INTERNATIONAL REAL ESTATE REVIEW

2018 Vol. 21 No. 2: pp. 227 – 250

Comparison of Two Affordable Housing Finance Channels

Chen L. Miller
Johns Hopkins University, Carey Business School, Baltimore, USA.
Email: cmil173@jhu.edu

This paper demonstrates, theoretically and empirically, that shared equity mortgages are a better affordable housing solution than high-leverage lending, in terms of both default reduction and cost to mortgage insurers. Their effectiveness in reducing strategic default is increased when shared equity contracts are conducted in expensive house price areas, during housing bubble periods, with long holding terms, or for borrowers with high expected returns. The paper develops numerical examples with the use of simulation and back-testing, which are applied to Los Angeles. The results show that Los Angeles could have avoided many of its strategic defaults in the recent recession if it had used a shared equity mortgage as an alternative to conventional low down-payment mortgages.

Keywords: Strategic defaults, Shared equity mortgage, Affordable housing

1. Introduction

In order to improve homeownership and affordability, the Federal Housing Administration (FHA) of the United States (U.S.) provides mortgage insurance on loans made by FHA-approved lenders. Homeowners who qualify for an FHA loan only need to have a down-payment that is as low as 3.5 percent of the house value. On average, the original loan to value (LTV) ratio for FHA loans is over 95 percent[1]. However, as Lekkas et al. (1993) and Deng et al. (1996) show, low down payments increase defaults and loss severity and the subsidy cost from taxpayers to borrowers, especially in housing downturns. As we observed after the recession in 2008-2009, the percentage of FHA loans that were 90 days or more past due or in foreclosure peaked at 9.02%[2] in Q4 of 2011. Due to massive defaults and foreclosures, the government suffered huge losses from insuring mortgages, and millions of people lost their homes.

As an alternative to the FHA program, equity sharing programs help homeowners to raise some of the down payment from investors, who in turn receive a share of the future house price. The previous literature on equity sharing programs has mainly focused on affordability, but not reduction in defaults. This paper demonstrates that equity sharing leaves borrowers better off in terms of reducing strategic defaults, especially in highly volatile house price areas and during housing bubble periods. Not only do borrowers have less incentive to strategically default, but under market equilibrium, investors or lenders should also have incentive to buy such products. This is Pareto optimum. The main contribution of this paper is to investigate mortgage contract design in market equilibrium from the perspective of reducing strategic default instead of viewing shared equity mortgages as a housing affordability solution. Such mortgage contracts might be appealing in places where the mortgage markets are very costly and foreclosure is expensive. For instance, the contracts might be useful in countries such as the United Kingdom (UK) and Austria which are developing legal structures for mortgages or cannot do long term fixed rate mortgages.

The literature on mortgage default has focused on two explanations on why borrowers default. One explanation is based on models of the implicit default and prepayment options possessed by the mortgage borrower. Under the "ruthless" or "strategic default" hypothesis, these option-based models generate predictions for default based on the current value of housing relative to the discounted value of future mortgage payments. The starting point for option-based models is the contingent claims model, which is developed by Jensen et al. (1972) and Cox et al. (1985). Foster and van Order (1984), Dokko and Edelstein (1991), Archer and Ling (1993), Kau et al. (1993, 1995), Archer et al.

[1] FY2013 Actuarial Review of MMIF Forward Mortgages, Exhibit IV-5.
[2] http://www.nationalmortgagenews.com/dailybriefing/FHA-Serious-Delinquency-Rate-Hits-three-Year-Low-1038093-1.html

(1996), Keenan and Kau (1995), Crawford and Rosenblatt (1995), Phillips et al. (1996), Deng et al. (2000), Bajari et al. (2008), and Ghent and Kudlyak (2009) among others, have applied this method to value mortgage contracts.

Another view is the “double trigger” hypothesis. Under this hypothesis, the probability that homeowners with negative equity will default is conditional on the financial and economic characteristics of the household, for example, a negative income shock or unexpected expense; Gerardi et al. (2007) and Foote et al. (2008) show evidence that support this view.

The main research questions of this paper are whether shared equity mortgages reduce strategic defaults, compared to conventional low down-payment mortgages, and if so, how does it work in an equilibrium market? This paper answers these questions in two parts: first, there is the theoretical part, in which a default model is built to investigate the default choice of a borrower, as well as investor and lender behaviors in the market context. Second, there is an empirical part, which develops a numerical example by using simulation schemes and a back-testing method.

We find that in order to ensure the effectiveness of default reduction, shared equity contracts require long holding terms, and work better in expensive house price areas or housing bubble periods, or for borrowers with high expected returns. In the numerical example in this paper, we show that homeowners in Los Angeles could have avoided much of the strategic defaults in the recent recession if they had used a shared equity mortgage as an alternative to the conventional low down-payment mortgage.

The rest of the paper is organized as follows. Section 2 develops the theoretical framework. Section 3 uses the theoretical model to interpret investor choices in offering shared equity products. Section 4 evaluates the size of the shared equity product that lenders would be willing to offer. Section 5 uses a numerical example of a house in Los Angeles, with a Monte Carlo simulation scheme and a back-testing method to support the hypothesis that shared equity mortgages or Home Appreciation Participation Notes (HAPNs) are a better contract than the current FHA mortgages, in terms of improved affordability, strategic default incentives, and attracting new sources of funding via investors. Section 6 concludes.

2. Theory of Default from Perspective of Borrower

2.1 Simple Two Period Model of Default

We use a model similar to that in Foote et al. (2008) to illustrate the decision of a borrower to default or continue to make mortgage payments. The major contribution is to expand the model to three states and a scenario in which borrowers use a shared equity mortgage.

We assume a two-period world ($t = 1,2$), with three possible future states of good, moderate and bad, which occur with a probability of p^G, p^M, p^B, respectively, and where $p^G + p^M + p^B = 1$. We assume that the borrower has purchased a home valued at P_1 with a mortgage in the first period. In the second period, the house is worth ${P_2}^G$ if the good state occurs, ${P_2}^M$ if the moderate state occurs, and ${P_2}^B$ if the bad state occurs.

In the first period, the borrower decides between making a mortgage payment and staying in the home, or stopping payment and defaulting. We assume that the borrower either sells the home in the second period or defaults on the mortgage. Also, we denote $Stigma_i$ as the transaction costs associated with default, such as penalties for bad credit score records, moving costs, etc., and these transaction costs differ across borrowers, which result in heterogeneity across households and indexed by i.

To measure whether or not borrower defaults, we compare the benefits of staying in the home to the cost of not doing so. If the cost of staying in the home is higher than the benefits of doing so, the (rational) borrower defaults.

