. Numerous programs, such as the Home Owners Preserving Equity (HOPE), Home Affordability Modification Program (HAMP), and Home Affordability Refinance Program (HARP), were introduced or enhanced to allow more aggressive modifications than traditionally observed prior to the crisis. Loan modification is believed to be a way to avoid foreclosure and to help borrowers keep their homes. However, the effectiveness of loan modification in preventing eventual foreclosure has not been quantified. In this paper, we use Federal Housing Administration (FHA) modified loans to analyze their re-default risk. We use loan-level data to trace the performance of loans with heavy modifications. We have three major empirical findings. First, the empirical model shows that modified loans tend to have much higher redefault risk than otherwise identical never-defaulted loans. Second, the re-default model shows that re-default hazard is less sensitive to traditional risk drivers, compared with non-modified loans. Third, the redefault risk declines initially with the magnitude of the payment reduction associated with the modification received. However, as the payment reduction becomes substantial, the probability of re-default increases. Our empirical results suggest payment reduction is most effective around the 10% to 30% level, in order to reduce re-default risk. The

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effect is relatively flat between the 30% to 40% level. Payment reduction beyond the 40% level increases re-default risk, controlling for all observable variables. These findings have profound implications in how lenders should design optimal modification policies.

Keywords

Residential Mortgage, Default Risk, Credit Risk

1. Introduction

During the recent housing recession and financial crisis, mortgage modification has been heavily promoted by the U.S. government as a way to stabilize the housing and the national banking systems. Numerous programs, such as the Home Owners Preserving Equity (HOPE), Home Affordability Modification Program (HAMP), and Home Affordability Refinance Program (HARP), were introduced or enhanced to allow more aggressive modifications than traditionally observed prior to the crisis. Loan modification is believed to provide benefits to the mortgage borrower, lender, and overall housing market in the following three aspects.

First, loan modification helps borrowers to avoid foreclosure and keep their homes. Brevoort and Cooper (2013) show that the average drop in credit score during foreclosure could be 150-250 points, and the post-foreclosure credit recovery of the borrower could be lengthy and painful. For subprime borrowers, it takes about 5-7 years for their credit score to recover back to the pre-foreclosure level. Prime borrowers may take 7-10 years to recover. The borrower groups associated with recent vintages have been hurt so badly that some of them may never recover from the trauma of foreclosure in terms of their credit profile. Thus foreclosure avoidance through loan modification definitely helps distressed borrowers to maintain better credit, and keep their house as shelter.

Second, loan modification provides the lender with an alternative other than going through foreclosure and the eventual real estate owned (REO) process. The loss given default (LGD) rate is an important factor in determining mortgage default risk. It is largely driven by the local house price movement, initial financial leverage ratio, foreclosure costs, lawyer fees, maintenance costs, and the time length for the REO sale. The whole process could be very costly, especially during the housing downturn of 2006-2012. Qi and Yang (2009) report that the LGD rate could have been as high as 49.2% for the highest current loan-to-value (CLTV) bucket for Federal Housing Administration (FHA) loans during 1990-2003. Chen et al. (2013) report that for some states, such as Michigan (MI) and Ohio (OH), the LGD rate of FHA loans could have

been as high as 80%-85% during the 2000-2012 period. Obviously with such high loss rates, even a loan modification with a 50% principal reduction is still cost effective, if moral hazard is not considered.

Third, loan modification reduces the shadow inventory of distressed homes in the foreclosure/REO pipeline. As mentioned earlier, the foreclosure process could be lengthy and take up to two, or even five years for some judicial states. As long as the foreclosure courts are backlogged with loans which are in the pipeline, the continuing supply of additional housing units will make the housing market weak. Also, this expectation will reduce the confidence of home builders. Removing those mortgage loans from the shadow inventory will definitely help the housing market achieve a healthy recovery.

As of March 2014, the HAMP official web site reported that nearly 2 million mortgage assistance actions, including 1.3 HAMP modifications have been performed.¹ This is much less than the originally planned 7-8 million target. The median monthly payment reduction is \$544, and homeowners have saved US\$25.5 billion since the HAMP modifications.

However, the effectiveness of loan modification in preventing eventual foreclosure has not been adequately quantified and appears to be inconclusive. A Fox Business report on May 2012 alleged that programs like HAMP modifications are only helping a few homeowners and have not been effective at dealing with the mortgage crisis.² The National Taxpayer Union has also argued that the HAMP has been grossly ineffective.³ Based on a mortgage metric report of the Office of the Comptroller of the Currency (OCC), U.S. Department of the Treasury for 2013 Q3, the re-default rate within 5 years was close to 70%.⁴ The following statements are quoted from the report.

"Servicers modified 3,288,717 mortgages from the beginning of 2008 through the end of the second quarter of 2013. At the end of the third quarter of 2013, 45.5 percent of these modifications were current or paid off. Another 6.3 percent were 30 to 59 days delinquent, and 11.1 percent were seriously delinquent. Another 5.1 percent were in the process of foreclosure, and 7.8 percent had completed the foreclosure process."⁵

In this paper, we use FHA modified loans to investigate the effectiveness of loan modification in preventing re-default. Loan-level data are used to trace the performance of loans with heavy modifications. The empirical results show that

¹ http://www.makinghomeaffordable.gov

² http://www.foxbusiness.com/industries/2012/05/02/mortgage-programs-target-many-help-few/

³ http://www.ntu.org/news-and-issues/government-reform/hamp-terminate.html

⁴ http://www.occ.gov/publications/publications-by-type/other-publications-

reports/index-mortgage-metrics.html

⁵ See Appendix for the detailed status of mortgages modified.

modified loans tend to have much higher re-default risk than otherwise identical never-defaulted loans. The re-default risk declines initially with the magnitude of the payment reduction associated with the modification received. However, as the payment reduction becomes substantial, the re-default probability increases. Our empirical results suggest payment reduction is most effective around the 10% to 30% level, in order to reduce the re-default risk. The effect is relatively flat between the 30% to 40% level. Payment reduction beyond the 40% level increases re-default risk, controlling for all observable variables. These findings should have profound implications for loan modification policy design, and change the conventional optimal modification strategy.

This paper is organized as follows. Section 2 offers an overview of the literature on mortgage default, and re-default of loan modifications. Section 3 briefly outlines the competing hazard risk model for mortgage termination, and the specification of multinomial logistic (MNL) models. Then we describe the summary statistics of the data, and provide some discussion on the model variable specification in Section 4. In Section 5, we present the empirical model results. Lastly, in Section 6, we summarize the major findings and provide some hypotheses for the seemingly unintuitive result, and discuss the policy implications of our research.

2. Literature Review

In this paper, we focus on the effectiveness of loan modification in preventing re-default. Therefore, in this section, we review the literature on mortgage default models and re-default of loan modifications, thus providing background information upon which our paper has been developed.

2.1 Residential Mortgage Default

There is an abundance of literature on mortgage default, even before the subprime meltdown followed by the global financial crisis. They can be broadly categorized into two types: option-based (or "structural") and hazard-based (or "reduced-form") default models.

The option-based default models follow the seminal work of Merton (1974), and formulate mortgage default as a put option, which may be exercised when the option is in-the-money, i.e., the collateral value is lower than the mortgage value, and the borrower can realize monetary gain by selling the property at a higher price (the mortgage value). Kau et al. (1992, 1993b) build option-based pricing models for fixed rate mortgages (FRMs) and adjustable rate mortgages (ARMs). Titman and Torous (1989) build similar models for commercial mortgages. However, some empirical works (Foster and Van Order 1984, 1985) suggest that mortgage borrowers do not default as efficiently as the option theory suggests.

In order to accommodate these empirical findings, Crawford and Rosenblatt (1995) extend the option-based default model to include transaction cost, which can be interpreted as loss of credit opportunities in the future, and some non-monetary factors, such as stigma associated with foreclosure record. Kau et al. (1993a) build contingent claim valuation models with both transaction costs and "suboptimal" exercising, and calculate the default probabilities of mortgage loans. Kau et al. (1994) also argue that even without transaction cost, the borrower would not default immediately when the collateral value drops below the mortgage value, due to time value of the option.

Vandell (1995) conducts a comprehensive survey on these option-based models, and still find that the predicted mortgage default rate is much higher than the default rate actually observed, even after including transaction cost, and sub-optimality. Buist and Yang (1996) characterize how stochastic household income partly determines the choice of a household in a rental or mortgage contract through time. Yang et al. (1998) extend the conventional two-factor (interest rate and house price) contingent claim model to a three-factor model to include the stochastic income factor, which affects both the capacity of the borrower to refinance and the ability to pay existing mortgage payment obligations.

Even with all these adjustments, option-based default models are still not widely used by industry practitioners, mainly because of three reasons. First, it is very time-consuming to solve the American option pricing problem with multiple factors. Second, it is hard to calibrate the default zone empirically with microeconomic data. Third, it is difficult to capture the real dynamics of random drivers and their correlations in an arbitrage free framework.

