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The (De)merits of using Integral Transforms in Predicting Structural Break Points

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The structural break points of returns and volatility are generally illustrated by using uni-and-multivariate time series models. Despite the elegance of uni-and-multivariate models, the interchangeability of different structural break points is not well accounted for in those models. This study uses integral transforms (Fourier and Laplace) to illustrate the interchangeability of structural break points of indices. Furthermore, structural break points are validated with commonly used unit root structural break tests [(i) augmented Dickey Fuller (ADF), (ii) ADF-generalized least squares (GLS), Phillips Perron (PP) 1988 and Zivot-Andrews (ZA) 1992 tests]. The results illustrate persistent interchangeability and interconnectedness patterns of structural break points throughout the time series. Moreover, the structural break points tests confirm the findings of the integral transforms.

Keywords:

Integral transforms, structural breaks

JEL: C35, G12

1. Introduction

Kola and Sebehela (2020) study volatility portfolio risks made up of four indices (bonds, commodities, equities and real estate) of Brazil, Russia, India, China and South Africa (hereafter the BRICS countries). Their study uses a multivariate model – a vector autoregressive (VAR) model. The results of the VAR model illustrate that volatility spillovers are in between and within different indices of the BRICS nations. Moreover, volatility spillovers have no specific direction in terms of which country they follow and where they form. The pattern of the latter can be observed when one looks at different indices irrespective of the BRICS country. That leads to the question of how one would hedge those volatility risks in a portfolio. On the other hand, the Markov-regime switching model illustrates interesting findings: (i) illiquid indices converge faster than liquid indices, and (ii) volatility spillover patterns of out-sample periods mirror the in-sample periods. Back to the main question in this paragraph: how can volatility spillovers be hedged?

Kola and Sebehela (2021) opine that the common and/or appropriate way of hedging volatility spillovers is by using discrete volatility models. A commonly used model for hedging discrete volatilities is the generalised autoregressive conditional heteroskedesticity (hereinafter GARCH). Despite the elegance and widely acceptable usage of GARCH(1,1), especially in practice, Kola and Sebehela (2021) illustrate its shortcomings. In order to increase the accuracy of calculating "accurate" volatilities, they recommend incorporating (i) correlation coefficients of debt and equity, (ii) equity parameters, (iii) risk premium, (iv) interest rates and (v) shock-stock markets. The results illustrate that Kola-Sebehela GARCH(1,1) (hereinafter KS-GARCH) outperforms GARCH(1,1). However, the KS-GARCH does not account for structural break points. Yet structural break points are central to the appropriate hedging of volatility spillovers. This study investigates how to appropriately predict those structural break points.

This line of research is nothing new; however, in the context of alternative portfolios of assets, this research study raises the following questions. First, are there any links among different indices that lead to structural breaks? Secondly, which asset classes precede others in terms of movement when there are structural breaks, and lastly, are structural breaks within or in between different asset classes in different markets? The novelty of this article is that it exactly responds to these questions. Many studies mainly use univariate models; (hereinafter ARCH) and GARCH, and multivariate models such as VAR processes to illustrate structural changes. This study uses Fourier and Laplace transforms to exemplify structural breaks. The former integrate values between negative and positive infinities while the latter integrate values between zero and positive infinity. Thus, the structural breaks of returns will be illustrated by using Fourier transform, as returns can be negative. The structural breaks of volatilities will be illustrated by using Laplace transform as volatilities cannot be negative. Fundamentally, integrals converge if there are no breaks but do not converge in the presence of break points. Another compelling reason that supports the usage of integral transforms is that normally, price changes lead to changes in returns. Thereafter, volatilities change. However, some studies separate the structural breaks of returns and volatilities, although logic tells us that there is an interconnected relationship. The study that is most related to this article is Enders and Holt (2012). They use transforms for spillover while we use two integral transforms to illustrate structural breaks. The results of Enders and Holt (2012) illustrate that first, the Fourier transform results show "the last upward break in the series" (p. 667). Secondly, the combination of frequencies and structural breaks do not affect the performance of Fourier transforms. Third, the Fourier transforms capture structural break points at lower confidence interval levels than the Bai-Perron test. Fourth, Fourier transforms capture structural break points even when there are shifts in the movement of the time series. Finally, even though Bai-Perron and Fourier transform break points reinforce support for each other, overall, Fourier transforms provide more insight into the structural break points.