2.2 Default Decision of Borrower under Conventional Mortgage Structure

With a conventional mortgage, the borrower has a nominal mortgage balance of M_1 in the first period, in which s/he pays $P_1 - M_1$ as the initial down payment, where $M_1 < P_1$. We also assume that ${P_2}^B < {P_2}^M < M_2 \leq {P_2}^G$, where M_2 is the remaining nominal mortgage balance in the second period.

The value of the house to the borrower, or the benefit of staying in the home, in the first period is given by,

$$V_1^H = rent_1 + \frac{1}{1+r_i}\left(p^G \times P_2^G + p^M \times P_2^M + p^B \times P_2^B\right) \tag{1}$$

where $rent_1$ is the cost of renting the house for one period, thus saving on rental payment is part of the benefits of staying in the home. The second component of the house value is the expected present discounted market value of the house in the second period, since we assume that the household will sell the home in the final period. The market house value in the second period is given by the weighted average of the price that occurs in that state. r_i is the cost of funding, which is the rate at which a borrower is willing to sacrifice future consumption for current consumption. The heterogeneity in the rate across different households results in different discount factors, indexed by i.

The value of the mortgage, or the cost of staying in the house, in the first period, is given by,

$$V_1^M = mpay_1 + \frac{1}{1+r_i}\left(p^G \times M_2 + p^M \times P_2^M + p^B \times P_2^B\right) \quad (2)$$

where $mpay_1$ is the mortgage payment that the borrower is required to make in the first period, and M_2 is the remaining balance of the mortgage in the second period, in which the borrower is required to repay after selling the house. Since we assume that ${P_2}^B < {P_2}^M < M_2 \leq {P_2}^G$, if the good state occurs, the borrower sells the house and pays off the mortgage. If the moderate or bad state occurs, a rational borrower defaults, and loses the house, because the debt from the remaining balance of the mortgage exceeds the house value. Thus, we see ${P_2}^M$ and ${P_2}^B$ substitute for M_2 in the moderate and bad states in Equation (2).

Thus, from the perspective of the borrower, the decision to default depends on the sign of the following expression, where we subtract (2) from (1) and add the default transactions costs.

$$\begin{aligned} &V_1^H - V_1^M + Stigma_i \\ &= \left(rent_1 - mpay_1\right) + \frac{1}{1+r_i} \times p^G \times \left(P_2^G - M_2\right) + Stigma_i \end{aligned} \quad (3)$$

The probability of defaulting is the probability that the above expression goes to negative.

In order to improve housing affordability, the FHA allows borrowers to make a down payment as low as 3.5 percent[3]. However, a low down payment increases the amount of the monthly mortgage payment. Also, the borrower is required to either pay a mortgage insurance premium in full upfront, or as a monthly payment. Since Equation (3) is a decreasing function of $mpay_1$, increased mortgage payments are more likely to cause Equation (3) to become negative, and increase default probability.

A lower down payment also means that the original mortgage balance M_1 is close to the initial house value P_1. In areas with houses that have high price variance, the probability that a house price will drop below the mortgage balance could be very high, thus making $probability\ ({P_2}^G \geq M_2)$ low. This

[3] United States Department of Housing and Urban Development (HUD), Mortgagee Letter 2008-23.

leads to a high probability of default according to Equation (3). This is consistent with the previous literature such as Deng et al. (1996), who argue that low down payment comes with the cost of high defaults.

As an alternative to low down payments, the FHA could work with third party investors, who provide borrowers with funds to increase their down payment in exchange for a share of the future house appreciation. The next section discusses how shared equity mortgages reduce the probability of borrower default.

2.3 Decision of Borrower to Default with Shared Equity Mortgages

With shared equity mortgages, investors would pay E_1 in the first period in exchange for λ percent of the house appreciation when the borrower sells the house in the final period, in which $0 < \lambda \leq 1$. Since the investor pays E_1 as part of the down payment, the nominal mortgage balance of the borrower in the first period is reduced to $M_1{}'$, where $M_1{}' = M_1 - E_1$.

The value of the house to the borrower, or the benefit of staying in the home, in the first period is given by,

$$\begin{aligned} V_1^H &= rent_1 + \frac{1}{1+r_i}\left(p^G \times P_2^G + p^M \times P_2^M + p^B \times P_2^B\right) \\ &\quad - \frac{1}{1+r_i} \times Max\left[-E_1, \lambda \times \left(p^G \times P_2^G + p^M \times P_2^M + p^B \times P_2^B - P_1\right)\right] \end{aligned} \tag{4}$$

The first two components are the same as those in Equation (1), and the third component is the discounted market value of the house appreciation that the borrower gives up in the final period. When the expected future house value exceeds the original house value, the third component is positive, and the borrower loses part of the capital gains.

However, when the expected future house value is less than the original house value, the third component is negative, and the borrower gains from the shared appreciation agreement because the investor shares the capital loss with the borrower. The maximal loss that the investor takes is E_1. In other words, in the extreme case that the house market collapses and investors lose all of their investment, the maximal amount that the borrower can benefit from the shared appreciation agreement in the second period is E_1.

The value of the mortgage or the cost of staying in the house in the first period is given by,

$$
\begin{aligned}
V_1^M &= mpay_1{}' + \frac{1}{1+r_i}\left(p^G \times (M_2{}' + E_1) + p^M \times (M_2{}' + E_1) + p^B \times P_2^B\right) \\
&= mpay_1{}' + \frac{1}{1+r_i}\left(p^G \times M_2 + p^M \times M_2 + p^B \times P_2^B\right)
\end{aligned}
\tag{5}
$$

where $mpay_1{}'$ is the mortgage payment that the borrower is required to make in the first period. Clearly this payment is less than the mortgage payment under the conventional mortgage structure, denoted as $mpay_1{}' < mpay_1$, due to the lower nominal mortgage balance in the first period. $M_2{}'$ is the remaining mortgage balance in the second period. E_1 is the down payment contribution of the investors in the first period. Intuitively, this is like a second lien but with more upside gains. Thus, the borrower is required to repay both the lender and investor after selling the house, which is denoted as $M_2{}' + E_1$. It is close to the remaining mortgage balance in the second period under the conventional mortgage structure because E_1 is the difference between the nominal mortgage balances under the two different mortgage structures.

We also assume that $P_2{}^B < M_2{}' \leq P_2{}^M < P_2{}^G$, and if the good or moderate state occurs, the borrower sells the house and pays off the mortgage and pays back the down payment contribution of the investors. If a bad state occurs, the borrower defaults and walks away from the house.