Due to the above limits, the other mortgage default model, i.e., hazard-based model, has gained substantial popularity recently, especially with the influx of large amounts of default data after the subprime crisis. The option-based model tries to solve for the boundary values of the state variables (such as interest rate, house price, income level) and identify areas of option exercise. Instead of solving for the boundaries of optimal (or suboptimal) exercise, the hazard-based model assumes that the mortgage could default (or prepay) at any time after origination, conditional on that it has not prepaid or defaulted yet. The hazard function in this model is generally defined as the product of a baseline hazard and a function of time-varying covariates, such as the Cox proportional hazard model. These covariates could include variables upon which the option value depends, such as probability of negative equity, refinance incentive, etc. However, they are not limited to those option related variables, and can include any other important factors, when deemed necessary or empirically sound, such as credit score, seasonal dummies, etc. The hazard-based model can be estimated relatively straight-forward and fit reasonable mortgage prepayment and default behaviors empirically. Also, it does not need to explicitly address the so-called "sub-optimality" found under the option-based model framework.

Several early empirical studies have applied the Cox proportional hazard model to evaluate mortgage default or prepayment risk (such as Green and Shoven, 1986; Schwartz and Torous, 1989; Quigley and Van Order, 1990, 1995). However, these models generally address prepayment and default separately, as if they are independent terminations. We know that these two termination events are mutually exclusive, thus making them competing hazards. Also factors that drive one event generally deter the other. For example, borrowers with high credit score are more likely to prepay, and less likely to default. Mortgages with a higher loan-to-value (LTV) ratio are more likely to default, and less likely to prepay. Thus these two events are highly inter-dependent.

In a series of papers, Deng et al. (1996, 2000) and Deng (1997) attempt to simultaneously estimate the prepayment and default risk of residential mortgages from micro level data. After that, the competing hazard risk model has been widely accepted as the standard modeling approach to estimate the prepayment and default behavior of residential mortgages, and many researchers have contributed to this line of literature, mainly on finding new explanatory variables or re-examining the traditional credit risk factors. For example, Keys et al. (2010) find that the securitization level during the subprime boom period increases the default hazard risk. Foote et al. (2010) find that affordability level at origination is not a significant default indicator, while expectation of future house price appreciation is significant. Gerardi et al. (2010) find that low financial literacy levels are definitely highly correlated with higher default risk. Krainer and Laderman (2011) suggest that tightening mortgage underwriting guidelines may contribute to low prepayments and high delinquency. Fuster and Willen (2013) try to incorporate payment size into the hazard function

In this paper, we follow the standard literature in using the most recent competing hazard risk model to estimate the default and prepay risks. Since we focus on loan modification and re-default, we review papers that specifically address the re-default risk of loan modification in the next section.

2.2 Residential Mortgage Modification Re-default

Haughwout et al. (2009) are one of the first to conduct research on subprime modification, which proceeded the government initiated HAMP. They find that the re-default rate declines with the magnitude of the reduction in the monthly payment, and declines relatively more with principal forgiveness, compared to interest rate reduction.

Voicu et al. (2012) draft a hazard-based framework that compares the performance of the HAMP with non-HAMP modifications, and find that the HAMP modifications are more successful than those of the non-HAMP. They also find payment reduction as the main determinant for loan modification redefault.

McCoy et al. (2012) examine the performance of new private-label mortgage loan modifications after 2009. They find that these more recent private-label loan modifications have a lower overall re-default rate, compared to similar modifications made in the pre-2009 years. They also report that having a fixed rate mortgage, higher credit score, and lower initial mortgage note rate all contribute to a lower re-default rate. Mortgage type, loan purpose, and documentation level all affect the success rate. They confirm that payment reduction via principal forgiveness is most effective, compared to arrears capitalization and rate reduction.

Payment reduction related research is conducted by Tracy and Wright (2012). They do not attempt to estimate the re-default rate for modified loans. Instead, they propose a competing risk model to estimate the sensitivity of default risk to downward adjustments of the monthly mortgage payments of borrowers for a large sample of prime ARMs. They find that payment reduction is a significant driver in reducing the default risk.

However, none of these previous research work has identified the increasing redefault rate, which is associated with excessive payment reduction. We believe that our paper is the first to identify this phenomenon.

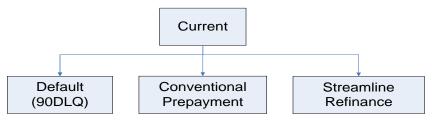
3. A Competing Hazard Risk Model for Post-Modification Performance

In this section, we provide a brief introduction on the competing hazard risk model framework used in this paper and the specification of our MNL models.

3.1 A Competing Hazard Risk Model for Mortgage Default

Our model framework is illustrated in the chart in Figure 1.

Figure 1 Competing Hazard for Current FHA Loans



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Following the standard practice of modeling mortgage termination, we establish the competing hazard risk model framework with the three following sources of termination.

Default: this event is defined as becoming 90 days delinquent (90DLQ) from the "current" status. Technically a mortgage is not terminated at this stage. It can go back to current, be modified, be prepaid, or go to foreclosure or short sale. However, we are mainly interested in borrower-driven events such as 90DLQ. The following events are largely driven by both the borrower and the lender, and heavily influenced by policy interventions and operational constraints, such as foreclosure moratorium, modification initiative, foreclosure lag, servicing capacity, backlogs in foreclosure court, etc. Thus we only model the competing hazard for loans in the "current" status, which could have selfcured from previous defaults, or have been modified in the past. The performance of defaulted loans will be another topic.

Conventional Prepayment: this event is defined as when the borrower pays off the mortgage via a property sale transaction, or refinance into a conventional mortgage, generally a government-sponsored enterprise (GSE) loan. FHA loans have an annual mortgage insurance premium (MIP), which generally results in a higher effective coupon rate (nominal mortgage note rate plus MIP). When house price appreciates and/or the loan amortizes, and the current LTV reaches 80%, FHA borrowers can refinance to GSE loans without paying the insurance premium. It is widely speculated that during the subprime boom period of 2004-2007, many FHA loans refinanced into subprime. However, after that subprime meltdown, most of the conventional prepayments are believed to take GSE refinance opportunities, especially as the FHA has drastically increased its MIP.

Streamline Refinance (SR): this event is defined as when the borrower refinances into another FHA via the SR program. This program allows current FHA borrowers to take advantage of lower interest rates, and exempt them from the traditional underwriting process, i.e., property appraisals and credit profile checks are not required.

The reason that we separate total prepayment into conventional prepayment and the SR is based on the following observations.

First, conventional prepayment and the SR are driven by different events. Conventional prepayment includes both housing turnover and rate refinance, while SR only includes rate refinance.

Second, conventional prepayment and SR have difference refinance rates. During a conventional refinance, the borrower is comparing the GSE mortgage rate with his/her effective coupon rate. For a borrower who is considering an SR opportunity, s/he is comparing the new effective coupon rate (new FHA mortgage rate plus new MIP) with his/her existing effective coupon rate. Third, conventional prepayment and SR are driven by different agency behaviors. Both the GSEs and FHA have raised their fees after the financial crisis, yet at different levels and different dates. The GSEs have sharply raised their delivery and guaranty fees after the conservatorship in 2008. The FHA raised their upfront MIP in 2008, and then their annual MIP in 2010, 2011, 2012, and 2013. In 2013, the FHA also changed the minimum MIP schedule and revoked the annual MIP expiration threshold at CLTV 78%. They also have special treatment for SR loans, based on prior mortgage endorsement date.

All these differences have made it extremely difficult to combine conventional prepayment and SR into one hazard function.

3.2 Specification of Multinomial Logistic Models

As summarized above, the competing hazard model framework attempts to model loan behaviors in current status. For loans currently at the start of the quarter, the competing risks are prepayment, transition to default status, or remaining current, as shown above in Figure 1. Competing risks include three possible types: default, SR, and other prepayment (PRE). This gives rise to four possible transition probabilities that require estimation.

We specified the MNL models of quarterly conditional probabilities for transitions from current to prepayment, default, or remaining current. The corresponding mathematical expressions for the conditional probabilities over the time interval from t to t+1 for loans starting in the current status in a quarter t to conventional prepayment, SR, default, or remaining current, respectively, in the subsequent quarter t + 1 are given by:

$$\pi_{PRE}^{CUR}(t) = \frac{e^{\alpha_{PRE}^{CUR} + X_{PRE}^{CUR}(t)\beta_{PRE}^{CUR}}}{1 + e^{\alpha_{PRE}^{CUR} + X_{PRE}^{CUR}(t)\beta_{PRE}^{CUR} + e^{\alpha_{SR}^{CUR} + X_{SR}^{CUR}(t)\beta_{SR}^{CUR}} + e^{\alpha_{DEF}^{CUR} + X_{DEF}^{CUR}(t)\beta_{DEF}^{CUR}}}$$
(1a)

$$\pi_{SR}^{CUR}(t) = \frac{e^{\alpha_{SR}^{CUR} + X_{SR}^{CUR}(t)\beta_{SR}^{CUR}}}{1 + e^{\alpha_{PRE}^{CUR} + X_{PRE}^{CUR}(t)\beta_{PRE}^{CUR}} + e^{\alpha_{SR}^{CUR} + X_{SR}^{CUR}(t)\beta_{SR}^{CUR}} + e^{\alpha_{DEF}^{CUR} + X_{DEF}^{CUR}(t)\beta_{DEF}^{CUR}}}$$
(1b)

$$\pi_{DEF}^{CUR}(t) = \frac{e^{\alpha_{DEF}^{CUR} + X_{DEF}^{CUR}(t)\beta_{DEF}^{CUR}}}{1 + e^{\alpha_{PRE}^{CUR} + X_{PRE}^{CUR}(t)\beta_{PRE}^{CUR}} + e^{\alpha_{SR}^{CUR} + X_{SR}^{CUR}(t)\beta_{SR}^{CUR}} + e^{\alpha_{DEF}^{CUR} + X_{DEF}^{CUR}(t)\beta_{DEF}^{CUR}}}$$
(1c)

$$\pi_{CUR}^{CUR}(t) = \frac{1}{1 + e^{\alpha_{PRE}^{CUR} + X_{PRE}^{CUR}(t)\beta_{PRE}^{CUR}} + e^{\alpha_{SR}^{CUR} + X_{SR}^{CUR}(t)\beta_{SR}^{CUR}} + e^{\alpha_{DEF}^{CUR} + X_{DEF}^{CUR}(t)\beta_{DEF}^{CUR}}}$$
(1d)

