

## Testing for Long Memory in REIT Returns

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This study examines the long memory properties of composite, equity, mortgage, and hybrid real estate investment trust (REIT) returns by using semi-parametric and wavelet estimators. In particular, this paper applies the GPH semi-parametric estimator, the Haar and the Daubechies wavelet procedures to investigate the long memory properties of REIT returns. The results from the various procedures reveal that composite, equity, mortgage, and hybrid REIT returns are long memory processes with anti-persistence. The existence of long memory suggests that the dynamics which govern the four return series contain predictable components. This finding indicates that the markets for composite, equity, mortgage, and hybrid REITs are inefficient. The fact that these markets are inefficient suggests that investors can devise profitable strategies by using historical data or past information.

### Keywords

REITs; Long Memory; Wavelets; Fractional Integration; NAREIT; Daubechies; Haar

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## 1. Introduction

The issues that surround real estate investment trusts (REITs) has gained increased attention from financial economists and practitioners. REITs are companies that invest in real estate and designed to reduce corporate income taxes. For companies to qualify as REITs, they must invest at least 75 percent of their assets in real estate and generate 75 percent or more of their gross incomes from investments in real estate. In addition, REIT companies are required to distribute 90 percent of their earnings to investors yearly. The three basic types of REITs are equity, mortgage, and hybrid. Equity REITs typically invest in or own real estate. They generate returns for investors through rent collection. Mortgage REITs on the other hand, are designed to lend money to owners and developers or invest in financial instruments that are secured by mortgages on real estate. Hybrid REITs are basically a combination of equity and mortgage REITs. REIT companies can be publically or privately held.

The rapid growth in REIT markets in the past decade and half has been attributed to the 1993 Revenue Reconciliation Act which removed the barriers that prevented institutional investors from actively participating in real estate. Before 1993, companies were required to meet two basic ownership rules in order to qualify for REIT designation. These rules include the “100 Shareholder Rule” and the “5/50 Rule”. The “100 Shareholder Rule” stipulated that a REIT company must be owned by 100 or more shareholders. The “5/50 Rule” maintained that 5 or fewer individuals cannot own more than 50 percent of the stocks of a REIT company. The 5/50 rule made REIT securities unattractive to institutional investors, such as pension funds. Prior to the enactment of the 1993 Revenue Reconciliation Act, a pension fund was treated as an individual investor relative to the 5/50 rule. To encourage the participation of institutional investors in the REIT markets, the 1993 Revenue Reconciliation Act altered the 5/50 rule. With the passage of the Act, beneficiaries of a pension fund are counted as individual investors instead of counting the pension fund itself as a single investor (Brandon, 1997; Crain et al., 2000; Craft, 2001; Fickes, 2006). The relaxation of the 5/50 rule translated into substantial growth for the REIT market as institutional ownership of their securities soared. Glascock et al. (2000) report that new equity capital raised by REIT companies rose from \$6.5 billion in 1992 to \$18.3 billion in 1993. According to the National Association of Real Estate Investment Trusts, Inc. (NAREIT) data, the market capitalization of REITs in 1992 and 1993 were respectively, \$15.91 billion, and \$32.16 billion. The equity market capitalization of the REIT industry peaked in 2006 at \$438.07 billion before decreasing to \$191.65 billion in 2008. Chan et al. (1998) document that institutional ownership of REIT securities ranged from 12% to 14% in the period of 1986 to 1992, and increased to 30% in 1995.

REIT markets provide investors with diversification opportunities on the assumption that real estate and financial markets are not highly correlated. In addition, real estate has been touted as a hedging instrument against inflation. Apgar (1986) and Ibbotson and Siegel (1983) suggest that real estate accounts for between 40 and 50 percent of total wealth in the United States. This estimate is predicated on the notion that most families in the United States own their own homes. In addition, they contend that the majority of commercial properties are still in the hands of investors. In comparison with financial markets, real estate markets are associated with high transaction costs and infrequent trading and hence are thought to be less efficient than the stock markets.

A clear understanding of long memory properties of REIT returns is important to investors who seek to exploit the existence of arbitrage in real estate markets. Asset prices and returns exhibit long-memory if persistent temporal dependence exists between their distant observations. Such series are characterized by distinct, but non-periodic cyclical patterns. The existence of long memory indicates nonlinear dependence in the first moment of the distribution and hence a potentially predictable component in the series dynamics. However the short-memory property reveals the low-order correlation structure of a time series. For short-memory series, observations separated by a long time span are nearly independent. The presence of a fractional structure has both theoretical and econometric modeling implications for asset prices.

The efficiency of stock markets has been greatly studied. However, the efficiency of the real estate markets has not gained much attention. The few studies that have investigated the efficiency of the real estate market have provided mixed results. Seck (1996) uses the variance ratio test and finds that both equity REITs and the S&P 500 markets are random walk processes. However, Ambrose et al. (1992) find that the markets for mortgage REITs, equity REITs and S&P 500 are highly correlated and hence do not provide investors with diversification opportunities. Kleiman, Payne and Sahu (2002) use conventional procedures which include the unit root, variance ratio and runs tests to examine the random walk behavior of international real estate markets for Europe, Asia and North America. Their study provides evidence that the real estate markets for Europe, Asia, and North America are random walk processes and hence weak-form efficient. Kuhle and Alvaay (2000) use data from 108 EREIT companies for the period of 1989 to 1998 to suggest that the market for equity REITs is inefficient.

Giliberto (1990) tests the residuals from REITs and real estate returns for correlation. He finds that the residuals from REITs and real estate returns are significantly correlated. Based on this finding, he concludes that both the returns for REITs and real estate markets are driven by a common factor. Goebel and Ma (1993) use a cointegration analysis to examine the relationship

between REIT returns and the net asset values (NAVs) of their underlying assets. They find that REIT returns and the values of their underlying assets are cointegrated. The existence of cointegration between the REIT returns and the underlying fundamentals is interpreted as evidence of price inefficiencies in the REIT markets. Nelling and Gyourko (1998) compare the predictability of REIT returns with those of small-cap and mid-cap firms. They find that REIT returns are predictable unlike the returns for small- and mid-cap firms. Based on this finding, they conclude that the REIT markets are inefficient. Stevenson (2002) examines the existence of mean reversion behavior in international real estate securities. He finds that international real estate securities do not exhibit mean reverting behavior. Jirasakuldech and Knight (2005) use serial correlation and variance ratio tests to examine the random walk behavior of REIT markets. They conclude that REIT markets are efficient and hence investors cannot devise profitable strategies.