Similarly, from the perspective of the borrower, the decision to default depends on the sign of the following expression, where we subtract (5) from (4) and add the default transactions costs.

$$
\begin{aligned}
&V_1^H - V_1^M + Stigma_i \\
&= \left(rent_1 - mpay_1{}'\right) + \frac{1}{1+r_i} \times \left(p^G \times (P_2^G - M_2) + p^M \times (P_2^M - M_2)\right) \\
&- \frac{1}{1+r_i} \times Max\left[-E_1, \lambda \times (p^G \times P_2^G + p^M \times P_2^M + p^B \times P_2^B - P_1)\right] + Stigma_i
\end{aligned}
\tag{6}
$$

Therefore, from the perspective of the borrower, a shared equity mortgage would reduce default probability if the following inequality is met, where subtracting (3) from (6) should be positive.

$$
\begin{aligned}
mpay_1 - mpay_1{}' &> \frac{1}{1+r_i} \times p^M \times (M_2 - P_2^M) \\
&+ \frac{1}{1+r_i} \times Max\left[-E_1, \lambda \times (p^G \times P_2^G + p^M \times P_2^M + p^B \times P_2^B - P_1)\right]
\end{aligned}
\tag{7}
$$

The left-hand side of the inequality indicates the borrower’s benefit from mortgage payment reduction, because the down payment is higher for a shared equity mortgage. The first component of the right-hand side of the inequality indicates the borrower's cost from an increased mortgage value. Since we assume $P_2{}^B < M_2{}' \leq P_2{}^M < M_2 \leq P_2{}^G$, if the moderate state occurs, the borrower with a conventional mortgage defaults while the borrower with a shared equity mortgage would not because the house value exceeds the remaining mortgage balance, which results in an increased mortgage value. The second component of the right-hand side of the inequality indicates the borrower's cost from the shared appreciation agreement if future house price increases, or the borrower's benefit from the shared appreciation agreement if future house price drops. Hence, the inequality implies that the benefit of mortgage payment reduction should exceed the value that the borrower gives upon exercising the put option when the moderate state occurs and the value of the partial capital gain that the borrower gives up due to the shared appreciation.

This inequality has policy implications for lenders in designing mortgage contracts. If the goal is to reduce defaults, the reduction of mortgage payment has to exceed the costs. One feasible way is to choose expensive housing areas, because the dollar payment amounts would be high. Another way is, given other parameters, to increase the down payment assistance E_1. This is because, on the one hand, it reduces the total mortgage balance, while increasingly reducing mortgage payments. On the other hand, it increases the maximal level that the borrower benefits from the shared appreciation agreement when future house price drops. However, from the perspective of the lender, there should be an upper bound for down payment assistance, which we discuss in Section 4. Moreover, inequality is more likely to be satisfied for borrowers with a high expected return, r_i. For example, troubled borrowers who are facing job loss or divorce have higher costs of borrowing, and the value of giving up future consumption is lower. Another example is a high inflation environment.

3. Expected Return from Perspective of Investor

In this section, we look at the decision of the investor to offer shared equity as an option. The key is to identify how investors can receive a higher return through a shared equity product. We assume a *k* period world. Investors can buy a house through a conventional mortgage in the first period, rent it out for *k* periods, and sell the house for capital gain in the final period. An alternative is to invest on shared equity products to obtain house appreciation.

In the first case, we denote ρ as the cumulative return from renting, u as the cumulative appreciation rate of house price, P_1 as the initial house price, E_1 as the down payment, $mpay$ as the mortgage payment for each period, and α_0 as the probability of mortgage default. The payoff through k periods is revenue from renting if default occurs or revenue from both renting and house appreciation if no default occurs.

The gross return of the investor in k periods financed by a conventional mortgage is given by,

$$\frac{Payoff}{principle} = \frac{P_1\left[\alpha_0 \times (\rho + 0) + (1-\alpha_0) \times (\rho + u)\right]}{E_1 + \sum mpay} \tag{8}$$

In the alternative case, we denote E_1 as the down payment assistance invested in a conventional mortgage, and in exchange, the investor gets λ percentage of the house appreciation in the final period, where $0 < \lambda \leq 1$. We also assume that the probability of mortgage default, denoted by α_1, would be less than that in the first case, which is discussed in Section 1. Then the investor gets nothing if default occurs or partial house appreciation if no default occurs.

The gross return of the investor in *k* periods through the shared equity product is given by,

$$\frac{Payoff}{principle} = \frac{P_1\left[\alpha_1 \times 0 + (1-\alpha_1) \times (\lambda \times u)\right]}{E_1} \tag{9}$$

If the investor gets a higher return through the shared equity product, the following inequality, subtracting (8) from (9), should be positive.

$$\lambda u(\alpha_0 - \alpha_1) + (1-\alpha_1)\lambda u \times \frac{\sum mpay}{E_1} > \rho + (1-\lambda)u(1-\alpha_0) \tag{10}$$

On the left-hand of the inequality, the first component measures appreciation due to the reduction in mortgage defaults. The second component indicates additional capital gains that the investor would have if s/he uses the funds paid for mortgage payments to invest in shared equity products. On the right-hand of the inequality, the first component indicates rent income that the investor loses due to no occupancy right. The second component indicates partial appreciation that the investor loses due to no ownership right. The following expression is given after we divide u on both sides,

$$\lambda(\alpha_0 - \alpha_1) + (1-\alpha_1)\lambda \times \frac{\sum mpay}{E_1} > \frac{\rho}{u} + (1-\lambda)(1-\alpha_0) \tag{11}$$

There are three methods that can be applied to increase the chances of meeting this inequality. The first is to increase the magnitude of the reduction in the probability of default, which brings us back to the discussion in Section 1. The second is to increase the ratio of the total mortgage payment over *k* periods to the down payment assistance amount. Since the total mortgage payment is an increasing function over time, the solution is to increase the holding period *k*. In reality, current policies like high transaction fees (around 6%) and high taxes imposed for house investment are incentives to investors for a longer holding period, which helps to make this inequality work. There is evidence that supports this solution. For example, on the shared appreciation agreement contract, FirstRex[4] required borrowers to hold their house at least for five years before selling the house, otherwise a large penalty is incurred. Long holding periods require finding long term investors, such as pension and retirement funds. Also, this method should be more effective than the first method, due to $1 - \alpha_1 > \alpha_0 - \alpha_1$. The third method is to offer the shared equity product in areas with abnormal house price growth. The first component on the right-hand side of the inequality (11) can be viewed as a proxy for expected house price because one measurement of expected growth is the price to rent ratio (like price to earnings ratio for a growth stock). One policy implication here is that this product can be introduced in a housing bubble market, and then spectators would go for shared equity products due to higher returns, and accordingly reduced house buyers would cool down the over-heated market.