We apply the approach developed by Begg and Gray (1984), in which we estimate separate binomial logistic (BNL) models for each possible transition type and then recombine the estimates to derive the MNL probabilities. Begg and Gray (1984) apply Bayes' Law for conditional probabilities to demonstrate that the values of parameters α_f^i and β_f^i estimated from separate BNL models are parametrically equivalent to those for the corresponding MNL model once appropriate calculations are performed. Assume that the conditional

probabilities for current-to-prepay and current-to-default transitions for separate BNL models for loans in the current status at the start of quarter t are given, respectively, by:

$$\Pi_{PRE}^{CUR}(t) = \frac{e^{\alpha_{PRE}^{CUR} + X_{PRE}^{CUR}(t)\beta_{PRE}^{CUR}}}{1 + e^{\alpha_{PRE}^{CUR} + X_{PRE}^{CUR}(t)\beta_{PRE}^{CUR}}}$$
(2a)

$$\Pi_{SR}^{CUR}(t) = \frac{e^{\alpha_{SR}^{CUR} + X_{SR}^{CUR}(t)\beta_{SR}^{CUR}}}{1 + e^{\alpha_{SR}^{CUR} + X_{SR}^{CUR}(t)\beta_{SR}^{CUR}}}$$
(2b)

$$\Pi_{DEF}^{CUR}(t) = \frac{e^{\alpha_{DEF}^{CUR} + X_{DEF}^{CUR}(t)\beta_{DEF}^{CUR}}}{1 + e^{\alpha_{DEF}^{CUR} + X_{DEF}^{CUR}(t)\beta_{DEF}^{CUR}}}$$
(2c)

where we have used upper-case Π to indicate the BNL probability and differentiate it from the lower-case π that was used above to denote the MNL probabilities. Estimation of the BNL probabilities in Equations (2a) - (2c) produces estimates of parameter α_f^i and β_f^i s that can be substituted directly into Equations (1a) - (1c) to derive the corresponding MNL probabilities.

4. Data, Variable and Summary Statistics

In this section, we provide an introduction on our sampling method and explain how we constructed the model variables, including the control variables. Lastly, we provide summary statistics on the sample.

4.1 Choice-Based Sampling Approach

All of the loan-level data and loan modification data from the FHA singlefamily data warehouse were extracted for the analysis. We focused on fixedrate 30-year fully underwritten purchase and refinance loans. This produced over 22 million single-family loans that originated between 1975 through to the second quarter of 2013. Among these loans, historical status transition records during 1996 and later years were reconstructed to estimate the loan status transition models. Our model estimation dataset did not include pre-1996 data due to the limited availability of reliable 90-day default episode data and major changes in FHA underwriting policies in 1996. The resulting dataset was used to generate loan-level transition event histories until the end of the observed data period.

In credit risk modeling, a choice-based sample is commonly used for large populations with relatively rare events of interest. We used a two-stage choicebased sampling process for estimating the transition equations where the sampling rates were determined by the terminal status of each loan and its status at each period. This sampling approach enhances the efficiency of model estimation, and is supported by the literature. A paper by Manski and Lerman (1977) in Econometrica titled "The Estimation of Choice Probabilities from Choice Based Samples" is one of the first papers to address the topic of choice-based samples. Before that, sampling was mainly used on independent variables, instead of dependent variables. Since the parameters of a probabilistic choice model are estimated conditional on the independent variables, the sampling technique generally does not produce bias. Manski and Lerman (1977) prove that for a general probabilistic choice model, the maximum likelihood estimation (MLE) estimator is consistent and converges to the un-sampled estimator when the choice-based samples are weighted correspondingly. Scott and Wild (1986) discuss a response-based sample in a logistic model framework, and find that although the weighted estimators might be less efficient, the sampling produces unbiased parameter estimates of the logistic coefficients. Xie and Manski (1988, 1989) argue that even though under the logistic model, the random sampling and response-based sampling maximum likelihood estimators coincide for all parameters except the intercept, modelers should avoid assuming the logistic model form and analyzing the responsebased samples without adjusting the sample weights. The weighted MLE estimates a constrained best predictor of the binary response.

Two-stage Choice-based Sampling

The first step in the two-stage sampling process is to over sample the bad loans, in which a bad loan is defined as a loan that has ever been 90-day delinquent:

- a. Loan-level sampling rate of good loans = 10%
- b. Loan-level sampling rate of bad loans = 100%

The second step is to over sample in the bad quarters, in which a bad quarter is defined as the quarter that a loan becomes a first-time 90-day delinquent and all subsequent performance quarters:

- c. Quarterly loan-level sampling rate of non-default quarters = 10%
- d. Quarterly loan-level sampling rate of default and subsequent quarters = 100%

With this two-stage sampling process, we calculated the following sampling probability matrix that shows the ultimate sampling probability for loan-quarter combinations. The corresponding weights that we used are the reciprocal of the probabilities of selection.

 Table 1
 Choice-based Sampling Probability Matrix

| Sampling Rate | Good Loan | Bad Loan |
|---------------|-----------|----------|
| Good Quarter | 10% | 10% |
| Bad Quarter | N/A | 100% |

We used loans that originated from 1996 through to 2012Q3 to estimate the status transition models that start in current then transition to other statuses, which correspond to the loan cohorts for which complete data were available on new 90-day default episodes. The data were used to generate quarterly loan-level event histories to the end of the sampling period or when the loan claimed, fully prepaid or matured.

4.2 Model Variables

In this section, we first discuss the major model variables, then the control variables.

4.2.1 Major Model Variables

Prior Loan Modification Indicator

We separated the loans which were self-cured or cured by loan modification, and for the latter, we introduced a prior loan modification indicator (Prior_mod). The prior loan modification indicator is equal to 1 after the flag of a loan modification cure is turned on, and remains at 1 until the termination or payoff of the loan. For example, if a loan receives a loan modification and is cured from default in its 20^{th} quarter, the prior loan modification indicator is equal to 1 and remains 1 starting from the 21^{st} quarter.

Loan Modification Payment Change

The purpose of loan modification is to change one or more of the terms of a loan. This allows the loan to be reinstated, and results in a payment the borrower can afford. Therefore, the percentage change of monthly payment resulting from a loan modification (Payment_rdct) will affect the capacity of the borrower to service the loan, and hence impact the future transition of the loan.

Since the financial crisis and the crash of the U.S. housing market, loan modification has been widely used to reduce foreclosures. At the beginning of the financial crisis, most loan modifications were in the form of forbearance, which resulted in monthly payment increases. In the subsequent years, modifications of the terms such as interest rate and amortization schedule became the most frequent types of modification. Within all the major types of loan modifications, forbearance is the only type which would result in monthly payment increases. As mentioned above, most of the forbearances occurred at the beginning of the financial crisis and the number of forbearances has become insignificant since 2010. Since forbearance is not expected to be a major modification type in the future time horizon, we floor the percentage of monthly payment change to zero so that the monthly payment change that results from forbearance will not impact the estimation and forecast of the model.

Details of the loan modification payment changes cannot be retrieved for some of the modified loans. In such a case, we created an indicator that specifies this missing information (*Payment_rdct_mis*).

Borrower Credit Scores

Borrower credit scores are an important predictor of claim and prepayment behavior. The FHA has relatively complete data on borrower Fair Isaac Corporation (FICO) scores for loans originated since May 2004. In addition, the FHA can retroactively obtain the credit history information of borrowers in selected samples of FHA loan applications that were submitted as far back as 1992.

Debt-to-Income Ratio

The debt-to-income (DTI) ratio measures the ratio of monthly debt payment to before-tax total household income at origination. There are two ratios available: the front-end ratio, which counts only mortgage-related housing costs, i.e., principal, interest, tax and insurance (PITI), and the back-end ratio, which includes payments for all other regular monthly debt, including car and student loans, and credit cards. We use the front-end ratio to capture the debt burden effect for the borrower, because it is better documented and measured more accurately than the back-end ratio.

Current Loan-to-Value Ratio

The CLTV is calculated as the origination loan-to-value (OLTV), divided by the appreciation factor since origination (i.e., inflating-or deflating-the denominator, the house price), adjusted for amortization. Empirical results show that the mortgage default rate is very sensitive to the CLTV ratio when the property value moves into the negative equity range (at a CLTV near or greater than 100%).

Loan-to-Value Ratio

The initial LTV is recorded in the data warehouse of the FHA. For fully underwritten mortgage products and SR loans with required appraisals, these LTV values are used directly to compute the CLTV.