Lee and Chiang (2004) examine the degree of substitutability between equity and mortgage REITs. They find that equity and mortgage REITs are random walk processes, which indicate that both assets are substitutes. Assaf (2006) uses fractional cointegration and long memory procedures to examine the long run relationship between stock and securitized property markets for Canada. He finds that the markets for stocks and securitized properties are cointegrated and hence concludes that both assets should not be included in the same portfolio in the long run.

From the literature review, it is evident that most of the studies on this issue have concentrated on the relationship between the REITs and stock markets. The few studies that have examined the long memory properties of REIT markets provide mixed results. The mixed results could be attributed to the fact that most of the previous studies apply the standard variance ratio and runs tests. These procedures tend to have low power against the most recent techniques, especially the wavelet procedures. Unlike the previous studies, the present study uses longer time series which run from January 1972 through June 2008. In addition, the study applies the semi-parametric fractional integration procedure (GPH) proposed by Geweke and Porter-Hudak (1983) and the Haar and Daubechies wavelets (Daubechies, 1988) to examine the long memory behavior of the composite, equity, mortgage, and hybrid REIT return series.

The remainder of the study is structured as follows. Section 2 provides the methodology of the study. Section 3 presents the data and the descriptive statistics for the four closed-end funds. Section 4 discusses the empirical results. Section 5 furnishes the summary and implications of the study.

## 2. Methodology

This study uses the KPSS (Kwiatkowski et al., 1992) unit root tests to ascertain the time series properties of composite, equity, mortgage, and hybrid REIT returns. The KPSS unit root test is based on the residuals from an ordinary least squares (OLS) regression of the series of interest on the exogenous variable ( $s$ ) which includes a constant, or a constant and a linear time trend. The series  $y$  is regressed on a constant  $r_0$  and then the sum of the residuals ( $S_t$ ) is calculated. The following OLS is implemented for the KPSS test:

$$y_t = r_0 + \varepsilon_t \tag{1}$$

The residual ( $\varepsilon$ ) obtained from equation (1) is used to calculate the Lagrange multiplier ( $LM$ ) statistic. The KPSS LM test statistic is based on the following expression:

$$LM = \sum_{t=1}^T S(t)^2 / (T^2 \lambda_0) \tag{2}$$

where,  $S_t = \sum_{i=1}^t \hat{\varepsilon}_i = \sum_{i=1}^t (y_i - r_0) = \sum (x_t - \bar{x})$ , for  $t = 1, 2, \dots, T$  and  $\lambda_0$  represents an estimator of the residual spectrum at zero frequency and  $T$  represents the number of observations in the sample. Unlike other standard unit root tests, the null hypothesis under the KPSS is that the series is stationary. The alternative hypothesis on the other hand, is that the series is non-stationary. The null hypothesis of stationarity is rejected in favor of the alternative, if the test statistic exceeds the critical value at the conventional levels.

### 2.1 Fractional Integration

The application of fractional integration techniques enables the researcher to ascertain both the short-term and long-term behavior of a given time series through an autoregressive fractionally integrated moving-average (ARFIMA). ARFIMA models have three parameters, which include  $p$ ,  $d$ , and  $q$ . The parameter  $p$  represents the number of lags,  $q$  stands for the moving average parameter, while  $d$  is the difference parameter. A random process  $x_t$  follows ARFIMA if:

$$\theta(L)(1-L)^d x_t = \Omega(L)\mu_t \tag{3}$$

where  $d$  represents the difference operator, which may assume any real value,  $L$  is the lag operator with roots outside the unit circle.  $\theta(L)$  and  $\Omega(L)$  represents the AR and MA components, respectively. In equation (3),  $\mu_t$  is white noise and assumed to be normally distributed with zero mean and variance  $\sigma_u^2$ .  $(1-L)^d$  is the differencing operator given by:

$$(1-L^d) = \sum_{j=0}^{\infty} \frac{\Gamma(j-d)L^j}{\Gamma(d)\Gamma(j+1)}(-d) \tag{4}$$

where  $\Gamma(\cdot)$  represents the gamma function. The random process  $x_t$  is stationary if  $d = 0$ . In this case, a shock to  $\mu_t$  decays geometrically. For an integer value of  $d = 1$ , the random process  $x_t$  is non-stationary (i.e. the series has a unit root). This implies that a shock to  $\mu_t$  persists into the future indefinitely. If the value of  $d$  is within the interval  $(0, 0.5)$ , the random process is stationary and exhibits long memory. This implies that the observations are not independent as they carry a memory of all past events, which is characterized as long memory or long-range dependence. Hosking (1981) shows that when  $d$  is within the interval  $(0, 0.5)$  and  $d \neq 0$ , the correlation function,  $\rho(\cdot)$ , of an ARFIMA process is proportional to  $j^{2d-1}$  as  $j$  approaches zero. As a result, the auto-correlations of the random process  $x_t$  decay hyperbolically to zero as  $j$  approaches zero. If the differencing parameter,  $d$ , is within the interval  $(-0.5, 0)$ , the random process exhibits long memory with anti-persistence. This implies that if the series were up in the previous period, they will likely be down in the next period. When  $d$  is within the interval  $(0.5, 1.0)$ , the process is non-stationary, but still mean reverting. This suggests that the process is persistent or trend enforcing and it is most likely for the trend in the last period to continue in the next period. For  $d \geq 1$ , this indicates that the process is non-stationary and non-mean reverting.