4. Maximum Down Payment Assistance Amount from Perspective of Lender

In this section, we look at the decisions of lenders to offer a shared equity option. We consider how it might reduce the possibility of foreclosure of borrowers with negative equity.

Consider a similar model to that in Foote et al. (2008), the lender has an outstanding loan and the value of that loan, conditional on the borrower not defaulting, is m. We assume that the house is worth P_H and costs γ dollars to foreclose on the borrower, so the lender recovers $P_H - \gamma$ if it chooses to foreclose on the borrower with a probability of α_0. The expected recovery is given by,

$$E(recovery) = \alpha_0(P_H - \gamma) + (1 - \alpha_0)m \tag{12}$$

With shared equity, the new value of the loan is m'. The difference between m and m' is the down payment assistance, E_1, from the shared equity agreement. The lender allows the borrower to repay E_1 with no interest payment, but to share the appreciation value, $\lambda(P_H - P_1)$, after selling the house. The shared equity option reduces the probability of foreclosure changes to α_1, where $\alpha_1 < \alpha_0$, and the risk-free rate is r. The expected recovery with shared equity options is given by,

$$E(recovery) = \alpha_1(P_H - \gamma) + (1 - \alpha_1)\left[m + \lambda(P_H - P_1)\right] - E_1(1 + r) \tag{13}$$

From the perspective of the lender, shared equity is the optimal choice if the following inequality holds, (where Equation (13) exceeds Equation (12)).

[4] FirstRex is the most popular shared equity product in the U.S market.

$$E_1(1+r) \le \lambda(P_H - P_1)(1-\alpha_1) + (\alpha_0 - \alpha_1)(m - P_H + \gamma) \qquad (14)$$

The left-hand side is the cost of the shared equity policy or the down payment assistance, while the right-hand side is the benefit or the first component, which indicates the payoff from shared appreciation. The second component indicates dollars saved due to the shared equity option, which is the product of the reduction in foreclosure rate and loss given foreclosure. To simplify the model, we ignore time value and assume payoffs from partial appreciation are proportional to down payment assistance, which is denoted as $\lambda(P_H - P_1) = \beta E_1$,where $\beta > -1$. The inequality (14) is written as,

$$E_1 < \frac{(\alpha_0 - \alpha_1)}{1-\beta(1-\alpha_1)}(m - P_H + \gamma) \qquad (15)$$

From the perspective of the lender, the inequality (15) gives an upper bound for down payment assistance. We can view $\frac{(\alpha_0-\alpha_1)}{1-\beta(1-\alpha_1)}$ as the leverage to decide on the upper bound of E_1 . If $\beta = 1$, this inequality becomes an upper bound for the forbearance amount. $\beta > 0$ means that the lender makes money from shared equity products, and the upper bound will be higher due to leverage and is an increasing function of β . If $\beta < 0$, the lender loses money. However, this does not mean that the lender has no incentive to offer this option, as the lender gains as long as the down payment assistance amount offered does not exceed its upper bound as implied by Equation (15).

5. Numerical Example of Houses in Los Angeles

In this section, a numerical example is introduced to illustrate that a shared equity mortgage is a better option for affordable housing than a low-down payment mortgage, in the sense that the former reduces the payment burden and incentive of borrowers to strategically default and bringing higher returns to investors.

Consider a borrower whose wealth allows a house purchase equal to 5% of the house price for a property in Los Angeles. S/he has two choices for financing. One is to get a mortgage with a nominal mortgage amount equal to 95% of the house price. Usually banks do not issue high loan-to-ratio mortgages unless the mortgage is guaranteed by the FHA (or a private insurer). Here it is assumed that the borrower is qualified for the mortgage guaranteed by the FHA. Another option is to give up future house appreciation in exchange for extra down payment. The normal mortgage amount is much less than 95% of the house price and the amount of extra down payment depends on value of the HAPN[5] that the investor is willing to buy.

[5] HAPN are shared equity products, designed by Cassidy et al. (2008).

Following a decomposition of house value differences as discussed by Cassidy et al. (2008), we calculate the HAPN value[6] based on H_0^I as Equation (16) shows below.

$$H_0 = H_0^C + H_0^I = \sum_{t=0}^{T}\left(\frac{R_t}{(1+k_c)^t} + \frac{H_0^C}{(1+k_c)^T}\right) + \left\{\frac{(H_T - H_0) + H_0^I}{(1+k_l)^T}\right\} \quad (16)$$

where H_0 is the house value at time 0, H_0^C is the portion of the house value due to living in the house at time 0, H_0^I is the portion of house value due to capital gain at time 0, R_t is the net rent at time *t*, k_c is the consumption cost of capital, H_T is the house value at time *T*, *T* is the length of the housing tenure, and k_l is the investment cost of capital. Here, house tenure is assumed to be 3 years, and projected house price in Year 3 is assumed to have the same growth rate as the past 3 years: *H(3)/H(0) =H(0)/H(-3).*

In this numerical example, we use historical information as inputs, such as house price, household income, rent, etc. in Los Angeles from 1980 to 2012. Table 1 provides the data sources.

Table 1 Source and Description of Variables

Variable	Source	Description
House Price	NAR; Moody's Analytics	Median Existing Single-Family Home Price, (Ths. $, SA); quarterly data from 1980Q1 to 2012Q4
Mortgage Rate	Moody's Analytics	Mortgage Rates Primary Market: 30-Year Commitment Rate-Fixed Rate, National
Household Income	Census of Employment and Wages	Average Annual Pay, all industries included, from year 2001 to 2011
Rent Index	Consumer Price Index-All Urban Consumers	Rent index of primary residence, monthly data from year 1950 to 2012
Gross Rent	Census	Median Gross Rents, published at 2000 at national and state levels
S&P 500 Index	Standard and Poor's	Average S&P 500 Stock Price Index (NSA), quarterly data from 1957-1-2 to 2012-5-24

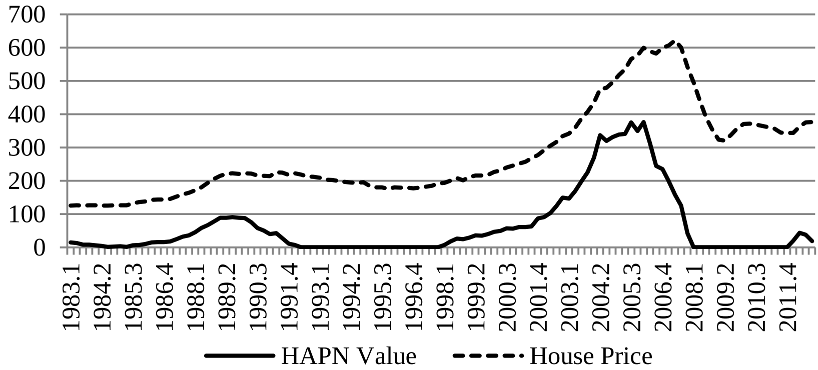
[6] The cost of capital is assumed to be the same value as the mortgage rate, because the mortgage rate covers the credit risk of the underlying mortgage, which is the same risk embedded in the HAPN.