Relative Loan Size

The relative loan size is proxied by the mortgage origination amount, divided by the average loan origination amount in the same state for the same fiscal year. Empirical results show that this variable is very significant in prepaymentrelated termination. This is consistent with the option theory, since loans with a higher loan size could achieve higher monetary savings, given the same relative mortgage spread.

Spread at Origination

The spread at origination (SATO) is measured as the spread between the mortgage note rate, C, and the prevailing mortgage rate, R, at the time of origination. It is widely regarded as the lender surcharge for additional borrower risk characteristics, which are not captured by standard underwriting hard data such as the FICO score, OLTV, DTI ratio, documentation level, etc. A high SATO loan is generally more risky, compared to a similar loan with a low

SATO. Some researchers also argue that a high SATO is an indicator of predatory lending, which also tends to increase credit risk.

$$SATO = C - R \tag{3}$$

Number of Quarters Since End of Last Default Episode

We use the number of quarters since the end of the latest default episode (CX_TIME) for transitions in the current status. The CX_TIME is set to zero at the origination of each loan until the end of its first default episode. It becomes 1 after the end of the default episode, and keeps increasing quarterly until the start of the next default. For example, if a loan experiences a second default episode, CX_TIME continues to increase until the start of the second default episode, it is reset to 1 and continues to accumulate until the next default.

Mortgage Premium (Refinance Incentive)

In this paper, we use the percentage difference between the monthly payment of a potential refinance $PMT_1(t)$ relative to the current payment $PMT_0(t)$ as the refinance incentive,

$$Refi_i(t) = 100 \times \frac{PMT_0(t) - PMT_1(t)}{PMT_0(t)}$$
(4)

This variable is an approximation to the call option value of the mortgage given by the difference between the present value of the "anticipated" future stream of mortgage payments discounted at the current market rate of interest and the present value of the mortgage evaluated at the current note rate. Additional details are given in Deng et al. (2000) and Calhoun and Deng (2002).

For transition into an FHA SR mortgage, we used the refinancing option for an FHA mortgage, by definition. For all other transitions, we used the payment from a market mortgage, which is assumed to be a GSE mortgage.

Also, we added the annual FHA MIP to the mortgage rate, in both the current FHA loan and the potential new FHA loan (for SR), as follows:

$$effect_coupon_rate(t) = C(t) + annual MIP(t)$$
(5)

where C(t) is the coupon rate for extant FHA loans.

For the effective GSE refinancing rate, we wanted to add the effective refinancing points to the contract rate, which translates the one-time points into an equivalent interest rate spread over time. The Federal Housing Finance Agency (FHFA) publishes both the contract rate and this effective rate, and we calculated the spread difference which was projected in our analysis. Therefore, we defined the effective refinancing cost avg_refi_cost as the spread between

an effective fixed-rate mortgage for 30 years (FRM 30) and the contract rate provided in the FHFA survey:

$$GSE_refi_rate(t) = R(t) + avg_refi_cost$$
(6)

Assuming that refinancing costs are the same for both GSE and FHA refinances, the effective rate for refinancing into an FHA loan is then built onto this GSE refinancing rate, by adding the average FHA to the GSE spread and the new annual MIP:

$$FHA_refi_rate(t) = GSE_refi_rate(t) + avg_FHA_GSE_sprd + annual_MIP$$
(7)

The payment on the current FHA loan is $PMT_0(t)$. Using the above effective refinance rates, we computed "effective" monthly mortgage payments for the current and the prospective new refinancing loans $PMT_1(t)$, which have a prefix that denotes whether they are the GSE or FHA loan option. The refinance incentive for a GSE refinancing loan is:

$$GSE_Refi_incentive(t) = 100 \times \frac{PMT_0(t) - GSE_PMT_t(t)}{PMT_0(t)}$$
(8)

The GSE refinance incentive variable is used in transitions other than the current-to-SR. The refinance incentive for a loan refinanced from the FHA in the transition current-to-SR is:

$$FHA_Refi_incentive(t) = 100 \times \frac{PMT_0(t) - FHA_PMT_1(t)}{PMT_0(t)}$$
(9)

Burnout Factor

A burnout factor is included to identify borrowers who have foregone opportunities to refinance. It is measured as the accumulation of the positive spreads between the coupon rate and new refinance mortgage rate throughout the life of the loan. The burnout factor is included to account for individual differences in propensity to prepay, often characterized as unobserved heterogeneity. In addition, unobservable differences in borrower equity at the loan level may give rise to heterogeneity that can impact both prepayment and default rates.

Credit Burnout

Burnout is a relatively well-understood concept in prepayment modeling. Borrowers who have forgone refinance opportunities in the past are less likely to refinance in the future. Similarly, borrowers who have forgone a default option and showed resilience by making uninterrupted payments in the past are less likely to default in the future. We used the cumulative number of quarters that a property has been "underwater" to proxy this effect.

Purchase-Only Home Price Index

In the calculation of the CLTV, we used the Purchase-Only (PO) Home Price Index (HPI) published by the FHFA. The PO Index is based on repeat sales at market price and does not use any appraised values. As such, it provides a more reliable measure of housing market conditions. Evidence cited below has found appraisal bias, albeit not from all appraisers. We wanted a house price series that accurately estimates CLTVs and thus the sale price of defaulted properties.

There is documented evidence of bias in residential appraisals so that the PO Index is a more accurate representation of market values. Chinloy et al. (1997) compare purchase prices against appraisals and find a two percent upward bias. In addition, they find that appraisal prices exceed purchase prices in 60 percent of the cases. They postulate that the existence of a moral hazard incentive to complete the deal might be the reason for the bias. More recent papers provide additional empirical support for the existence of appraisal bias, i.e., Agarwal et al. (2012), Tzioumis (2013), and Zhu and Pace (2012). Another reason for using the PO HPI is that in recent years, industry practices are leaning toward the PO HPI. The most commonly used indices, such as the Case-Shiller home price index and CoreLogic HPI, are all constructed based on a purchase-only methodology.

Home Price Volatility

The option theory predicts that the put (default) option value increases when the volatility of the collateral increases, with everything else being equal. The empirical results show that the marginal effect of home price volatility on default behavior is generally positive, which is consistent with the option theory. An easier way to interpret this phenomenon is that the home price volatility measures our uncertainty in calculating the updated property value; higher volatility would introduce more error on both positive and negative sides. However, the loss introduced on the negative side is not compensated by the gain on the positive side, due to the asymmetric nature of mortgage credit risk.

The home price volatility (*sigma_parm_a*) is the same as the measurement of parameter "*a*" calculated in the probability of negative equity, which indicates uncertainty with regard to the dispersion of individual house price appreciation rates around the market average, represented by the local-level HPI. The parameter "*a*" is estimated by the FHFA by applying the three-stage weighted-repeat-sales methodology advanced by Case and Shiller (1987, 1989).

Home Price Appreciation

The home price enters the model via two variables, each of which has a different interpretation. Home price appreciation since origination (at the metro/non-metro area level) determines the CLTV ratio, which is used to measure the current equity in the property. Short-term house price appreciation, which proxies for expectations of future house price movements, is also used. The rationale for this variable is that borrowers make their decisions not only on the realized historical information, but also on their expectations about future house

price appreciation. Short-term home price appreciation, HPA2y(t), is calculated as the projected house price index one year ahead, HPI(t+4), divided by the historical house price index one year ago, HPI(t-4), measured at both the national and metropolitan statistical area (MSA) levels, HPI(i):

$$HPA2Y(t,i) = \frac{HPI(t+4,i)}{HPI(t-1,t)}$$
(10)

When historical observations are used to estimate the transition equations, actual four-quarter-ahead observations are used to measure this variable. For simulations along future HPA/interest rate paths, the same measurement is made, by using the projected HPAs four-quarters ahead.

The variable $hpa2y_n = min(0, hpa2y)$ differentiates the response when the anticipated HPA is negative compared to positive.

Unemployment Rate

There is ample literature that indicates job loss, or loss of income, is one of the major trigger events for mortgage default. The natural choice of macroeconomic variables to capture this effect is the unemployment rate. However, during the period of 1994-2008, when the U.S. economy grew at a steady rate and only experienced a minor recession, the variation in the unemployment rate was extremely small, which makes it difficult to demonstrate that it is a significant factor: the national unemployment rate in that period was almost always between 4% and 6%. That is part of the reason why previous attempts to use this variable showed that it is not statistically significant. After 2008, the unemployment rate rose rapidly, and consequently, we found that this variable is both statistically and economically significant in the default behavior of borrowers.

We use two types of unemployment rates: the short-term unemployment rate change, $Delta_UE(t)$, and a relative unemployment rate, $Relative_UE(t)$. The former is measured as the change in the unemployment rate level between the last quarter and that three quarters ago, which indicates the direction of change in unemployment. The latter is measured as the ratio between the unemployment rate level in last quarter, UE(t-1), and the moving average over the last 10 years, $UE_10yr_avg(t)$, which indicates the current inventory of unemployment. For example, although the quarterly change in the unemployment rate did not vary much after 2008, the relative unemployment rate continued to climb due to the recession. The formulas for computing these two measures are:

$$Delta_UE(t) = UE(t-1) - UE(t-3)$$
(11)

$$Relative_UE(t) = \frac{UE(t-1)}{UE_10yr_avg(t)}$$
(12)

4.2.2 Control Variables

Yield Curve Slope

Expectations about future interest rates and differences in short-term and longterm borrowing rates associated with the slope of the Treasury yield curve influence the choice between ARM and FRM loans and the timing of refinancing. We used the spread of the 10-year Constant Maturity Treasury (CMT) yield over the 1-year CMT yield to measure the slope of the Treasury yield curve.