**2.2 GPH Semi-Parametric Fractional Estimator**

This paper uses the GPH procedure to obtain the long memory parameter ( $d$ ) for the four return series. To complement the results from the GPH, the study implements a modified GPH technique (MGPH) advanced by Reinsen (1994). Under the MGPH procedure, the differencing parameter ( $d$ ) is obtained through the least squares regression based on the smoothed periodogram by using the Parzen window. The GPH and MGPH tests are based on the following regression equations, respectively:

$$\log[I(w_j)] = a_0 + a_1 \log \left[ 2 \sin \left( \frac{w_j}{2} \right) \right]^2 + e_j, \tag{5}$$

$$\log[f_s I(w_j)] = a_0 + a_1 \log \left[ 2 \sin \left( \frac{w_j}{2} \right) \right]^2 + \mu_j \tag{6}$$

In equations (5) and (6),  $w_j = 2\pi j/n$ ,  $j = 0, 1, 2, \dots, m$ , represents the set of harmonic frequencies,  $I(w_j)$  is the sample periodogram,  $f_{w_j}$  and  $f_s w_j$  are the spectral density and smoothed periodogram of the process,  $\log[I(w_j)/f(w_j)] - E \log[I(w_j)/f(w_j)]$  and  $e = \log [f_s(w_j)/f(w_j)]$ ,  $\hat{d} = (\hat{a}/2)$ . The standard error of  $\hat{d}$  solely depends on  $m$  as follows:  $\sqrt{m}(\hat{d} - d) \approx N(0, \pi^2/24)$ . Under the GPH and MGPH procedures, the null hypothesis that the long memory parameter ( $d$ ) is equal to one (i.e.  $d = 1$ ) is tested against the alternative that  $d$  is not equal to 1 (i.e.  $d \neq 1$ ).

### 2.3 Wavelet OLS Estimator

To obtain consistent estimates of the long memory parameter, the paper applies the wavelet OLS estimator procedures proposed by Jensen (1999). According to Jensen (1999), the wavelet procedures are preferred over the Fourier analysis because they have the ability to jointly localize a process both in time and scale. In other words, the wavelets can zoom in on the behavior of a process at a specific point in time. Similarly, they can zoom out to uncover any long and smooth qualities of a given time series. Under the wavelet procedures, the covariance stationary process such as  $x_t$  is given as a linear combination of sine and cosine functions in the frequency domain as follows:

$$f(x) = \alpha_0 + \sum_{k=1}^{\infty} (a_k \cos 2\pi k_x + b_k \sin 2\pi k_x) \tag{7}$$

Unfortunately, most economic and financial time series do not exhibit smooth rhythmic cycles required by functions that involve sine and cosines, such as equation (7). For this reason, Jensen (1999) proposed an alternative wavelet transformation, whereby the function  $f(x)$  on  $[0, 1]$  interval is expressed in the wavelet domain as:

$$f(x) = \alpha_0 + \sum_{i=0}^{\infty} \sum_{j=0}^{2^i-1} \alpha_{ij} \Psi(2^i x - j) \tag{8}$$

In equation (8),  $\Psi(x)$  represents the mother wavelet which can be expressed as:

$$\Psi(x) = \begin{cases} 1, & \text{if } 0 \leq x < \frac{1}{2} \\ 1, & \text{if } \frac{1}{2} \leq x < 1 \\ 0, & \text{otherwise} \end{cases} \tag{9}$$

Tkacz (2001) points out that the functions of  $\Psi_{jk}(x) = \Psi(2^j x - k)$  for  $j \geq 0$  and  $0 \leq k < 2^j$  are orthogonal. In short, the functions of  $\Psi_{jk}(x)$  form the basis functions of all square-integrable functions  $L_2$  within the interval  $[0, 1]$ . In equation (8),  $j$  represents the dilation index which compresses the function  $\Psi(x)$ , and  $k$  is the translation index responsible for shifting the function  $\Psi(x)$ . According to Tkacz (2001), any such basis in  $L_2(R)$  is a wavelet. Equation (9) is known as the Haar wavelet.

This study utilizes both the Haar and the Daubechies (1988) wavelet frameworks which have been shown by Jensen (1999) and Tkacz (2001) to have better qualities than the semi-parametric estimators which include the GPH and the log periodogram regression (LPR) (Robinson,1995). The wavelet OLS estimators tend to have smaller mean squared error and as such, have better predictive power than both the GPH and LPR procedures. In addition, the Daubechies procedure supports wavelets whereby each wavelet represents different degrees of smoothing of the step function. Jessen (1999)

shows that as the scaling coefficient approaches zero (i.e.  $j \rightarrow 0$ ), the wavelet coefficients  $\alpha_{ij}$  in equation (8) are distributed as  $N(0, \sigma^2 2^{-2id})$ , when  $|d| < 0.5$ . If we represent the variance of the wavelet coefficient at scale  $z$  by  $H(z) = \sigma^2 2^{-2d(z)}$ , this variance equation can be rewritten in log-linear form in terms of the differencing parameter  $d$  as follows:

$$\ln H(z) = \ln[\sigma^2] - d \ln[2^{-2(z)}] \quad (10)$$

where  $H(z)$  represents the wavelet coefficient variance at scale  $z$ . For estimation purposes, equation (10) can be rewritten as:

$$\ln H(z) = \alpha - d \ln[2^{-2(z)}] + \mu \quad (11)$$

where  $\alpha$  is the intercept and  $\mu$  represents the error term. Jensen (1999) and Tkacz (2001) document that a consistent estimate of the long memory parameter can be obtained from equation (11) through the application of the OLS procedure when  $d \in (-0.50, 0.50)$ . The wavelet equation is usually estimated over a moving window whose width is a power of two.

### 3. Data and Descriptive Statistics

The study employs monthly data on returns for composite, equity, mortgage, and hybrid REITS. The data are obtained from the website of NAREIT at <http://www.nareit.com/library/index.cfm>. The data span the time period of January 1972 through June 2008. To assess the impact of the 1993 Revenue Reconciliation Act on the REIT market, the sample period is divided into two. The first sample period runs from February 1972 through December 1992. The second sample period runs from January 1993 through June 2008.