5.1 Affordability Comparison

Here we use the minimum down payment and income requirements as measurements of affordability. We assume a 95% LTV and 43% debt to income (DTI) ratio allowed for qualifying for a mortgage. We examine over time the minimum down payment amount required if the borrower chooses to use an FHA loan or an HAPN mortgage, and also given the budget of the borrower, the minimal annual income amount required if the borrower chooses to use an FHA loan or HAPN mortgage.

In order to decide on the nominal mortgage balance for an HAPN mortgage, first we need to calculate the HAPN value. The HAPN value will increase during house booms and decrease during house recessions. Figure 1 illustrates the HAPN value in a housing cycle. We see in the housing downturn, for example, from 1992 to 1997 and after 2008, investors would withdraw from investing in housing, and the HAPN would be worth nothing. However, in a housing upturn, such as in 2006, the HAPN would be worth $376,000, or almost 60% of the house price. That means the house prices are abnormally high. Borrowers are required to have annual income of $99,000 to qualify for a 95% LTV FHA mortgage, while the median household income is only $48,517, which means median-income families cannot afford a house in 2006, and people who can afford would be the ones with neither income constraints nor down payment constraints, like investors.

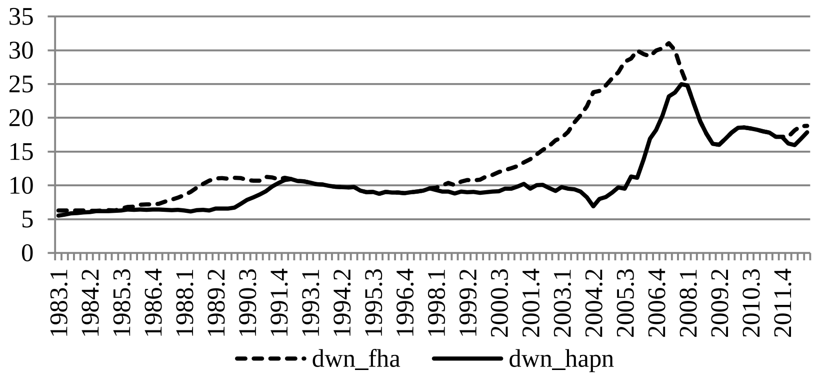
Figure 1 House Price and HAPN Value (in thousands of USD)



The HAPN mortgage significantly reduces the payment burden for the borrower, especially during a housing boom. For example, when house price increased to over $600,000 in 2007, the down payment amount required for an HAPN mortgage was $20,000, or 33% less than the down payment required for FHA loans. Figure 2 shows the minimum down payment amount required for a qualified FHA loan and HAPN structure loan, given house price is realized with its historical information. Since the HAPN reduces the nominal mortgage balance in exchange for future appreciation, this leads to a reduction in monthly mortgage payments.

However, this reduction only happens in a normal or booming housing market, and not recessions. Figure 2 shows that the gap becomes wider in 2003-2007, but narrows to zero in 1992-1997. The reason is that house price drops in 1990-1995 as shown in Figure 1 lead to the assumption that future house growth is based on past observed information. Then the investor opts out of the HAPN investment in 1992-1997, the HAPN is worth nothing, and there is no difference between an FHA loan and HAPN mortgage. However, in 2003-2007, HAPN is worth a lot more, so that the borrower with an HAPN mortgage benefits from a much lower nominal mortgage balance.

Figure 2 Down Payment Requirements (in thousands of USD)

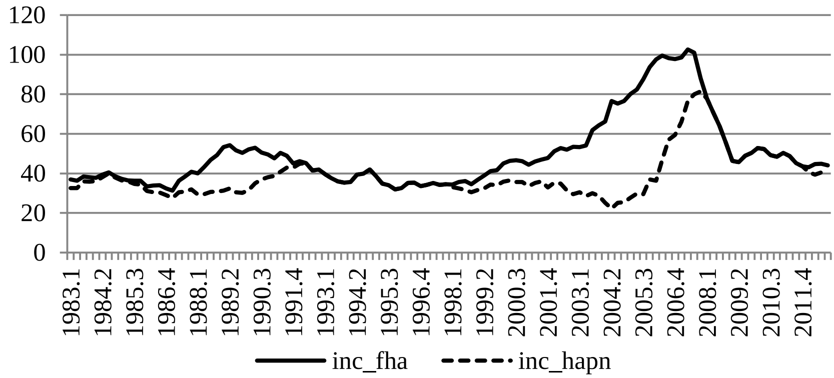


Similarly, the HAPN structure eases the income constraints of low income borrowers, especially during housing booms. For example, when house prices were over $600,000 in 2007, the borrower who chose an HAPN mortgage was required to have an annual income of $66,000, which is 33% less than the income requirements for an FHA loan. Since the annual median income was less than $55,000 before 2011, people living in Los Angeles could not afford a house after 2001 through FHA loans, but could still afford one until 2006 through an HAPN mortgage based on the annual requirements shown in Figure 3.

5.2 Mortgage Default Comparison

Here we examine the probability of default for the two mortgage structures. To capture ruthless default as an option, house price declines accompanied by declines in household income is a necessary condition. Following Yang et al. (1998), the probability of default can be approximately measured by combining

Figure 3 Annual Income Requirement (in thousands of USD)



the probability of negative equity and shortage of income. Considering a borrower living in Los Angeles who is purchasing a house in Q1 of 2007. We check the conditional disposable income distributions and equity distributions by using Monte Carlo simulations for house price and household income. The reason that we choose the 2007 book of business is because 2007 is the worst performing vintage[7] in the FHA portfolio due to the crash after 2008. We test how mortgages would have performed with an HAPN structure rather than from an FHA program.