The results counter conventional findings on structural break points. First, there are hidden structural break points in indices (i.e. bonds, commodities, equities and real estate). Moreover, hidden structural break points outnumber evident structural break points. Fundamentally, structural break points of index returns surpass those of index volatilities. Moreover, movements in index returns influence those in index volatilities; to the best of our knowledge, this is the first study that illustrates systematic patterns in stock returns and volatilities. This might be because indices are more encompassing than stock prices. That is, more parameters (i.e. inflation, dividends, debentures, economic growth, etc.) are included in indices but not in stock prices. Secondly, economic, political and governance structures influence the financial markets of countries; this is consistent with findings in previous market microstructure studies. Thirdly, integral transforms reveal more structural break points than conventional models. This is because there can be twenty or more structural break points. Lastly, structural break points from liquid (illiquid) indices are (not) systematic; this is a rare finding. Some of possible reasons for the latter are (i) acceptable standard marginal error of valuation (Crosby et al. 1998, and Capozza and Israelsen 2007), (inii) uncertainty in the form of bid-offer spreads in real estate investment trusts (REITs) tends to be high (Marcato and Ward 2007), (iii) calendar anomalies are more common in common stocks than REITs (Akbulut et al. 2015) and (iv) securitised real estate tends to be integrated across different regions (Al-Abduljader 2019).

On the demerits side of using integral transforms to identify structural breaks, the main demerit is that unlike Davis et al. (2006) and Chan et al. (2014), first, integral transforms never presents structural breaks of a set of vectors. Secondly, integral transforms cannot show the best combination of structural break points; however, the highest and lowest structural break points can be selected from graphs. Third, integral transforms are continuous in nature; therefore, using them to illustrate discrete structural break points is a challenge. Finally, there are no techniques that are similar to integral transforms; hence, validation and/or verification of the structural break points of integral transforms is an open-ended answer.

The article proceeds as follows. Section 2 reviews previous studies on structural breaks. Section 3 is on modelling. Section 4 focuses on the data and Section 5 is the analysis. The final section concludes the study.

2. Literature Review

The literature review of this article is structured as follows: first, the study presents on how earlier studies describe multiple structural breaks and thereafter, the study synthesises how different sectors (i.e. bonds, commodities, equities and listed real estate) present sectorial structural break points.

2.1 General Multiple Structural Break Points

Davis et al. (2006) studied structural break points based on nonstationary time series. The heart of their study is to find the best combination of structural break points. However, Kola and Sebehela (2020, 2021) proceed beyond the best combination of structural break points. They use integral transforms that account for continuity and flexibility. Davis et al. (2006) mainly use piecewise autoregressive (AR) processes, which are discrete. They construct a minimum description length (MDL) model, starting from the 'derivation of MDL'. Then, they present a consistent scenario and finally, provide a general description of the model. In the derivation of MDL, the key is that current break points are part of the filtration of past break points. To put it simply, structural break points are independent and identically distributed (iid) random variables. In integral transforms, structural break points are not necessarily (iid) random variables. In terms of consistency, Davis et al. (2006) assume that there must be sufficient observations for structural break points to occur and furthermore, true break points are assumed in advance. The integral transform chooses the break points through a convergence process. Generally speaking, Davis et al. (2006) set an initial point, which gives rise to other subsequent points. They called their vectors *chromosomes* because of the latter phenomenon. Every time when one moves to the subsequent structural break point, the current break point is initialised at the time in order to obtain the next break point from the subsequent break point. They use Darwin's *Theory of Natural Selection*, where preceding break points give rise to subsequent improved structural break points. When using integral transforms, the theory would be loosely termed *theoretical <u>convergence</u> through natural selection*. The keyword that distinguishes Darwin's theory from the integral transforms theory is convergence, which usually makes the process faster. The process of initialising every point in order to detect the next break point is computationally consuming and complex. The latter phenomenon in integral transforms is irrelevant.

The results in Davis et al. (2006) show that structural break points are interrelated at different points in time. Secondly, multiple points are finite at some later point in time. In integral transforms, break points are infinite pretty much in the entire time series. Then, Davis et al. (2006) proceed to test a ‘piecewise stationary process with dyadic structure’.