Table 1 displays the descriptive statistics for composite, equity, mortgage, and hybrid REIT returns. The mean return values are 0.89, 1.09, 0.67, and 0.54 percent, respectively. The minimum and maximum values indicate that the return series have fluctuated greatly during the period under consideration. For example, the return for composite REIT ranges from a minimum of -17.94 percent to a maximum of 30.81 percent. The returns for mortgage REIT exhibits the greatest variability (6.08%) from the mean as indicated by the standard deviation. In contrast, equity REIT returns with a standard deviation of 4.07 percent shows the least deviation from the mean. The return series for composite and hybrid REITs are positively skewed, while those for equity and mortgage REITs are negatively skewed. The composite, equity, hybrid, and mortgage REIT returns display excess kurtosis. However, the excess kurtosis for equity REIT returns is less pronounced than those for composite, hybrid and mortgage REITs. Based on the Jarque-Bera statistics, the null hypothesis that the return series are normally distributed is rejected at the 1 percent significance level in all of the cases.

**Table 1 Summary Statistics (February 1972 - June 2008)**

	COMPR	EQUITY	HYBR	MORTR
<b>Mean</b>	0.89	1.09	0.67	0.54
<b>Median</b>	1.02	1.24	0.88	0.73
<b>Maximum</b>	30.81	14.08	40.46	38.40
<b>Minimum</b>	-17.94	-15.24	-24.48	-24.11
<b>Standard Deviation</b>	4.48	4.07	5.87	6.08
<b>Skewness</b>	0.02	-0.43	0.04	-0.24
<b>Kurtosis</b>	8.69	4.71	9.80	8.08
<b>Jarque-Bera</b>	591.17***	66.58***	843.14***	475.03***
<b>Probability</b>	0.00	0.00	0.00	0.00
<b>Observations</b>	438	438	438	438

**Note:** \*\*\* Indicates significance at the 1% level. COMPR = composite REIT returns, EQUITY = equity REIT returns, HYBR = hybrid REIT returns, MORTR = mortgage REIT returns.

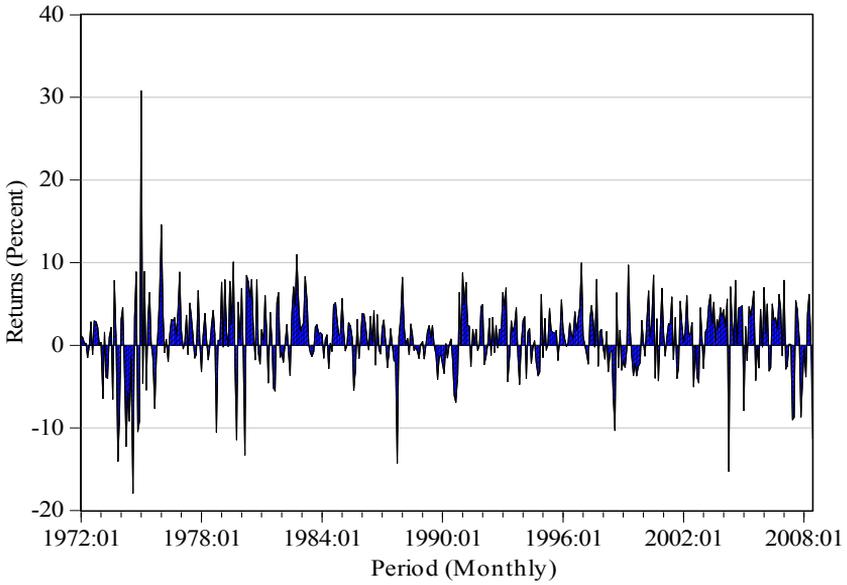
**Table 2 Spearman's Correlation Coefficients between The REIT Returns (February 1972 - June 2008)**

	COMPR	EQUITY	MORTR	HYBR
<b>COMPR</b>	1.000			
<b>EQUITY</b>	.907***	1.000		
<b>MORTR</b>	.735***	.569***	1.000	
<b>HYBR</b>	.795***	.636***	.650***	1.000

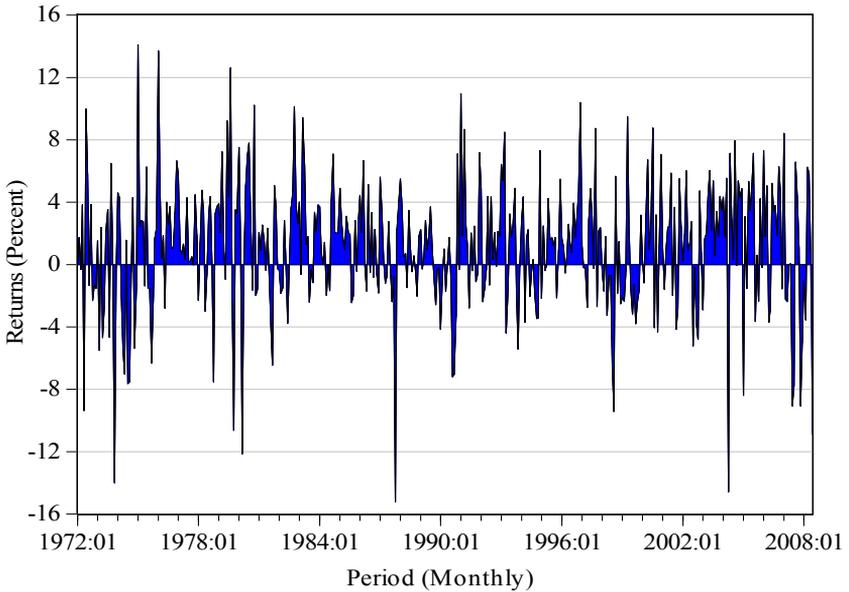
**Note:** \*\*\* Correlation is significant at the 1% level (2-tailed). COMPR = composite REIT returns, EQUITY = equity REIT returns, HYBR = hybrid REIT returns, MORTR = mortgage REIT returns.

Figures 1 through 4 plot the various REIT returns. These graphs reveal that the returns for composite, equity, hybrid, and mortgage REITs are highly volatile. This observation is consistent with the minimum and maximum statistics reported in Table 1. Table 2 displays the Pearson's correlation coefficients between the return series. The results suggest that the relationships between the various return series are positive and statistically significant at the 1 percent level. The highest correlation is recorded between composite and equity REIT returns and the least is recorded between equity and mortgage REIT returns.