FHA Score Indicator

The FHA adopted a number of changes in 2005 with potential impacts on underwriting, including implementation of its TOTAL scorecard. So this dummy variable is defined as unity if the loan originated after 2004, and zero otherwise.

Seasonality Indicators

The season of an event observation quarter is defined as the season of the year which corresponds to the calendar quarter, where Season 1 = winter (January, February, and March), Season 2 = spring (April, May, and June), Season 3 = summer (July, August, and September), and Season 4 = fall (October, November, and December). All categorical (0-1 dummy) variables take on the value of 1 for the specified quarter, and one of the categories is omitted as the reference category.

4.3 Summary Statistics

Table 2 provides the distribution of payment reductions for modified loans. This is the key variable of focus in this paper. Each column presents the sample dispersion of the payment reduction variable under the corresponding transition. In order to investigate the changing effect of payment reduction on re-default, prepay and SR probability, we constructed 6 dummies based on the continuous payment reduction variable. Although the way that these dummies are constructed sounds arbitrary, we can see from Table 2 that there are enough observations for each dummy to generate reliable estimation results.

| | | Number of Observations Under Each | | | | |
|-----------------|---------------|-----------------------------------|------------|------------|--|--|
| | | Transition | | | | |
| | Payment | Current to | Current to | Current to | | |
| | reduction (%) | Default | Prepay | Streamline | | |
| | | | | Refinance | | |
| Payment_rdct_d1 | 0 < -10 % | 121,275 | 105,145 | 105,029 | | |
| Payment_rdct_d2 | 10% - 20% | 102,085 | 92,106 | 92,010 | | |
| Payment_rdct_d3 | 20% - 30% | 36,284 | 32,979 | 32,942 | | |
| Payment_rdct_d4 | 30% - 40% | 15,322 | 13,941 | 13,929 | | |
| Payment_rdct_d5 | 40% - 50% | 3,799 | 3,291 | 3,287 | | |
| Payment_rdct_d6 | 50% and above | 2,772 | 2,241 | 2,239 | | |

 Table 2
 Distribution of Loan Modification Payment Reduction

Table 3 presents the summary statistics for the model variables in the current to default transition. After two-stage choice-based sampling, there are 10,642,828 observations in total. The top panel describes the loan characteristic variables, i.e., the LTV ratio and credit score information. The second panel provides information on the macro-economic variables, such as housing price appreciation, unemployment rate and the 10 year and 1 year CMT bond yield curves. The major model and control variables are listed in the third panel. The summary statistics presented in Table 3 show that all of those variables have reasonable dispersion.

In Section 5, we provide two subsample estimations for the current to default transition. The first subsample is with all loans that have zero payment reduction, and the second subsample is with loans that have a positive payment reduction. Table 4 shows the summary statistics for these two subsamples.

5. Empirical Model Results

In this section, we present and discuss our empirical findings. Table 5 shows the estimation results when the continuous payment reduction variable is used in the regression. The coefficient of payment_rdct is negative. However, Figure 2, which plots the actual and predicted default likelihood at each payment reduction value, shows that the effect of payment reduction to default is not monotonic as we had first thought. The figure shows that the actual default likelihood decreases with the payment reduction but then increases after some point. With a very large payment reduction, we can see that the default likelihood decreases again. Nevertheless, Figure 2 shows that the effect of payment reduction to default likelihood may not be monotone, and using a discontinuous payment reduction variable might be a good choice to capture this non-monotone effect, i.e., use payment reduction dummies.

| | Description | MIN | MAX | MEAN | STD |
|----------------|--|------------|------------|------------|------------|
| Loan Character | ristic Variable | | | | |
| Ν | Total observations | 10,642,828 | 10,642,828 | 10,642,828 | 10,642,828 |
| LTV | Loan to value ratio | 50 | 110 | 94.82 | 6.25 |
| credit_score | Credit score | 300 | 850 | 596.16 | 43.16 |
| Macro-econom | ic Variable | | | | |
| hpa2y_n | Housing price appreciation at national level | -50.56 | 0.00 | -2.95 | 6.01 |
| delta_ue | Unemployment rate change in last two quarters | -14.50 | 12.05 | 0.14 | 0.81 |
| ycslope | Yield curve slope measured as difference of 10 year | -0.36 | 3.35 | 1.73 | 1.16 |
| | CMT to 1 year CMT rates | | | | |
| Major and Con | trol Variable | | | | |
| loansize | Relative loan size | 4.29 | 475.83 | 93.41 | 33.06 |
| sato | Spread at origination | -5.42 | 3.45 | 0.21 | 0.60 |
| ratio_tmp_tei | Front-end ratio | 0.10 | 100.00 | 24.80 | 7.90 |
| LTV_current | Current LTV | 0.11 | 2.45 | 0.76 | 0.20 |
| age | Mortgage age function | 0 | 66 | 20.16 | 14.16 |
| burnout | Burnout factor. Cumulative amount of quarterly | 0 | 65.1 | 18.33 | 19.69 |
| | positive refinance incentives | | | | |
| c_burnout | Credit burnout factor. Prior cumulative number of | 0 | 14 | 0.58 | 2.16 |
| | quarters default option is underwater | | | | |
| cx_time | Number of quarters since end of last default episode | 0 | 64 | 5.40 | 7.85 |
| GSE_refi_inc | GSE refinance incentive | -70.29 | 43.12 | 14.01 | 8.30 |

 Table 3
 Summary Statistics for Model Variables in Current to Default Transition

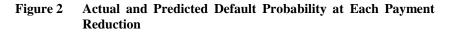
| | Loans without positive principal reduction | | | | Loans with positive principal reduction | | | |
|----------------|--|------------|------------|------------|---|---------|---------|---------|
| | MIN | MAX | MEAN | STD | MIN | MAX | MEAN | STD |
| Loan Character | istic Variable | | | | | | | |
| Ν | 10,361,268 | 10,361,268 | 10,361,268 | 10,361,268 | 281,537 | 281,537 | 281,537 | 281,537 |
| LTV | 50 | 110 | 94.87 | 6.21 | 50 | 109.85 | 93.15 | 7.40 |
| credit_score | 300 | 850 | 596.11 | 43.09 | 300 | 850 | 597.95 | 45.66 |
| Macro-economi | c Variable | | | | | | | |
| hpa2y_n | -50.56 | 0.00 | -2.94 | 6.04 | -50.56 | 0.00 | -3.27 | 4.67 |
| delta_ue | -14.50 | 12.05 | 0.15 | 0.81 | -3.65 | 8.64 | -0.18 | 0.72 |
| ycslope | -0.36 | 3.35 | 1.71 | 1.17 | -0.36 | 3.35 | 2.43 | 0.56 |
| Major and Cont | rol Variable | | | | | | | |
| loansize | 4.29 | 475.83 | 93.18 | 32.94 | 12.03 | 356.65 | 101.61 | 36.02 |
| Sato | -5.42 | 3.45 | 0.21 | 0.61 | -4.05 | 3.11 | 0.30 | 0.45 |
| ratio_tmp_tei | 0.10 | 100.00 | 24.70 | 7.90 | 0.10 | 100.00 | 26.80 | 8.20 |
| LTV_current | 0.11 | 2.45 | 0.76 | 0.20 | 0.16 | 2.41 | 0.88 | 0.23 |
| age | 0 | 66 | 19.99 | 14.15 | 4 | 66 | 24.77 | 13.82 |
| burnout | 0 | 65.1 | 18.12 | 19.65 | 0 | 65.1 | 26.21 | 19.53 |
| c_burnout | 0 | 14 | 0.52 | 2.04 | 0 | 14 | 2.62 | 4.43 |
| cx_time | 0 | 64 | 5.44 | 7.94 | 1 | 23 | 3.73 | 2.67 |
| GSE_refi_inc | -70.29 | 43.12 | 13.82 | 8.28 | -38.16 | 40.25 | 21.27 | 5.36 |

 Table 4
 Summary Statistics for Subsample Estimations in Current to Default Transition

| | Coefficient | Wald chi2 | P-value |
|------------------|-------------|-----------|---------|
| Intercept | -0.2717 | 319.63 | <.0001 |
| age | 0.0194 | 19586.60 | <.0001 |
| burnout | -0.0097 | 8829.75 | <.0001 |
| c_burnout | 0.0358 | 6711.32 | <.0001 |
| credit_score | -0.0095 | 309370.16 | <.0001 |
| credit_score_000 | -0.1898 | 1739.77 | <.0001 |
| credit_score_999 | -0.6433 | 81892.19 | <.0001 |
| cx_time | 0.0322 | 35954.77 | <.0001 |
| delta_ue | 0.1411 | 18695.87 | <.0001 |
| dti000 | -0.0185 | 5.30 | 0.02 |
| FHA_score | -0.1768 | 4216.07 | <.0001 |
| GSE_refi_inc | 0.0441 | 65965.24 | <.0001 |
| hpa2y_n | -0.0102 | 5250.29 | <.0001 |
| Payment_rdct_mis | -0.3154 | 1849.62 | <.0001 |
| Prior_mod | 1.5749 | 58542.13 | <.0001 |
| loansize | 0.0007 | 994.96 | <.0001 |
| LTV | 0.0003 | 6.31 | 0.01 |
| LTV_current | 0.8079 | 10330.46 | <.0001 |
| Payment_rdct | -2.5621 | 2346.62 | <.0001 |
| ratio_tmp_tei | 0.0207 | 53389.45 | <.0001 |
| Sato | 0.1719 | 7499.86 | <.0001 |
| season_fall | 0.2754 | 16783.18 | <.0001 |
| season_spring | -0.0460 | 408.26 | <.0001 |
| season_summer | 0.1782 | 6479.83 | <.0001 |
| ycslope | -0.0005 | 0.30 | 0.58 |