In a simulated setting, both house value *h* and income *y* are assumed to follow a geometric Brownian Motion (lognormal process). The processes are expressed as:

$$\frac{dh}{h} = u_h dt + \sigma_h dz_h$$
$$\frac{dy}{y} = u_y dt + \sigma_y dz_y \qquad (17)$$

where u_h and u_y are the mean growth rates (trends) for their respective series; σ_h and σ_y are the standard deviations (volatilities) of the series; and z_h and z_y are possibly correlated standard Wiener Processes.

For a mortgage contract with initial values h_0 and y_0, the values for processes at time *t* are normally distributed with means and variances:

$$E[\ln h_t \mid \ln h_0] = \ln h_0 + t\left(u_h - \frac{\sigma_h^2}{2}\right)$$

[7] FHA mortgage performance in different vintages can be found in "Actuarial Review of the Mutual Mortgage Insurance Fund- 2012 Report".

$$V[\ln h_t \mid \ln h_0] = t\sigma_h^2$$
$$E[\ln y_t \mid \ln y_0] = \ln y_0 + t\left(u_y - \frac{\sigma_y^2}{2}\right) \tag{18}$$
$$V[\ln y_t \mid \ln y_0] = t\sigma_y^2$$

All of the parameters use historical average information; parameters of the baseline house-price-process[8] are assumed to be $u_h = 0.037$ and $\sigma_h = 0.123$, and parameters of the baseline income-process[9] are assumed to be $u_h = 0.029$ and $\sigma_h = 0.014$.

First, a borrower with an HAPN mortgage is less likely to experience income shortage as opposed to a borrower with an FHA loan. Figure 4 shows the conditional distribution of income within two standard deviations and the annual mortgage payment of FHA and HAPN loans.

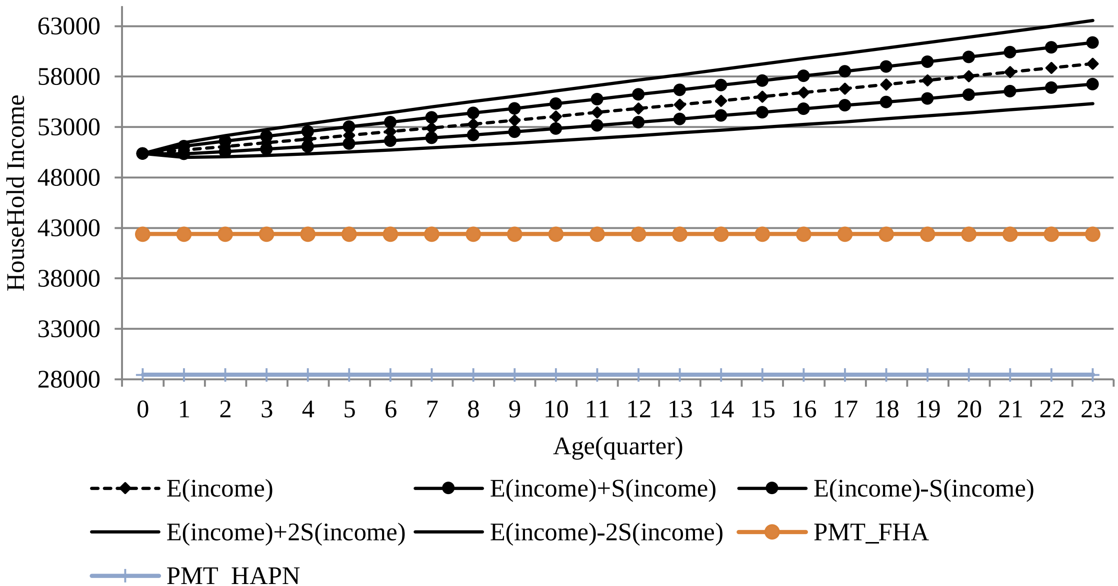
The disposal income is measured as the difference between income and mortgage expenses. We see that the expenses of an HAPN mortgage are much lower than those of an FHA loan, which leaves more future disposal income and reduced probability of income shortage. For example, if a borrower has a pay cut during the recession, and his/her annual income falls to two standard deviations below expectation in Q4 of 2012, then the average disposal income per month is $1,076 if s/he purchases a home through an FHA loan in 2007. However, if this borrower finances his/her house through an HAPN mortgage, the disposal income per month under the same condition is $2,239.

Next, a borrower with an HAPN mortgage is less likely to experience negative equity. Figure 5 shows the conditional distribution of the equity position of homeowners in the two years after the origination date. The range of the two conditional standard deviations of house equity if financed with an FHA loan is much larger than that if financed with an HAPN mortgage, which means for homeowners who choose an HAPN mortgage, significant house price drops do not cause them to suffer negative equity, and significant house price increases do not help them to grow equity. Thus, HAPN mortgages indicate lower probability of negative equity and lower incentive to default compared to FHA loans. For example, homeowners who have an FHA loan suffer negative equity of more than $100,000 after Q1 of 2008, while a homeowner with an HAPN mortgage does not suffer negative equity until Q3 of 2008 when house price is under two standard deviations away from the expected future path.

---

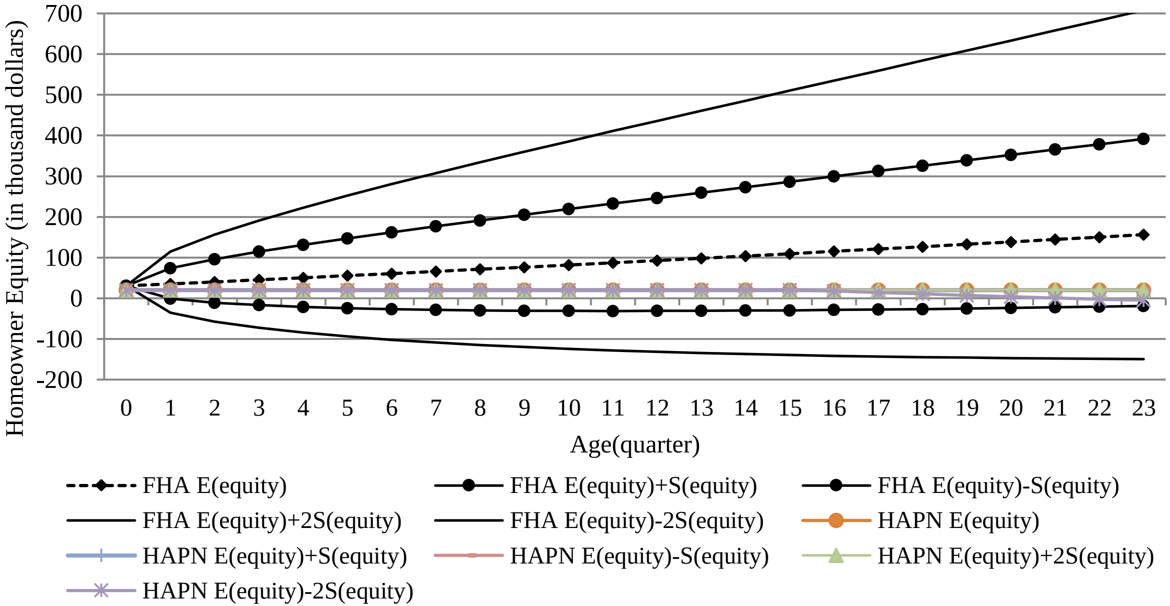
[8] The housing parameter values are based on the National Association of Realtors (NAR) median house price database for Los Angeles on an average annualized basis over 1980-2012.
[9] The income parameter values are based on the United States census dataset for Los Angeles on an average annualized basis over 2001-2011.