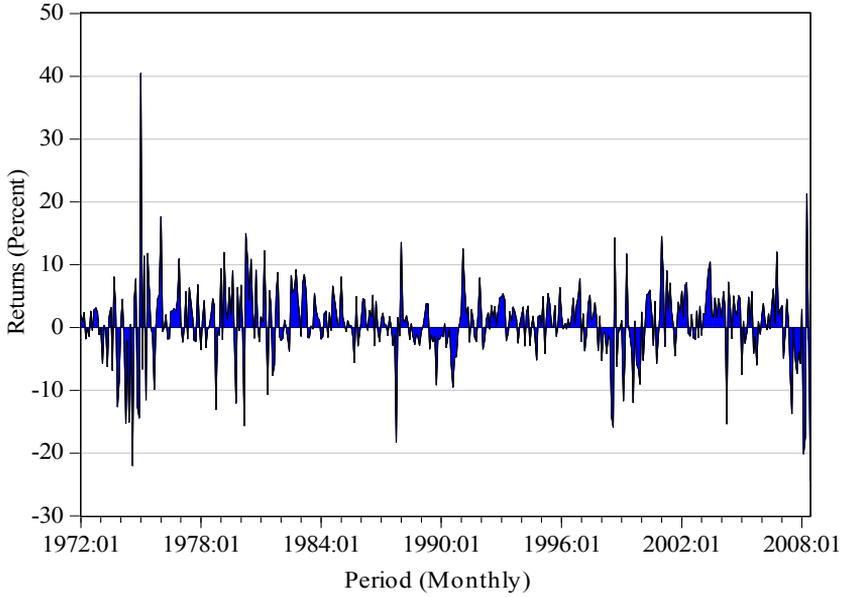
**Figure 1 REIT Composite Returns**



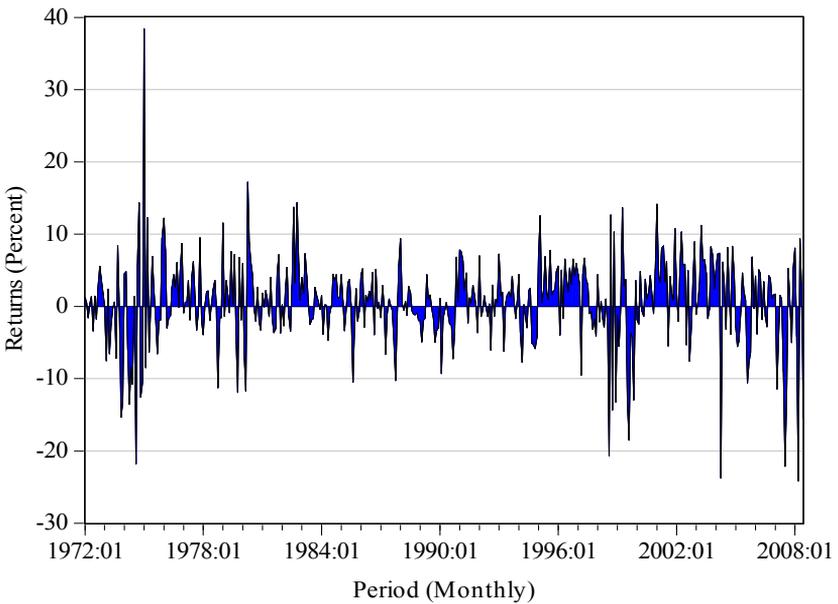
**Figure 2 REIT Equity Returns**



**Figure 3 REIT Hybrid Returns**



**Figure 4 REIT Mortgage Returns**



#### 4. Empirical Results

The empirical analysis of the study begins with the examination of the time series properties of the return series for composite, equity, hybrid, and mortgage REITs since fractional integration tests must be applied on stationary time series. Table 3 presents the KPSS unit root test results based on the first differences of the return series. The KPSS unit root tests were conducted with a constant, and with a constant and linear time trend. The results suggest that the null hypothesis of stationarity should not be rejected at the 5 percent level of significance. In each of the cases, the KPSS test statistic is less than the critical value at the 5 percent level, with a constant and with a constant and linear time trend.

**Table 3 KPSS Based On First Differences Of The Return Series (February 1972 - June 2008)**

Series	<i>t</i> -Stat	Lag(s)	5%CV
<i>Panel A Test with Constant</i>			
COMPR	0.0042	5	0.463
EQUITY	0.0291	5	0.463
MORTR	0.0233	5	0.463
HYBR	0.0452	5	0.463
<i>Panel B Test with Constant and Trend</i>			
COMPR	0.0138	5	0.146
EQUITY	0.0143	5	0.146
MORTR	0.0135	5	0.146
HYBR	0.0222	5	0.146

**Note:** Under the KPSS unit root test,  $H_0: X_t \sim I(0)$  while  $H_a: X_t \sim I(1)$ . COMPR = composite REIT returns, EQUITY = equity REIT returns, HYBR = hybrid REIT returns, MORTR = mortgage REIT returns.

Conventional unit root procedures which include the augmented Dickey-Fuller (Dickey and Fuller 1981), Phillips-Perron (Phillips-Perron, 1988) and KPSS are mainly designed to determine whether time series are level [i.e.  $I(1)$ ] or first difference stationary [i.e.  $I(1)$ ]. In short, standard unit root tests lack the ability to differentiate integer order of integration from fractional order of integration. Furthermore, Diebold and Rudebusche (1991), Sowell (1990), and Hassler and Wolters (1994) have shown that the conventional unit root tests tend to have low power against fractional alternatives. The rejection of the null hypothesis of level or trend stationarity by the KPSS unit root test could be an indication that the REIT return series are neither  $I(0)$  nor  $I(1)$  but instead, fractionally differenced processes [i.e.  $I(d)$ ].