They apply Auto-PARM to 200 realizations, and show that Auto-PARM can provide the correct number of segments for 96% of the realizations. Then, their sensitivity analysis shows minimal errors. The total run time is 1.53 seconds. Note that segment is modelled as an AR(1) process. The time-dependent MA(2) process has a time run of 3.76 seconds. For a human speech signal of 5762 observations, the total run time is 18.02 seconds. Fundamentally, Davis et al. (2006) illustrate multiple structural break points to a finite number. In integral transforms, the total number of break points depends on the length of the time series; however, integral transforms tend to detect more structural break points than the break points detected in Davis et al. (2006). Kola and Sebehela (2020; 2021) use a longer time series than that used by Davis et al. (2006).

Chan et al. (2014) carry out a study that is similar to that of Davis et al. (2006) except fundamentally, they increase the number of structural break points. In addition, Chan et al. (2014) account for regime changes by using a structural break autoregressive (SBAR) model. Chan et al. (2014) opine that the application of a SBAR model is computationally inefficient. Integral transforms are complex but once there is an understanding on how to handle integral transforms, then using integral transforms can be a smooth process. In detecting structural break points, Chan et al. (2014) incorporate the principles of the least absolute shrinkage and selection operator (LASSO) method. Similar to Davis et al. (2006), Chan et al. (2014) fix the points of the structural breaks. In their estimation procedure, Chan et al. (2014) start from one-step group LASSO estimations. Then, they move to a two-step estimation procedure and finally, deal with the computational issues. In the one-step group LASSO estimation, fundamental and two variables influence their modelling and in the two-step estimation procedure, they assume the “best possible subset” is arbitrary. Non-equilibrium applications are fundamentally flawed and have proven challenging (see Smith 1976). When the sample is large, Chan et al. (2014) show that further calculation in the form of a backward elimination algorithm (BEA) can be applied at that stage. For calculation purposes, they assume that an iteration process would solve every challenge including optimisation of the points of structural breaks. However, Marcato et al. (2018) show that the iteration process does not necessarily account for every single point of every used parameter. Thus by extension, there is always some error involved when running iteration processes.

The results of Chan et al. (2014) based on 200 observations where white noise, $\epsilon_t \sim iid\ N(0,1)$, show "correct segmentation for 96% of the realizations" (p.595) when estimation is based on a two-step procedure. The standard error is 0.004. Based on a long-time series, the calculation of the estimates requires 20 seconds with good accuracy. They further conduct an analysis (electroencephalogram, hereafter EEG and LASSO on the S&P500 index) in order to strengthen their earlier findings. First, the EEG records 100 Hz in 5 minutes and 28 seconds. Finally, LASSO places 612 securities out from S&P500 index backed by subprime residential mortgages. Nevertheless, even though Chan et al. (2014) uses a more sophisticated technique compared to Davis et al. (2006), they face limitations in terms of estimating the structural break points.

2.2 Sectorial Structural Break Points

Maghyereh and Awartani (2016) differ from other bonds studies in the sense that they focus on Sukuk (i.e. Islamic finance) and its relationship with the bonds markets. Sukuk finance follows the strict laws of Islam for investing. Thus, any product that is forbidden by Islam laws, means that this product or related products should not be the object of investment. According to Maghyereh and Awartani (2016), the Sukuk markets are similar to the bonds markets. Thus, ideally the inclusion of Sukuk investments in the bonds markets should not lead to diversification benefits. The crux of their study is to compare Sukuk products with those in the bonds markets in terms of returns and volatilities. The other interesting part of their study is that they include two more products in their study: global equities and global Islamic stocks. They outline the key differences between Sukuk investments and the bonds markets. They conclude that actual volatility transmissions between the two are more important as opposed to key differences. However, they state that the differences influence the volatility patterns.