Table 5Estimation Results of Current to Default Transition with
Continuous Payment Reduction Variables

Table 5 presents the estimation results for the current to default transition, where we use payment reduction dummies instead of the continuous payment reduction variable. Interestingly, with a moderate payment reduction, the default likelihood decreases. However, when the payment reduction increases or with positive incremental in payment reduction, the change in magnitude of the default likelihood is positive. In other words, with more payment reduction, the default likelihood increases. The above conclusion is inferred from the evidence in that the coefficients of the payment reduction dummy are increasing. In addition, the coefficient of prior_mod is positive, thus implying that with everything else equal, the loan with payment modification is more likely to default. The prior_mod is a dummy variable with a value of one if the loan has been modified. Figure 3 shows that payment reduction dummies capture the non-monotone effect of the payment reduction to default likelihood.



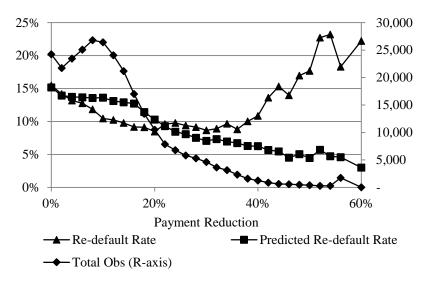
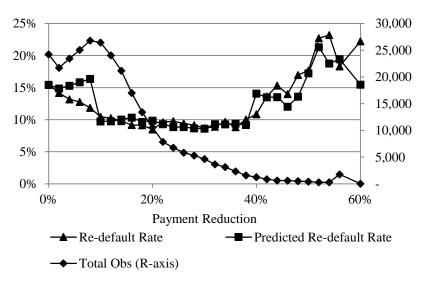


Figure 3 Actual and Predicted Default Probabilities at Each Payment Reduction



The left panel of Table 7 shows the regression results on the subsample which includes all loans with zero payment reduction, and the right panel of Table 7 presents the results on all loans with a positive payment reduction. Table 7 shows that traditional credit risk variables, such as credit score, negative equity level, DTI ratio, etc., have a weaker effect on the modified loans, compared with loans that have not been modified.

| | Coefficient | Wald chi2 | P-value |
|------------------|-------------|-----------|---------|
| Intercept | -0.2673 | 309.21 | <.0001 |
| age | 0.0194 | 19472.45 | <.0001 |
| burnout | -0.0097 | 8855.95 | <.0001 |
| c_burnout | 0.0361 | 6822.50 | <.0001 |
| credit_score | -0.0095 | 309655.84 | <.0001 |
| credit_score_000 | -0.1900 | 1743.41 | <.0001 |
| credit_score_999 | -0.6433 | 81911.07 | <.0001 |
| cx_time | 0.0323 | 36103.62 | <.0001 |
| delta_ue | 0.1411 | 18673.54 | <.0001 |
| dti000 | -0.0186 | 5.34 | 0.02 |
| FHA_score | -0.1768 | 4212.31 | <.0001 |
| GSE_refi_inc | 0.0442 | 66170.75 | <.0001 |
| hpa2y_n | -0.0102 | 5235.79 | <.0001 |
| Payment_rdct_mis | -0.3184 | 2141.05 | <.0001 |
| Prior_mod | 1.5777 | 69246.28 | <.0001 |
| loansize | 0.0007 | 1019.37 | <.0001 |
| LTV | 0.0003 | 5.77 | 0.02 |
| LTV_current | 0.8073 | 10311.24 | <.0001 |
| Payment_rdct_d2 | -0.6890 | 3167.61 | <.0001 |
| Payment_rdct_d3 | -0.5892 | 915.87 | <.0001 |
| Payment_rdct_d4 | -0.5754 | 387.36 | <.0001 |
| Payment_rdct_d5 | -0.1162 | 5.60 | 0.02 |
| Payment_rdct_d6 | 0.2471 | 24.26 | <.0001 |
| ratio_tmp_tei | 0.0207 | 53320.20 | <.0001 |
| Sato | 0.1720 | 7507.91 | <.0001 |
| season_fall | 0.2754 | 16780.63 | <.0001 |
| season_spring | -0.0460 | 407.59 | <.0001 |
| season_summer | 0.1782 | 6479.58 | <.0001 |
| ycslope | -0.0007 | 0.62 | 0.43 |

Table 6EstimationResultsofCurrenttoDefaultTransitionwith Dummy Payment Reduction Variables

| | Loans with no | Loans with no positive payment reduction | | | ositive paymer | t reduction |
|------------------|---------------|--|---------|-------------|----------------|-------------|
| Variable | Coefficient | Wald chi2 | P-value | Coefficient | Wald chi2 | P-value |
| | | | | | | |
| Intercept | -0.1889 | 151.94 | <.0001 | -1.0176 | 58.37 | <.0001 |
| age | 0.0192 | 18712.99 | <.0001 | 0.0001 | 0.01 | 0.93 |
| burnout | -0.0098 | 8860.17 | <.0001 | 0.0028 | 10.50 | 0.00 |
| c_burnout | 0.0397 | 7848.02 | <.0001 | 0.0035 | 2.64 | 0.10 |
| credit_score | -0.0097 | 312491.78 | <.0001 | -0.0025 | 309.28 | <.0001 |
| credit_score_000 | -0.1959 | 1823.41 | <.0001 | 0.0177 | 0.25 | 0.62 |
| credit_score_999 | -0.6511 | 83028.89 | <.0001 | -0.0263 | 1.45 | 0.23 |
| cx_time | 0.0331 | 37802.77 | <.0001 | -0.0757 | 763.15 | <.0001 |
| delta_ue | 0.1408 | 18350.74 | <.0001 | 0.0925 | 103.20 | <.0001 |
| dti000 | -0.0184 | 5.18 | 0.02 | -0.1224 | 2.25 | 0.13 |
| FHA_score | -0.1786 | 4236.46 | <.0001 | -0.0056 | 0.04 | 0.85 |
| GSE_refi_inc | 0.0444 | 65831.51 | <.0001 | 0.0101 | 20.06 | <.0001 |
| hpa2y_n | -0.0099 | 4901.55 | <.0001 | -0.0093 | 42.64 | <.0001 |
| Payment_rdct_mis | -0.4903 | 3180.45 | <.0001 | | | |
| Prior_mod | 1.7432 | 47272.20 | <.0001 | | | |
| loansize | 0.0008 | 1058.75 | <.0001 | 0.0001 | 0.34 | 0.56 |
| LTV | 0.0002 | 3.21 | 0.07 | 0.0024 | 6.50 | 0.01 |
| LTV_current | 0.8083 | 10060.73 | <.0001 | 0.3184 | 35.40 | <.0001 |

 Table 7
 Estimation Results of Current to Default Transition under Different Sub-Samples

(Continued...)

| | Loans with no | positive paymen | t reduction | Loans with p | ositive paymer | nt reduction |
|-----------------|---------------|-----------------|-------------|--------------|----------------|--------------|
| Variable | Coefficient | Wald chi2 | P-value | Coefficient | Wald chi2 | P-value |
| Payment_rdct_d2 | | | | -0.4037 | 808.94 | <.0001 |
| Payment_rdct_d3 | | | | -0.5102 | 534.83 | <.0001 |
| Payment_rdct_d4 | | | | -0.5326 | 283.02 | <.0001 |
| Payment_rdct_d5 | | | | -0.0928 | 3.51 | 0.06 |
| Payment_rdct_d6 | | | | 0.3356 | 45.72 | <.0001 |
| ratio_tmp_tei | 0.0209 | 53612.97 | <.0001 | 0.0075 | 94.73 | <.0001 |
| Sato | 0.1719 | 7396.46 | <.0001 | 0.0546 | 8.76 | 0.00 |
| season_fall | 0.2755 | 16513.77 | <.0001 | 0.2607 | 230.49 | <.0001 |
| season_spring | -0.0434 | 356.58 | <.0001 | -0.1764 | 100.67 | <.0001 |
| season_summer | 0.1785 | 6390.78 | <.0001 | 0.1188 | 45.13 | <.0001 |
| ycslope | -0.0025 | 7.37 | 0.01 | -0.0671 | 21.94 | <.0001 |

(Table 7 Continued)

The coefficients of prior_mod in Table 6 and the left panel of Table 7 are positive. The results in Table 6 are based on sample pooling with zero and positive payment reductions, and the sample on the left panel of Table 7 is on zero payment reduction loans. There are two groups of loans with zero payment reduction. The first group has no loan modification, which is the dominant group. The second group has a negative loan payment reduction, which is a very small group. As we discussed in Section 4.2, loans with a negative payment reduction are mostly forbearance, which is not expected to be a major modification type in the future time horizon, so we floor the percentage of monthly payment change to zero. However, the first group has a prior_mod equal to 0, while the second group has a prior_mod equal to 1. Therefore, the positive coefficient of prior_mod in Table 7 implies that all loans with a negative payment reduction are more likely to default than identical non-modified loans.