**Table 4 Fractional Integration Test Results (February 1972- June 2008)**

Series	$d [u = 0.55]$	$t$ -stat	$d [u = 0.60]$	$t$ -stat
<i>Panel A Tests Based on the Geweke and Porter-Hudak Estimator</i>				
COMPR	-0.238***	-8.636	-0.317***	-10.989
EQUITR	-0.247***	-8.704	-0.342***	-11.202
MORTR	-0.290***	-9.000	-0.436***	-11.986
HYBR	-0.004***	-7.008	-0.206***	-10.64
<i>Panel B Tests Based on the Reinsen Estimator with Lag Parzen Window</i>				
COMPR	-0.269***	-15.461	-0.419***	-20.687
EQUITR	-0.283***	-15.643	-0.458***	-21.246
MORTR	-0.307***	-15.933	-0.444***	-21.046
HYBR	-0.085***	-13.225	-0.280***	-18.661

**Note:** \*\*\* indicates significance at the 1% level. The ordinates ( $m$ ) were determined by  $T^h$ , where  $T$  represents the total number of observations. The null hypothesis ( $H_0$ ) is that  $d = 1$ . COMPR = composite REIT returns, EQUITY = equity REIT returns, HYBR = hybrid REIT returns, MORTR = mortgage REIT returns.

Based on the results from the KPSS unit root procedures, the long memory tests of GPH and the wavelet procedures were implemented by using the first differences of the composite, equity, hybrid, and mortgage REIT returns. Table 4 presents the results from the semi-parametric difference estimators with the following windows —  $\mu$ : 0.55 and 0.60, for full sample period running from February 1972 through June 2008. Panel A of Table 4 displays the results from the GPH procedure. In all of the cases, the null hypothesis that  $d = 1$  is rejected at the 1 percent significance level. When  $\mu = 0.55$ , the fractional differencing parameter ranges from a high of -0.004 for hybrid to a low of -0.290 for mortgage REITs. At this bandwidth (i.e.  $\mu = 0.55$ ), the results suggest that the composite, equity, mortgage, and hybrid return series are long memory processes, although the differencing parameters are all negative. Similarly, for  $\mu = 0.60$ , the results provide evidence of long memory for all of the return series at the 1 percent level of significance. The long memory parameters range from -0.436 for mortgage to -0.206 for hybrid REITs. Again, in each case, the null hypothesis that  $d = 1$  is rejected at the 1 percent level of significance.

Panel B of Table 4 presents the results from the Reinsen estimator (MPGH) with lag Parzen windows for composite, equity, hybrid and mortgage REIT returns. For  $u = 0.55$ , the differencing parameters varied from -0.307 for mortgage to -0.085 for hybrid REIT returns. The long memory parameter is negative and statistically significant in each case. Similar results are indicated when  $u = 0.60$ . The fact that the fractional differencing parameters are less than -0.5 indicates that all of the REIT return series exhibit long memory with anti-persistence.

**Table 4A Fractional Integration Test Results (February 1972 - December 1992)**

Series	$d [u = 0.55]$	$t$ -stat	$d [u = 0.60]$	$t$ -stat
<b>Panel A Tests Based on the Geweke and Porter-Hudak Estimator</b>				
COMPR	-0.852***	-10.518	-0.882***	-12.801
EQUITR	-0.991***	-11.309	-0.965***	-13.364
MORTR	-0.849***	-10.504	-0.847***	-12.600
HYBR	-0.762***	-10.010	-0.843***	-12.531
<b>Panel B Tests Based on the Reinsen Estimator with Lag Parzen Window</b>				
COMPR	-0.812***	-18.032	-0.822***	-21.645
EQUITR	-0.912***	-18.969	-0.924***	-22.855
MORTR	-0.746***	-17.327	-0.749***	-20.773
HYBR	-0.818***	-18.039	-0.837***	-21.820

**Note:** \*\*\* indicates significance at the 1% level. The ordinates ( $m$ ) were determined by  $T^m$ , where  $T$  represents the total number of observations. The null hypothesis ( $H_0$ ) is that  $d=1$ . COMPR = composite REIT returns, EQUITY = equity REIT returns, HYBR = hybrid REIT returns, MORTR = mortgage REIT returns.

**Table 4B Fractional Integration Test Results (January 1993-June 2008)**

Series	$d [u = 0.55]$	$t$ -stat	$d [u = 0.60]$	$t$ -stat
<b>Panel A Tests Based on the Geweke and Porter-Hudak Estimator</b>				
COMPR	-0.282***	-6.753	-0.442***	-8.646
EQUITR	-0.262***	-6.474	-0.447***	-8.680
MORTR	-0.459***	-7.483	-0.489***	-8.929
HYBR	-0.224***	-6.276	-0.288***	-7.724
<b>Panel B Tests Based on the Reinsen Estimator with Lag Parzen Window</b>				
COMPR	-0.311***	-11.743	-0.457***	-15.265
EQUITR	-0.310***	-11.742	-0.469***	-15.385
MORTR	-0.448***	-12.967	-0.500***	-15.808
HYBR	-0.191***	-10.668	-0.265***	-13.254

**Note:** \*\*\* indicates significance at the 1% level. The ordinates ( $m$ ) were determined by  $T^m$ , where  $T$  represents the total number of observations. The null hypothesis ( $H_0$ ) is that  $d=1$ . COMPR = composite REIT returns, EQUITY = equity REIT returns, HYBR = hybrid REIT returns, MORTR = mortgage REIT returns.

We next turn to the results obtained from the wavelet OLS. The wavelet analysis involves the implementation of the Haar and the Daubechies wavelets with 4, 12 and 20 smoothing parameters. The utilization of different smoothing parameters was important to ensure the robustness of the results from the wavelet OLS estimators. Table 5 displays the results from the Haar and Daubechies 4, 12 and 20 wavelet procedures for the full sample period

which runs from February 1972 through June 2008. Similar to the results from the GPH and MPMG procedures, the results from the wavelet OLS estimators overwhelmingly provide evidence of long memory with anti-persistence. The test statistics associated with the composite, equity, mortgage, and hybrid REIT return series are statistically significant at least at the 5 percent level and within the interval  $(-0.5 \leq d < 0)$ . For example, the wavelet OLS estimators from the Haar procedure are -0.364, -0.455, -0.373, and -0.303, respectively for composite, equity, mortgage, and hybrid REIT returns. These differencing parameters are all statistically significant at least at the 5 percent level. The wavelet OLS estimators from the Daubechies with 4 smoothing parameters (i.e. Daubechies -4) are -0.321, -0.336, -0.363, and -0.231, respectively for composite, equity, mortgage, and hybrid REIT returns. The test statistics are all statistically significant at least at the 5 percent level. Similar results were obtained by using Daubechies with 12 and 20 smoothing parameters. In all of the cases, the fractional differencing parameters are negative and statistically significant which indicate the existence of long memory with anti-persistence. This finding suggests that the autocorrelations of the REIT return series are negative and decreases gradually over time. The fact that the wavelet estimators for the four return series fall within the interval  $(-0.50$  and  $0)$  suggests that the series do not have infinite conditional variance.