Maghyereh and Awartani (2016) use a VAR model with a number of variables. According to them, a standard Cholesky decomposition has shortcomings in exemplifying how spillovers are affected by market orderings. The timeframe of the data ranges from 30th September 2005 to 24th February 2014. They concur that the global financial crisis started from September 2008 to December 2009. This study could pick up the structural breaks that might demonstrate when this financial crisis started. They choose a period characterised by other sub events: the European Sovereign Crisis period from April 2010 to June 2012, and the Arab Spring from December 2010 to 2015. The data used for the analysis are from four indices: the Dow Jones Citigroup Sukuk Index, Dow Jones Citigroup Global Bond Index, Dow Jones Islamic Stock Market Index and Dow Jones Global Stock Market Index.

The first set of results in Maghyereh and Awartani (2016) based on the Ljung-Box test shows that Sukuk returns are highly correlated with bonds. For the forecasting part, the aggregated shocks and total spillover seem to be lower than expected. Thus, the world bonds markets spillover 3.9% to the Sukuk while Sukuk only spillovers 0.2%. The volatility spills are high between Sukuk and equities than between Sukuk and bonds. This has to do with similarities between similar assets. Generally, during market downturns, correlation tends to be higher between different assets than similar assets. Partly, this has to do with the law of gravity. According to Maghyereh and Awartani (2016), Sukuk is overall the net receiver of shocks. The VAR model illustrates various structural break points including the overall patterns of spillovers. As part of their robustness testing, they explore dynamic conditional correlations based on a dynamic conditional correlation model of GARCH (DCC-GARCH). They include dummy variables for the structural breaks in the DCC-GARCH. The results confirm that there is weak correlation between Sukuk and bonds and equities, even in the presence of structural breaks. Testing for jump and co-jumps confirms the presence of structural breaks.

Shalini and Prasanna (2016) explore structural breaks in the Indian commodity markets during the global financial crisis. In analysing structural breaks, they focus on the properties of volatility dynamics including: (i) persistence, (ii) leverage effect, and (iii) long memory. Long memory should usually decrease with maturity with time to expiration. The heart of the argument in Shalini and Prasanna (2016) is that complex channelization is misunderstood. Interestingly, India is one of the major consumers of commodities and related products. In the structural breaks, they focus on how different commodity regimes influence break points using Markov regime switching models. Other techniques that are used to illustrate structural breaks include iterated cumulative sum of squares (ICSS), wavelet analysis, and wavelet exponential generalised autoregressive conditional heteroscedasticity (hereafter wavelet-EGARCH). The data are the spot prices of commodities including energy and precious, industrial and agricultural metals for the period of 2005 to 2012. Note that they use logarithmic prices.

The results of Shalini and Prasanna (2016) show first, the Markov regime model shows that structural breaks are influenced by different regimes. Furthermore, the first regimes are longer than the second regimes for most of the commodities. Interestingly, it takes longer for most prices to converge to their long-term averages. One possible reason is that India uses most commodities; generally, prices of commodities should converge much quicker. For energy, structural breaks are evident during the 2008-2009 global crisis period except for natural gas. The same phenomenon is evident for both industrial and precious metals during the same period. However, according to Shalini and Prasanna (2016) industrial and precious metals have different convergence periods. Agricultural products are characterised by numerous structural breaks than most other commodities analysed in their study. The indices show evidence of the structural breaks.

Shalini and Prasanna (2016) then proceed to explore the volatility dynamics. They find that energy has higher persistence which increases to negative over time. Post crisis period, the results show that energy does not show any significant signs of persistence or a leverage effect. According to Shalini and Prasanna (2016), energy shocks are short lived. The results of the metals are mixed. They argue that this is because metals have different investment and industry uses. Moreover, some metals are substitutes while others have complimentary usage. Agricultural products have significant persistence levels except for castor oil which is consumed in large quantities in India. The highly traded and constituted indices such as S&P500 and Nifty 500 have high volatility persistence. Finally, long-term memory is not evident in most commodities.