Figure 4 **Disposal Income Distribution (USD)**



Note: E(y) is the exponential of the conditional expected natural log of future household income. E(y)+/-xS(y) is the exponential of conditional expected natural log of future household income plus or minus x times the natural log of conditional standard deviation of future household income.

Figure 5 Homeowner Equity Distribution (in thousands of USD)

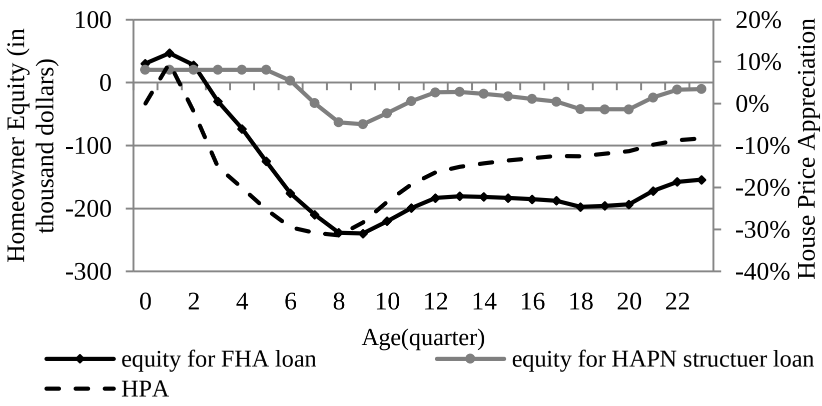


Note: E(h) is the exponential of the conditional expected natural log of future house price. E(h)+/-xS(h) is the exponential of conditional expected natural log of future house price plus or minus x times the natural log of conditional standard deviation of future house price. Conditional distribution of homeowner equity is calculated based on distribution of future house price path.

Last, Figure 6 supports the hypothesis that the 2007 book of business could have a better performance if homeowners used HAPN mortgages instead of FHA loans. Instead of simulating house price after loan origination, we examine the scenario in which house price realized its actual levels. In reality, house price dropped more than 30% 2 years after loan origination in 2007. Homeowners with an FHA loan had strong incentive to default, because they were exposed to negative equity starting 9 months after origination and suffered a loss of $125,209 15 months after origination, and $239,000 at 27 months. If homeowners used an HAPN mortgage, they would have experienced negative equity starting from 21 months and suffered a maximal loss of $66,000 at a loan age of 27 months.

However, borrowers do not default right away when the equity position becomes negative due to default related costs. Based on argument in Bhutta et al. (2010) that the median borrower does not strategically default until equity falls to -62 percent of the value of their home, our example indicates homeowners who use an FHA loan would default after Q1 of 2009 while homeowners with an HAPN mortgage do not default during the 2007-2012 recession. Thus, massive defaults could have been avoided with HAPN mortgages during the housing recession.

Figure 6 Homeowner Equity for 2007 Book of Business, Back Tested (in thousands of USD)



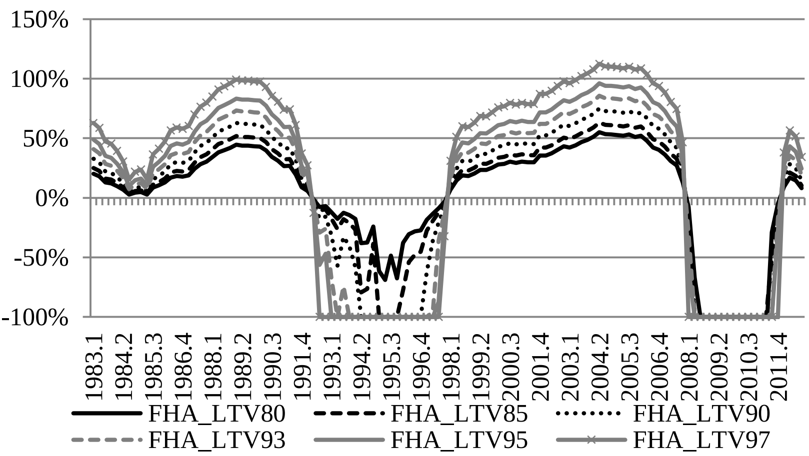
5.3 Housing Speculation Comparison

Consider a housing speculator who wants to gain appreciation by flipping houses within 3 years. S/he can finance the investment through an FHA loan, or just invest in an HAPN. Assuming that house price realized its historical value in 1983-2012, we examine speculation activities under two different channels by measuring the annual return of investors from house appreciation.

Figure 7 shows the annual return of investors if financed with different leverage levels via FHA loans. Clearly high leverage gives investors the highest return, but also the highest volatility. For example, the highest leverage of 97% LTV gives investors maximum return up to 113% in 2004, while a low leverage of 80% LTV only gives a maximum return of 53%. Also, based on the results in Table 2, investors would finance investment with at least a 94% LTV mortgage if they want to beat the market with an annual return of 7.78%. High leverage lending programs would definitely attract investors because of their goal of profit maximization. Especially during housing booming periods, the FHA program would induce much housing speculation by investors. HAPN mortgages differentiates homeowners and housing speculators. Speculative activity only happens in an investment market, and the crash of the investment market does not affect the primary residence of homeowners. Also investors cannot take advantage of government subsidy programs to conduct speculation.

As an alternative, HAPN also attracts investors. Table 2 shows the average annual return and standard deviation on housing investment by using different financing methods from 1983 to 2012. Although the average annual return for an HAPN (15.14%) is lower than that of a housing investment financed with a 97% LTV mortgage, the variance is also lower. In other words, a lower return to variance ratio for an HAPN investment (0.28) makes it more attractable to investors, compared to a housing investment financed with high leverage FHA lending programs.