**Table 5 Wavelet Fractional Integration Test Results (February 1972 - June 2008)**

REITs	HAAR	Daubechies-4	Daubechies-12	Daubechies-20
COMPR	-0.364*** (4.075)	-0.321*** (4.040)	-0.402*** (4.820)	-0.470*** (6.623)
EQUITR	-0.455*** (3.851)	-0.336*** (4.090)	-0.416*** (4.820)	-0.489*** (6.658)
MORTR	-0.373*** (3.966)	-0.363*** (4.467)	-0.446*** (4.797)	-0.500** (5.494)
HYBR	-0.305** (3.273)	-0.251** (2.793)	-0.319*** (3.455)	-0.385*** (4.902)

**Note:** \*\* and \*\*\* indicate significance at the 5 and 1% level, respectively. The figures in parentheses are the absolute *t*-values. COMPR= composite REIT returns, EQUITY = equity REIT returns, HYBR = hybrid REIT returns, MORTR = mortgage REIT returns.

We surmise from the results from the various procedures that the composite, equity, mortgage, and hybrid REIT return series are long memory processes. The finding that REIT returns are long memory processes indicates that REIT markets are inefficient. This finding is inconsistent with Jirasakuldech and Knight (2005). The contradictory results could be attributed to the fact that Jirasakuldech and Knight (2005) used serial correlation and variance ratio

tests while the present study applied more powerful techniques, namely, the Haar and the Daubechies wavelet OLS estimators.

Table 4A displays the results from the semi-parametric difference estimators with the following windows —  $\mu$ : 0.55 and 0.60, for the first sub-period running from February 1972 to December 1992. Panel A of Table 4A presents the results from the GPH procedure. In all of the cases, the null hypothesis that  $d = 1$  is rejected at the 1 percent significance level. When  $\mu = 0.55$ , the fractional differencing parameters ranged from a high of -0.762 for hybrid to a low of -0.991 for equity REITs. Similarly, for  $\mu = 0.60$ , the results provide evidence of long memory for all of the return series at the 1 percent level of significance. The long memory parameters range from -0.912 for equity to -0.7466 for mortgage REITs. In each case, the null hypothesis that  $d = 1$  is rejected at the 1 percent level of significance. In all of the cases, the results from the GPH procedures suggest that the composite, equity, mortgage, and hybrid return series are not stationary, but still mean reverting, as the differencing parameters are all negative and within the  $(-0.5 \leq d < -1.0)$ . These results suggest that the processes which govern the four return series are characterized by anti-persistence.

Panel B of Table 4A displays the results from the Reinsen estimator with lag Parzen window (MPGH) for composite, equity, hybrid and mortgage REIT returns, for the first sub-period (February 1972 to December 1992). The results show that for  $u = 0.55$ , the differencing parameters varied from -0.912 for equity to -0.746 for mortgage REIT returns. The long memory parameters are negative and statistically significant in all of the cases. Similar results are indicated when  $u = 0.60$ . The fact that the fractional differencing parameters are within the interval  $(-0.5 \leq d < -1.0)$  indicates that the REIT return series are non-stationary, but nevertheless mean reverting with anti-persistence for the sub-period which spans from February 1972 through June 2008.

Table 5A displays the results from the Haar and the Daubechies 4, 12 and 20 wavelet procedures for the first sub-period (February 1972 to December 1992). The results from the Haar wavelet procedure suggest that the composite, equity, and mortgage REIT returns are non-stationary, but still mean reverting, as the test statistics are within the interval  $(-0.5 \leq d < -1.0)$ . However, the result suggest that the hybrid REIT return is a long memory process with anti-persistence, as the test statistic (-0.499) is within the interval  $(-0.5 \leq d < 0)$ .

Similarly, the results from the Daubechies wavelet OLS estimators provide evidence against long memory for the composite, equity, and mortgage REITs return series. For Daubechies with 4 smoothing parameters (i.e. Daubechies-4), the test statistics are -0.808, -0.747, -0.713, and -0.453, respectively for composite, equity, and mortgage REIT returns. However, for hybrid REIT returns, the differencing parameter (-0.453) obtained from the Daubechies-4

lies within the range  $(-0.5 \leq d < 0)$  and is statistically significant. Similar results are obtained by using Daubechies with 12 and 20 smoothing parameters. In all of the cases, the fractional differencing parameters for composite, equity, and mortgage REIT returns are within the interval  $(-0.5 \leq d < -1.0)$  and statistically significant, at least at the 5 percent level. These results suggest that the composite, equity, and mortgage REIT return series are non-stationary, but nevertheless mean reverting with anti-persistence. In contrast, the differencing parameters for hybrid REIT returns are within the interval  $(-0.5 \leq d < 0)$  and statistically significant, at least at the 5 percent level, which indicates the existence of long memory with anti-persistence.

**Table 5A Wavelet Fractional Integration Test Results (February 1972 - December 1992)**

REITs	HAAR	Daubechies-4	Daubechies-12	Daubechies-20
COMPR	-0.755*** (5.897)	-0.808*** (6.146)	-0.837*** (7.513)	-0.813*** (8.089)
EQUITR	-0.678*** (5.841)	-0.747*** (11.473)	-0.907*** (3.982)	-0.835*** (6.510)
MORTR	-0.592*** (4.351)	-0.713*** (4.312)	-0.694*** (4.650)	-0.597*** (5.483)
HYBR	-0.499** (3.561)	-0.453*** (4.743)	-0.4772*** (5.305)	-0.498*** (5.662)

**Note:** \*\* and \*\*\* indicate significance at the 5 and 1% level, respectively. The figures in parentheses are the absolute *t*-values. COMPR = composite REIT returns, EQUITY= equity REIT returns, HYBR = hybrid REIT returns, MORTR = mortgage REIT returns.