Yin (2019) investigates parameter instability and model selection when forecasting equity premium in the equities markets. The study is based on Goyal and Welch (2008), which shows poor results for out-of-sample forecasts. Yin (2019) indicates that the poor results might be due to the uncertainty of model selection and parameter instability surrounding the data generation process. A stable linear regression model is used, $Y_t = \beta_0 + \beta_1 X_{t-1}$ where Y_t is the premium and the parameter vector β is subject to a one-time discrete break at an unknown time τ bounded from both ends of the sample. For the structural break detection, the study uses a combination of SupF-type tests in Andrews (1993), Andrews and Ploberger (1994) and Hansen (2001). Methods such as window estimations after detecting breaks are generally used to construct forecasts with estimated break dates; however, only observations detected after the latest structural break are used to train the predictive model. According to Yin (2019), break dates and size estimates are very likely to be imprecise, particularly when the sample size is small relative to the number of parameters. Yin (2019) proposes the use of the estimator in Pesaran et al. (2013); that, is, robust optimal weights to uniformly distribute break dates and avoid uncertainty regarding timing of parameter instability.

The empirical findings in Yin (2019) show that structural break tests reject the null hypothesis of no structural breaks and overall the structural break tests indicate that only a subset of linear predictive regression models are subject to parameter instability. The study concludes that structural breaks cannot fully explain the weak predictive performance for all forecasting models considered in Goyal and Welch (2008). Güloğlu et al. (2016) consider structural breaks when modelling volatility transmissions with the DCC GARCH model. Particularly, in a GARCH setting, the presence of structural breaks in conditional variance could lead to the overestimation of parameters and cause the sum of the estimated GARCH parameters to converge to one. The testing of the structural breaks in the conditional variance obtained from the GARCH for the DCC GARCH model is important as misleading results can distort the second estimation. A Kappa-2 ($k2$) statistic is used to test for structural breaks and is the best fit where there is non-normality and autoregressive conditional heteroscedasticity (ARCH) effects. Similar to the Bartlett kernel, a Kappa-2 statistic takes into consideration non-parametric estimators.

Zhou (2016) tests the two hypotheses for the leverage and volatility feedback effects in the real estate market for both linear and non-linear contexts. The econometric methodology applied considers the fact that the leverage effect is return driven whereas the volatility feedback effect is volatility driven. The study first uses a linear Granger causality test implemented in a VAR framework. In order to detect the nonlinear causality, the study follows the procedure in Baek and Brock (1992) who use a nonparametric setting. The findings from the linear Granger causality test indicate that both leverage and volatility feedback effects are present while the nonlinear test also indicates the same but in a nonlinear fashion. The study further examines nonlinearity causality and considers multiple sources such as regime switching, structural breaks and outliers. In testing for structural breaks, Zhou (2016) uses the Weighted Double maximum test in Qu and Perron (2007) which is different from other structural break tests in that the test can only be applied to single-equation regressions.

The findings in Zhou (2016) indicate that structural breaks are amongst the many different sources of linearity. On the other hand, the leverage and volatility feedback effects in the REIT market tend to be nonlinear. Moreover, the study finds that the leverage effect has the dominant casual effect. Liow et al. (2011) explore the importance of using a multiple-regime time-varying asymmetric variance and covariance approach in modelling real estate securities. They first use the methodology in Bai and Perron (2003) to identify multiple regime changes in the return and volatility series in REITs. Relative to other structural break tests, they favour the methodology in Bai and Perron (2003) as it allows for general specifications when calculating test statistics and confidence interval levels for the break dates and regression coefficients. These specifications include autocorrelations and heteroskedasticity in the regression coefficients as well as different matrices for the regressor in different regimes.

In addition, Liow et al. (2011) use a multivariate regime-dependant asymmetric dynamic covariance (MRDADC) model to detect the presence of mean-volatility linkages in real estate markets. Their findings indicate the presence of structural breaks in real estate markets and using the $SupF_{i,T}(l+1|l)$ statistics, no evidence of more than one break is found in the series. The MRDADC findings indicate that there are volatility-linkages across real estate markets which have further important implications for real estate valuations and portfolios.

3. Modelling

This study proposes the use of integral transforms (i.e. Fourier and Laplace) because first, there is flexibility found in integral transforms. Secondly, integral transforms capture time-varying intercepts in any absolute integrable function of time. Third, integral transforms capture sharp breaks, where standard structural break techniques fall short. Fourth, Fourier transforms are more global in their approach than being local focused. Finally, Fourier transforms capture all of the structural breaks, whether they are from low or high frequency data.