In order to see whether loans with positive payment reduction are more likely to default compared to identical non-modified loans, we design the following regression. The regression sample is based on loans with positive loan modification and without loan modification. Therefore, the prior_mod is turned on if the loan has a positive loan modification, and zero for the rest of loans. Table 7 presents the results for this regression. As we can see, the coefficient of prior_mod is positive, which clearly suggests that loans with a positive loan modification are more likely to default compared to identical loans with no modification.

The coefficients for the payment reduction dummy in Table 8 increase and then decrease at high levels of payment reduction. This evidence supports the observation in Figure 2 in that the actual default likelihood shows a trend of decline with payment reduction at first, but then increases after some point, and then declines again at very high payment reductions.

6. Policy Implications

In this section, we provide some discussion on our empirical results, and then move to the potential policy implications of our findings under two optimal modification strategies: first, optimal modification with a re-default rate that is monotonous, and second, optimal modification with a re-default rate that is non-monotonous.

6.1 Discussion of Empirical Results

Based on the empirical model results, we came up with the following findings. First, modified mortgages re-default at a much higher rate, and the re-default rate is still driven by many traditional credit risk variables, such as credit score, negative equity level, etc. Second, traditional credit risk variables have weaker effects on modified loans, compared with loans that have not been modified. Third, the re-default rate is sensitive to payment reduction, but the relationship is not monotonous, which suggests that some latent credit risk variable might be responsible for this phenomenon.

| | Coefficient | Wald chi2 | P-value |
|------------------|-------------|----------------|---------|
| Intercept | -0.1603 | 69670644.70 | <.0001 |
| age | /0.0194 | 376738180.00 | <.0001 |
| burnout | -0.0090 | 84900364.90 | <.0001 |
| c_burnout | 0.0377 | 26850720.70 | <.0001 |
| credit_score | -0.0097 | 92691800000.00 | <.0001 |
| credit_score_000 | -0.2169 | 4082843.74 | <.0001 |
| credit_score_999 | -0.6936 | 590165275.00 | <.0001 |
| cx_time | 0.0422 | 152117423.00 | <.0001 |
| delta_ue | 0.1394 | 43321170.50 | <.0001 |
| dti000 | -0.0095 | 2253.41 | <.0001 |
| FHA_score | -0.1940 | 37182864.10 | <.0001 |
| GSE_refi_inc | 0.0438 | 1286043969.00 | <.0001 |
| hpa2y_n | -0.0095 | 13224755.10 | <.0001 |
| Prior_mod | 1.3275 | 80737155.70 | <.0001 |
| loansize | 0.0008 | 16310284.70 | <.0001 |
| LTV | -0.0006 | 8352876.90 | <.0001 |
| LTV_current | 0.9029 | 1660814852.00 | <.0001 |
| Payment_rdct_d2 | -0.4263 | 2798712.45 | <.0001 |
| Payment_rdct_d3 | -0.3031 | 478591.21 | <.0001 |
| Payment_rdct_d4 | -0.2825 | 174847.04 | <.0001 |
| Payment_rdct_d5 | 0.0906 | 5782.67 | <.0001 |
| Payment_rdct_d6 | -14.1846 | 112.44 | <.0001 |
| ratio_tmp_tei | 0.0213 | 911409613.00 | <.0001 |
| Sato | 0.1720 | 27638802.10 | <.0001 |
| season_fall | 0.2687 | 57492994.80 | <.0001 |
| season_spring | -0.0434 | 1141567.93 | <.0001 |
| season_summer | 0.1745 | 20664445.70 | <.0001 |
| ycslope | -0.0008 | 8899.58 | <.0001 |

Table 8Estimation Results of Current to Default Transition with Sub-
Sample

The explanations for the first two findings are relatively straight-forward. Modified loans are defaulted at least once and then cured by payment reduction. Therefore, those loans, which have default experience, are more likely to default than identical non-default loans. Empirically, the following credit risk attributes are generally considered to be predictive for mortgage default. First, there are borrower characteristics, such as credit score, income level, income stability, other debt obligations, DTI ratio, reasons for financial distress, etc. Second, there are collateral characteristics, such as housing price, amount of (negative) equity in the house, local housing market dynamics, local housing market volatility, etc. Third, there are the mortgage loan characteristics, i.e., rate reset and payment shock in ARM.

Some of the credit attributes can be accurately measured and are very useful in predicting mortgage default. The credit score is generally required at the time of mortgage underwriting, which measures the probability that a borrower will become seriously delinquent on any of his/her credit lines within the next 18 months. ⁶ Although it is not designed to measure the default probability for mortgage loans, it is highly predictive in predicting mortgage default. The DTI ratio is another commonly used origination variable, which measures the ability of the borrower to pay. A higher DTI ratio means lower disposable income, and hence a higher default rate.

However, at the time of loan modification, the borrower would generally have been deeply in delinquency. If the mortgage payment did not change in the past, such as in the case of fixed rate and normal amortization mortgages, the borrower has mostly experienced some form of income reduction, such as unemployment/ underemployment, and/or financial distress from other life events, such as divorce, illness, etc. For adjustable rate, interest only (IO), or negative amortization mortgage, the payment could be adjusted upward, sometimes significantly. Under this situation, even if the borrower does not face income loss, or other financial distress, his/her ability to pay could be severely impaired, due to the incoming mortgage payment.

Although we generally observe many of these characteristics at the time of mortgage underwriting/origination, and can use them in common default models, some of them could be outdated and no longer indicative of the credit risk of the borrower at the time of mortgage modification. For example, the credit score at the time of origination is not very helpful for re-default prediction, since the borrower would have become seriously delinquent at that time, and the credit score would have been seriously impaired. Also, the origination DTI ratio is no longer valid since the borrower could have experienced income loss.

The explanation for the last finding is a little bit more tricky. The general belief for mortgage modification performance is that the re-default rate generally decreases monotonously as the payment reduction increases. Intuitively, this makes a lot of sense since more payment reduction means less impact on the residual income of the borrower, and a lower default risk for the loan modification.

⁶ http://www.savvyoncredit.com/credit-score-measure/

Loan modification is generally justified when there is "imminent" default, which is to say, the borrower will surely default if there is no modification. Mathematically, it basically means the re-default rate will be 100% if there is a 0% payment reduction. In the opposite case, if the payment reduction is 100%, i.e., the borrower is exempt from making any payments post-modification, the re-default rate will be 0%. Thus, conventional wisdom tells us that the re-default rate should be a monotonous function, with regard to payment reduction, and the relationship should look like Figure 4.

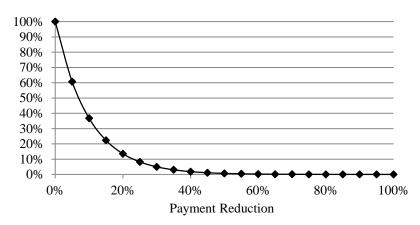


Figure 4 Stylized Re-default Rate vs. Payment Reduction

However, one major assumption that we made about the monotonous relationship between the re-default rate and payment reduction is that everything else is equal.

We could not directly observe some of the variables, even at the time of origination, such as income volatility. The income volatility contributes to a default probability in a similar way that the asset probability contributes to default probability in a traditional structural credit risk model. For example, in the seminal paper by Merton (1974) on defaultable bond pricing in the option pricing framework, default occurs when the asset value drops below the liability at the time of bond maturity. Moreover, the probability of default is driven by the three following factors. The first factor is the financial leverage of a firm. Higher leverage indicates that the firm could lose its equity position easily, and increases the default probability. The second factor is the risk free rate. A higher risk free rate means a higher drift term for the asset value in a risk-free world, and reduces the default probability. The third factor is asset volatility. Higher asset volatility widens the asset return distribution, and increases the default probability.

For consumer credit modeling, the asset level (collateral price) is still an important factor in determining default, since it measures the willingness of the borrower to pay. However, consumers generally do not default immediately after the asset value (house price) drops below the mortgage balance, due to various reasons, such as attachment to the property, shelter needs, concerns about foreclosure records, etc. It is commonly believed that the borrower would likely default when both of the following conditions are met: inability and unwillingness to pay.

The collateral value drives the willingness of the borrower to pay. When the borrower has positive equity in the house, even when s/he faces income loss and becomes unable to make the routine mortgage payment, s/he can still sell the house and avoid default. The income level drives the ability of the borrower to pay. When the borrower has adequate disposable income to pay, even when the house is underwater, s/he may choose to continue to keep the mortgage current. There have been many recent discussions on strategic default, such as the borrower is able to pay, but chooses not to, purely because of the negative equity position. However, if we look at the big picture, the overall majority of the mortgage borrowers with negative equity are still making their payments. When the borrower is both unwilling **AND** unable to pay, default probability, higher income volatility also drives up the default probability.

When we examine the common practice of loan modification, it generally follows the waterfall of rate reduction, term extension and principal forbearance/forgiveness.