Table 4B presents the results from the GPH and MGPH semi-parametric estimators for the second sub-period (January 1993 – June 2008) with the following windows —  $\mu$ : 0.55 and 0.60. Panel A of Table 4B presents the results from the GPH procedure. In all of the cases, the null hypothesis that  $d=1$  is rejected at the 1 percent significance level. For  $\mu = 0.55$ , the fractional differencing parameters ranged from a high of -0.224 for hybrid to a low of -0.459 for mortgage REIT returns. At this bandwidth (i.e.  $\mu = 0.55$ ), the results reveal that the composite, equity, mortgage, and hybrid return series are long memory processes. Similarly, for  $\mu = 0.60$ , the results provide evidence of long memory for all of the return series at the 1 percent level of significance. The long memory parameters ranged from -0.489 for mortgage to -0.288 for hybrid REITs. Again, in each case, the null hypothesis that  $d = 1$  is rejected at the 1 percent level.

Panel B of Table 4B presents the results from the MGPH for composite, equity, hybrid and mortgage REIT returns for the second sub-period. For  $\mu=0.55$ , the differencing parameters varied from -0.448 for mortgage to -0.191

for hybrid REIT returns. The long memory parameter is negative and statistically significant in each case. Similar results are obtained for  $u = 0.60$ . Table 5B furnishes the results from the Haar and the Daubechies 4, 12 and 20 procedures for the second sub-period. Similar to the results from the GPH and MGPH procedures, the results from the wavelet OLS estimators overwhelmingly provide evidence of long memory with anti-persistence for composite, equity, mortgage, and hybrid REIT returns. For example, the results from the Haar estimators are -0.428, -0.437, -0.487, and -0.312 respectively, for the composite, equity, mortgage, and hybrid REIT return series. The results from Daubechies-4 are -0.360, -0.376, -0.412, and -0.226, respectively. Similar results were obtained by using Daubechies with 12 and 20 smoothing parameters. The test statistics associated with the composite, equity, mortgage, and hybrid REIT return series are all statistically significant at least at the 5 percent level and within the interval  $(-0.5 \leq d < 0)$ . The fact that the differencing parameters are negative and statistically significant indicates that the four REIT return series are long memory processes with anti-persistence. In all, the results from the GPH, MGPH, Haar and Daubechies procedures for the second sub-period (January 1993 - June 2008) suggest that the composite, equity, mortgage, and hybrid REIT return series are long memory processes.

**Table 5B Wavelet Fractional Integration Test Results (January 1993 - June 2008)**

REITs	HAAR	Daubechies-4	Daubechies-12	Daubechies-20
COMPR	-0.428*** (4.572)	-0.360** (3.138)	-0.407*** (3.475)	-0.442*** (4.655)
EQUITR	-0.437*** (4.865)	-0.376*** (3.354)	-0.423*** (3.512)	-0.457*** (4.718)
MORTR	-0.487*** (4.629)	-0.412** (2.344)	-0.452*** (3.900)	-0.498** (4.057)
HYBR	-0.312** (2.544)	-0.226* (2.212)	-0.232* (2.263)	-0.279** (3.147)

**Note:** \*\* and \*\*\* indicate significance at the 5 and 1% level, respectively. The figures in parentheses are the absolute  $t$ -values. COMPR = composite REIT returns, EQUITR = equity REIT returns, HYBR = hybrid REIT returns, MORTR = mortgage REIT returns.

## 5. Summary and Implications

This paper has used fractional integration approaches to examine the long memory properties of composite, equity, mortgage, and hybrid REIT returns. In particular, the study applies the KPSS unit root procedure to test the stationarity of the four return series. The GPH, MGPH, Haar and Daubechies wavelet estimators are used to determine whether the composite, equity,

mortgage, and hybrid REIT returns possess short or long memory. The results from the KPSS unit root tests indicate that all the return series are first difference stationary. The results from the GPH and MGPH fractional integration procedures indicate that the composite, equity, mortgage, and hybrid REIT returns are fractionally integrated with anti-persistence. Similarly, the results from the Haar and the Debauchies wavelet OLS estimators provide evidence in support of the notion that the returns for composite, equity, mortgage, and hybrid REITs are long memory processes with anti-persistence. In each case, the estimated long memory parameter falls within the interval between  $-0.5$  and  $0$ . These results indicate that the autocorrelations associated with the four return series dissipate very slowly and are negative in sign. The sample was divided into two sub-periods to assess the effect of the 1993 Revenue Reconciliation Act on REIT returns. The first sample spans the period from February 1972 through December 1992, while the second sample runs from January 1993 to June 2008.

The results for the first sub-period reveal that composite, equity, and mortgage REIT returns are not long memory processes and hence, non-stationary. However, the results indicate that the hybrid REIT returns display long memory properties and thus are mean reverting with anti-persistence. Interestingly, the results from the various fractional integration techniques employed by the study for the second sub-period reveal that the four REIT return series including those for composite, equity, hybrid, and mortgage REITs are long memory processes with anti-persistence. This finding indicates that for the second sub-period, the four return series are stationary and therefore mean reverting. Taken together, the results from the study indicate that the 1993 Revenue Reconciliation Act has a significant impact on the time series properties of the REIT returns. The results for the full sample period suggest that composite, equity, hybrid, and mortgage REIT returns are long memory processes with anti-persistence. These findings imply that the four return series should be modeled in a nonlinear fashion. The existence of long memory further suggests that the dynamics that govern the REIT returns contain predictable components, which indicate that the markets for composite, equity, hybrid, and mortgage REIT are inefficient. This finding implies that investors can use past information or historical data to formulate profitable strategies. In addition, the fit of models for composite, equity, hybrid, and mortgage REIT returns can be enhanced by allowing for long memory in the data.

## **Acknowledgement**

The authors are grateful to Dr. Ko Wang, the editor of the journal and anonymous referees for their helpful comments and suggestions which helped to improve the quality of this paper. The usual disclaimer applies.

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