This study assumes that volatilities and returns in Fourier and Laplace transforms, respectively, are represented by an s parameter. The returns and volatilities are based on logarithms for continuity because suppose function (f) is continuous and piecewise smooth. Given that returns and volatilities are calculated based on logarithms, keeping them (i.e. returns and volatilities) in that state is ideal. Therefore, both $X(t)$ and $f(s)$ are e^{st}. The exponent (e^{t}) keeps functions in the logarithmic state. Note that in the e^{st}, time is not accounted for so that it does not influence structural breaks. Therefore, t is left out and e^{st} is simply written as e^{s}.The $f\colon \mathbb{R} \to \mathbb{R}$ is integrable over the real line $\mathbb{R}$, then the Fourier transform $\mathcal{F}[f(x)] := \hat{f}\colon \mathbb{R} \to \mathbb{R}$ of f is defined by:

$$\hat{f}(x) := \mathcal{F}[f(s)](x) := \int_{-\infty}^{+\infty} e^{-isx} f(s)d(s) \tag{1a}$$

$$:= \mathcal{F}[e^{s}](x) := -\sqrt{\pi}e^{s/2} \tag{1b}$$

Furthermore, the inverse Fourier transform is given by the following formula:

$$f(x) = \mathcal{F}^{-1}\big[\hat{f}(s)\big](x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \mathcal{F}(s)e^{isx}ds \tag{2a}$$

$$= s \quad \underbrace{\hat{f}(x)}_{\substack{Fourier \\ Transform}} \tag{2b}$$

The formula for the Laplace transform is:

$$\bar{X}(s) := \mathcal{L}[X(t)](s) := \int_{-\infty}^{+\infty} e^{-st}X(t)dt \tag{3a}$$

$$= \mathcal{L}[e^{at}](s) = \frac{1}{s-a} \tag{3b}$$

Also, the formula for the inverse Laplace is:

$$f(t) = \mathcal{L}^{-1}[\mathcal{F}(s)] = \mathcal{L}^{-1}\left[\frac{1}{s-a}\right] \tag{4a}$$

$$= \mathcal{L}^{-1}\left[\frac{1}{s-a}\right] = e^{st} = e^{s} \tag{4b}$$

where function $X(t)$ is defined for $t \geq 0$. Function $\bar{X}(s)$ is defined for all values such that the integral converges and the function is derived with respect to any variable of s. Eqs. (1b) and (2b) will be used to calculate the structural break points of returns, thus illustrating both algebraic and derivative structural breaks. Eqs. (3b) and (4b) will be used to exemplify structural break points of volatilities, thus illustrating both algebraic and derivative structural breaks. Note, it is assumed that structural breaks occur at the maturity of some event. Similar inferences are drawn by Gurrib (2008). Note that Fourier and Laplace transforms are derivatives while their inverses are algebraic formulas. In the context of this study, the inverses illustrate the structural break points while Fourier and Laplace transforms show the structural break point underpinned by the preceding structural break point. Note that for Eqs. (1b), (2b), (3b) and (4b) s is chosen such that the transforms converge. Sebehela (2014) uses the same approach. For formulas that have an a parameter like Eq. (3b), the a parameter is the long-term average given that both returns and volatilities converge to their long-term average (Straub and Werning 2020). Given that Eqs. (1b) and (3b) are Fourier and Laplace transforms, respectively, which are derivative formulas, and then the calculated solutions/answers will be forward looking. For example, solutions will predict future structural break points. Eqs. (2b) and (4b) are inverse Fourier and inverse Laplace transforms, respectively, which are algebraic formulas, and then the calculated solutions will be current events.

4. Data

The data in this study are based on four indices (i.e., bonds, commodities, equities and listed real estate) of the BRICS countries. According to Nayyar (2016), the similarities of the BRICS nations are that: (i) the political structure-ruling parties stay in power for at least 10 years without much challenge, although, we have recently seen the rise of opposition parties or citizens, (ii) the country governing-ruling elites combine free market and socialism; i.e., mixed economies (privatisation of government owned entities is extremely rare) and (iii) economic policies-ruling parties champion economic direction and by extension, the economies of the countries. Based on these similarities, there is a linkage between the political economy and capital markets (Haggard et al. 2008). However, connecting the relationship between political economy and structural break points is beyond the scope of this study.