According to Fannie Mae, the rule of thumb for loan modifications is to limit the new DTI ratio at 31%.⁷ Thus the loan modification agent will first try to lower the mortgage rate to reduce the new DTI ratio of the borrower to that level. The borrower will need to go through income verification to prove that s/he did not lie about his/her new level of income. The rate reduction generally has a floor rate of 2%. If the DTI ratio cannot be lowered to the target level even after excessive rate reduction, the modification agent will try to extend the mortgage term. However, the mortgage term cannot be extended for more than 40 years, and the benefits of payment reduction from a 30-year mortgage to a 40-year mortgage are limited. If both rate reduction and term extension do not do the trick, the modification agent will consider principal forbearance and/or forgiveness. Both principal forbearance and forgiveness will put aside some principal of which the borrower does not need to make principal and interest payment. It works as if the principal of the mortgage has been reduced. This approach can theoretically lower the mortgage payment, and hence the DTI ratio to any level. The difference between principal forbearance and principal forgiveness is that the former still attaches the principal forborn as a second lien, which becomes due when the house is sold and there is residual revenue

⁷ http://www.makinghomeaffordable.gov/tools/payment-reduction/Pages/default.aspx

after the first lien mortgage is paid off, while the latter writes off the forgiven principal completely.

Under such strict constraints on the DTI ratio, we can reasonably assume that a higher payment reduction implies higher income loss, thus higher income volatility. This could very likely create a U-shaped re-default rate of modified mortgage loans.

6.2 Optimal Modification of Re-Default Rate that is Monotonous

From the perspective of the lender, a 0% re-default rate is not the "optimality" criterion. The objective of the lender is to maximize the present value (PV) of the loan modification. Ignoring discounting and re-default timing, the PV of a modified loan can be written as below, by following a standard defaultable bond pricing formula. *LGD* is the loss given default rate, and measured by:

$$PV = PD \times (1 + LGD) \times UPB_M + (1 - PD) \times UPB_M$$
(13)

where PD is the probability of default, and we can write it as a function of payment reduction:

$$PD = f(PR)$$

 UPB_M is the unpaid principal balance (*UPB*) after modification, and can be written as a function of payment reduction as well, assuming that the payment is reduced via principal forgiveness:

$$UPB_M = UPB_D(1 - PR) \tag{14}$$

where UPB_D is the UPB at the time of default.

Thus the PV can be written as:

$$PV = f(PR) \times (1 - LGD) \times UPB_D(1 - PR) + (1 - f(PR)) \times UPB_D \times (1 - PR)$$

= $UPB_D(1 - PR) \times (1 - f(PR) \times LGD)$ (15)

Obviously, when the payment reduction is 100%, the PV is zero; when the payment reduction is 0%, and re-default rate is 100%, the PV is $UPB_D * (1 - LGD)$, which are thus purely determined by the *LGD*. Figure 5 shows the stylized PV, as a function of payment reduction.

In order to find the optimal modification strategy, or payment reduction level, we need to take the first order derivative of the above formula, with regard to the variable PR. When the first order derivative equals zero, we will find the optimal modification strategy which maximizes the PV of the lender from the loan modification.

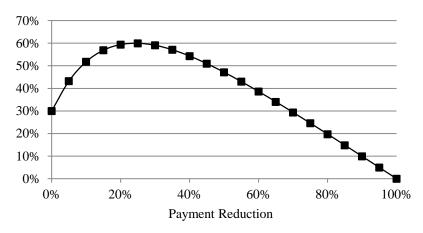
Re-Default Risk of Modified Mortgages 33

$$\frac{dPV}{dPR} = UPB_D(-1) \times (1 - f(PR) \times LGD) + UPB_D(1 - PR) \times \left(-\frac{\partial f(PR)}{\partial PR} \times LGD\right) = 0$$
(16)

Also, the optimality condition follows as below:

$$(1 - f(PR) * LGD) = (1 - PR) * \left(-\frac{\partial f(PR)}{\partial PR} * LGD\right)$$
(17)

Figure 5 Stylized Present Value vs. Payment Reduction



Assuming f(PR) is a monotonous decreasing function results in the three following conclusions, as demonstrated in Figure 5:

- PV is a concave function of *PR*, and
- There exists a single optimal point, where PV is maximized by the *PR* that satisfies the above optimality condition; and
- At the optimal point, the change in PV is relatively moderate with regard to the *PR*, which means that even if the optimality is violated within a small range, the difference from the optimal PV is not substantial.

6.3 Optimal Mortgage Modification with Re-Default Rate that is Non-Monotonous

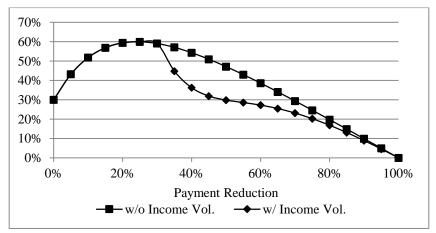
If we introduce the magnitude of the payment reduction as a proxy for income volatility, the additional benefit of a marginal payment reduction might be offset by the increased income volatility, as demonstrated in the following stylized example of a re-default rate with income volatility.

Following the logic to derive the optimal modification strategy in Section 6.2, we plot the PV as a function of payment reduction, with income volatility; see Figure 7. Under this new assumption, it can be seen that:

- PV is NO LONGER a concave function of *PR*, and;
- There still exists a single optimal point, which may not overlap with the optimal point under the optimality condition without income volatility; and
- At the optimal point, the change in PV is relatively sensitive with regard to the *PR*, which means that even if the optimality is violated within a small range, the difference from the optimal PV could be significant.

Figure 6 Stylized Re-default Rate with Income Volatility

Figure 7 Stylized PV with Income Volatility



7. Conclusion and Future Research

In this paper, we use FHA modified loans to investigate the effectiveness of loan modification in preventing re-default. Loan-level data are used to trace the performance of loans with heavy modifications. The empirical results show that modified loans tend to have much higher re-default risk than otherwise identical never-defaulted loans. Also, the loan modification re-default rate is less sensitive to traditional credit risk drivers, compared to never-modified loans. The re-default risk declines initially with the magnitude of the payment reduction associated with the modification received. However, as the payment reduction becomes substantial, the re-default probability increases.

The last finding is the first time that such a phenomenon is being identified. This not only changes our intuition about the relationship between re-default rate and payment reduction, but also makes us re-think the best way to modify distressed residential mortgages.

We plan to further explore the impact of income volatility on mortgage default behavior. Yang et al. (1998) has built a theoretical framework and incorporated borrower income as a random driver for mortgage termination. Yet they have not included empirical data to support the results. Diaz-Serrano (2005) finds that borrowers with higher income volatility may not be able to accumulate precautionary savings to meet mortgage payments when shocks in income occur. However, the study uses macroeconomic data at the national level, which is very likely affected by other latent macro factors. We will try to establish a plausible theoretical framework with income volatility explicitly imbedded, and locate microeconomic level indicators for income volatility to empirically support the theory.

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Appendix

| | Total | Current | 30-59 Days Delinquent | Seriously Delinquent | Foreclosures in Process | Completed Foreclosures | Paid Off | No Longer in the Portfolio* |
|-----------------------|-------------|-------------|-----------------------------|-------------------------|----------------------------|---------------------------|-------------|-----------------------------------|
| 2008 | 443,294 | 21.1% | 4.3% | 9.2% | 5.4% | 16.7% | 4.5% | 38.9% |
| 2009 | 593,884 | 31.2% | 5.3% | 11.5% | 6.2% | 12.7% | 4.1% | 28.9% |
| 2010 | 955,422 | 40.0% | 5.9% | 11.0% | 5.5% | 8.2% | 3.1% | 26.2% |
| 2011 | 569,553 | 47.4% | 6.6% | 12.3% | 5.5% | 4.0% | 2.4% | 21.7% |
| 2012 | 479,820 | 61.5% | 8.0% | 12.3% | 3.9% | 0.9% | 1.2% | 12.2% |
| 2013 | 246,744 | 70.4% | 8.9% | 9.4% | 1.2% | 0.1% | 0.5% | 9.5% |
| Total | 3,288,717 | 42.6% | 6.3% | 11.1% | 5.1% | 7.8% | 2.9% | 24.3% |
| HAMP Modification P | Performance | Compared | with Other M | odifications* | * | | | |
| Other Modifications | 1,774,830 | 46.2% | 7.3% | 13.3% | 5.5% | 6.3% | 2.8% | 18.6% |
| HAMP Modifications | 732,747 | 53.8% | 5.4% | 7.2% | 3.3% | 3.4% | 1.7% | 25.3% |
| Modifications That Re | duced Payme | ents by 10 | Percent or Mo | ore | | | | |
| | 2,083,687 | 48.7% | 6.3% | 9.6% | 4.1% | 5.2% | 2.1% | 23.9% |
| Modifications That Re | duced Payme | ents by Les | s Than 10 Per | rcent | | | | |
| | 1,205,030 | 31.9% | 6.2% | 13.8% | 6.7% | 12.1% | 4.2% | 25.0% |

Appendix I Status of Mortgages Modified from 2008-20132Q

Notes: *Processing constraints prevented some servicers from reporting the reason for removal from the portfolio.

**Modifications used to compare with HAMP modifications include only those implemented from the third quarter of 2009 through to the second quarter of 2013.

Source: http://www.occ.gov/publications/publications-by-type/other-publications-reports/index-mortgage-metrics.html