Figure 7 Housing Investment Return Using Different Leverage Financing



Additionally, we ran time series regressions of HAPN returns on S&P 500 returns from 1983-2012, and the beta is around 2.21 as can be observed in Table 3, which indicates that investors are very attracted to HAPN.

Table 2 Average Annual Return of Standard Deviation of Housing Investment

	FHA_LTV80	FHA_LTV85	FHA_LTV90
return	3.40%	3.31%	4.68%
standard deviation	44.77%	51.92%	61.10%
return/variance ratio	0.08	0.06	0.08
	FHA_LTV93	FHA_LTV94	FHA_LTA95
return	7.15%	8.83%	11.14%
standard deviation	68.48%	71.30%	74.31%
return/variance ratio	0.10	0.12	0.15
	FHA_LTV96	FHA_LTV97	HAPN
return	13.77%	18.50%	15.14%
standard deviation	78.74%	83.25%	53.84%
return/variance ratio	0.17	0.22	0.28
	SP500		
return	7.78%		
standard deviation	9.76%		
return/variance ratio	0.80		

Table 3 Regression of HAPN Annual Return

Variable Name	Coefficient	t-statistic	Pr>|t|
Annual return of S&P500	2.2137	3.6000	0.0006
Intercept	0.0568	0.9100	0.3680
Number of Observations=77 Adj R-Sq=0.1363 PR>|F|=0.0006			

6. Conclusion

Conventional high leverage lending has some major disadvantages; for instance, homeowners need to trade off their income affordability with wealth affordability, have the first loss position due to house price risk, and have to take on risk if they cannot make a large down payment. Shared appreciation mortgages mitigate those weaknesses because they separate the value of capital gains from the value of occupancy rights, so that house price risk can be partially transferred to investors, and homeowners can protect their equity during the housing recession.

In order to demonstrate that shared equity mortgages are better in terms of default reduction, this paper provides a theoretical part, in which a default model is built to investigate the default choice of a borrower, as well as investor and lender behaviors in the market context, and an empirical part, which develops a numerical example by using simulation schemes and a back-testing method. Theoretically, through a two-period default model, we find that the effectiveness of reducing strategic default increases when shared equity contracts are conducted in expensive house price areas or housing bubble periods, or with long holding terms, or borrowers with high expected returns. Then through numerical examples, using simulation and back-testing, we show that homeowners in Los Angeles could have avoided strategic defaults in the recent crisis if they had used shared equity mortgages as an alternative to conventional low down-payment mortgages. This would have mitigated their lost wealth.

References

Archer, W. and Ling, D. (1993). Pricing Mortgage-Backed Securities: Integrating Optimal Call and Empirical Models of Prepayment. *American Real Estate and Urban Economics Association Journal*, 21(4), Winter, 373- 404.

Archer, W., Ling, D. and McGill, G. (1996). The Effect of Income and Collateral Constraints on Residential Mortgage Terminations. *Regional Science and Urban Economics*, 26(3), 235-261.

Bajari,P., Chu, C.S., and Park, M. (2008) An Empirical Model of Subprime Mortgage Default From 2000 to 2007. Working Paper No. 14625. The National Bureau of Economic Research.

Bhutta, N., Dokko, J. and Shan, H. (2010). The Depth of Negative Equity and Mortgage Default Decisions. Working paper, Federal Reserve Board, Washington, D.C.

Cassidy H. J., Dennis, B. and Yang, T. T. (2008). Home Appreciation Participation Notes: A Solution to Housing Affordability and the Current Mortgage Crisis. *International Real Estate Review*, 11(2), 126-141.

Cox, J, Ingersoll, J. and Ross, S. (1985). A Theory of the Term Structure of Interest Rates. *Econometrica*, 385-408.

Crawford, G. and Rosenblatt, E. (1995). Efficient Mortgage Default Option Exercise: Evidence from Loss Severity. *Journal of Real Estate Research,*10(5), 543-556.

Deng, Y., Quigley, J. M. and Van Order, R. (1996). Mortgage Default and Low Downpayment Loans: the Cost of Public Subsidy. *Regional Science and Urban Economics*, 26(3–4), 263–285.

Deng, Y., Quigley, J. M. and Van Order, R. (2000). Mortgage Terminations, Heterogeneity and the Exercise of Mortgage Options. *Econometrica*, 68(2), 275–307.

Dokko, Y. & Edelstein, R.H. (1991). Interest Rate Risk and Optimal Design of Mortgage Instruments. *The Journal of Real Estate Finance and Economics,* 4(1), 59-48.

Foster, C. and Van Order, R. (1984). An Option-based Model of Mortgage Default. *Housing Finance Review*, 3(4), 351-372.

Foote, C., Gerardi, K. and Willen, P. (2008). Negative Equity and Foreclosure: Theory and Evidence. *Journal of Urban Economics*, 6(2), 234–245.

Gerardi, K., Shapiro, A. and Willen, P. (2007). Subprime Outcomes: Risky Mortgages, Homeownership Experiences, and Foreclosures. Working Paper 07-15. Federal Reserve Bank of Boston.

Ghent, A. C. and Kudlyak, M. (2009). Recourse and Residential Mortgage Default: Theory and Evidence from U.S.states. Working Paper, Federal Reserve Bank of Richmond, 09-10.

Jensen, M.C., Black, F. and Scholes, M.S. (1972). The Capital Asset Pricing Model. Studies in the Theory of Capital Markets.

Kau, J., Keenan, D. and Kim, T. (1993). Transactions Costs, Suboptimal Termination and Default Probabilities. *Journal of the American Real Estate and Urban Economics Association*, 21(3), 247-263.

Kau, J., Keenan, D., Muller, W. and Epperson, J. (1995). The Valuation at Origination of Fixed-Rate Mortgages with Default and Prepayment. *Journal of Real Estate Finance and Economics*, 11(1), 5-36.

Kau, J., Keenan, D., Muller, W. and Epperson, J. (1992). A Generalized Valuation Model for Fixed-Rate Residential Mortgages. *Journal of Money, Credit and Banking*, 24(3), 279-299.

Keenan, D. and Kau, J. (1995). An Overview of the Option-Theoretic Pricing of Mortgages. *Journal of Housing Research*, 6(2), 217-244.

Lekkas, V., Quigley, J. M. and Van Order, R. (1993). Loan Loss Severity and Optimal Mortgage Default. *AREUEA Journal*, 21(4), 353–371.

Phillips, R. A., Rosenblatt, E. and Vanderhoff, J. H. (1996). The probability of fixed- and adjustable-rate mortgage termination. *Journal of Real Estate Finance and Economics*,13(2), 95–104.