The weekly data are for the five BRICS countries (bonds, commodities, general equities and real estate) for the period of 1 January 2007 to 31 December 2017 obtained from Bloomberg. The out-sample ranges from 2007 to 2017 and in-sample from 2012 to 2017. The study starts at 2007 because at that point, the data for all five countries are crisp. The use of weekly data ameliorates concerns over non-synchronicities and bid-ask effects in the daily data. The phenomenon of using returns to illustrate the descriptive nature of volatility spillovers is synonymous with Mensi et al. (2018). The returns are logarithm returns which are consistent with the VAR model. All returns are from the indices of the five countries. The indices are as follows. The general equities are Brazil IBRX 50 for Brazil, Moex Russian index for Russia, Nifty 50 for India, SSE50 for China and JSE top 40 index for South Africa. For the listed real estate, IMOB is used for Brazil and the index is created based on the PIKK Group of Russia. The PJSC LSR Group, World Trade Centre "ordinary shares" and World Trade Centre "preferred shares" because Russia does not have a listed real estate index. The market capitalisations of those firms were aggregated over time. For India, Nifty Realty is used, SHROP for China and All Property Index (J803) for South Africa. The commodities are as follows: BM&F BOVESPA for Brazil, MICEX Oil and Gas Index of the Moscow Exchange for Russia, Nifty Commodities for India, CCI for China and JCGMSAG (gold mining index) for South Africa. The bonds are based on Brazilian 8 7/8 04/15/24 bond for Brazil, Russia-RFLB 08/29/18 bond for Russia, India-Nifty 10yr benchmark for India, GTUSDCN15yr bond for China and SAGB 10 ½ 12/26 bond for South Africa. According to Skinzi and Refenes (2006), one of the advantages of modelling volatility shocks with indices is that shocks are captured as both endogenous and exogenous variables.

5. Analysis

5.1 Volatility Spillovers

In order to detect structural break points, this study first presents the behavioural patterns of the volatilities from four indices (i.e. bonds, commodities, equities and listed real estate) of the BRICS nations. The results of the spillovers have been largely illustrated and elaborated in Kola and Sebehela (2021); therefore, these are summarised versions of Kola and Sebehela (2020). The results of a Cholesky decomposition must be read jointly with the vector autoregressive [VAR(1,1)] model. The reason why Kola and Sebehela (2020) use the VAR model is that it is a multivariate model, where different variables/parameters are shown as one portfolio. In that setting, in-between and across patterns and/ or illustrations from various inputs are captured. Moreover, the spillovers (i.e. those that are in-between and across) are easily captured in the model. Third, the results from the VAR model are read in conjunction with the Cholesky decomposition. As a stand-alone technique, the Cholesky decomposition is more than just lower-upper decomposition for solving systems of linear equations (see Aghamiry et al.

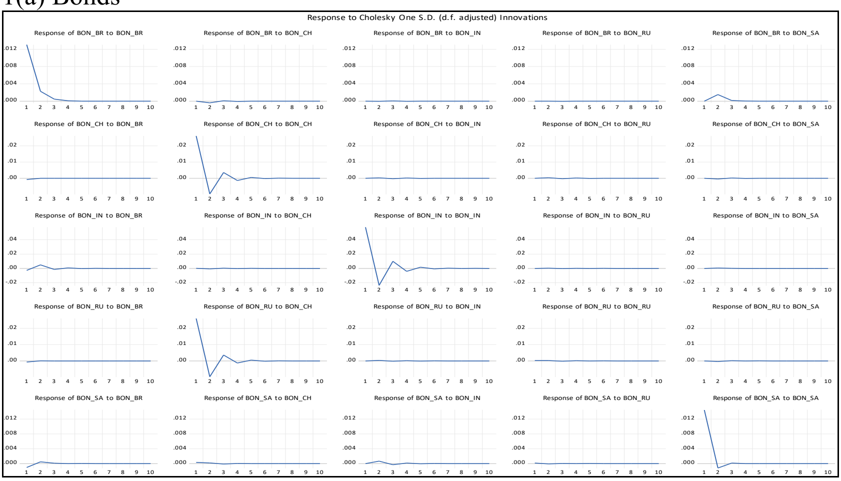
2021). The multiplicity and applicability of simultaneously using the VAR model with other techniques such as a Cholesky decomposition, gives much more insight into the results and better interprets the results.

The results in Figure 1 and Table 1 show that there are volatility spillovers. A possible reason for why there are spillovers might be the presence of structural break points. The spillovers are statistically significant as highlighted in grey for some of the indices of the BRICS nations. A salient point from Figure 1 and Table 1 is that reaction to shocks in one country emanate from lagged shocks in that country, and cause either positive and/or negative reactions, while shocks between different countries are largely positive. From the hedge funds narrative, positive shocks are more profitable than negative shocks. This would imply that transatlantic relationships should be encouraged as they have a positive effect on financial markets. Feng et al. (2019) show that despite the negative effects from internal shocks, they still motivate individuals to focus on opportunities.

The results of the in-sample (i.e., Figure 2 and Table 2) are indeed similar to those of the out-sample. Largely, the period of 2012 to 2019 was bullish except for the 2007/2009 subprime crisis. The close similarity between the in-sample and out-sample volatility spillovers confirm that forecasting performance, as illustrated by the out-sample (2007-2017) and estimation performance, as illustrated by the in-sample (2012-2017), is quite robust. Generally, it is very rare to find studies that confirm similarities of out-and-in sample results. Fundamentally, the issue of structural break points is likely to lead to volatile spillovers in this study.

Figure 1 Out-Sample Cholesky Decomposition

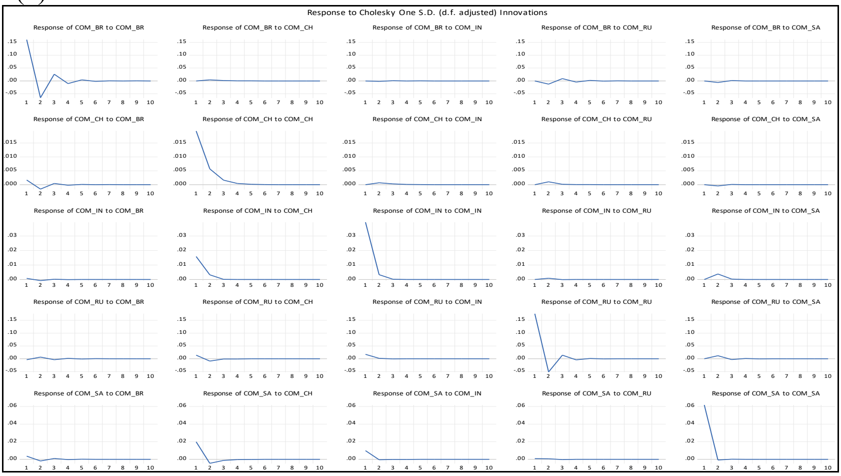
1(a) Bonds



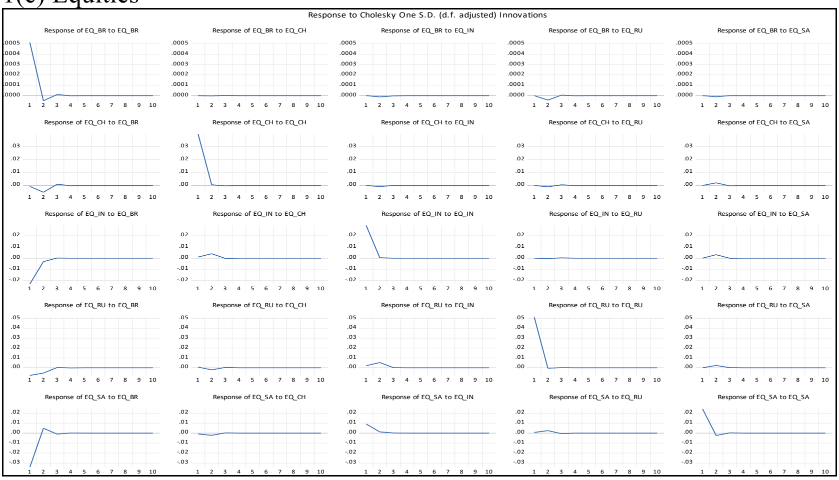
(Continued…)

(Figure 1 Continued)

1(b) Commodities



1(c) Equities



(Continued…)

(Figure 1 Continued)

1(d) Real Estate

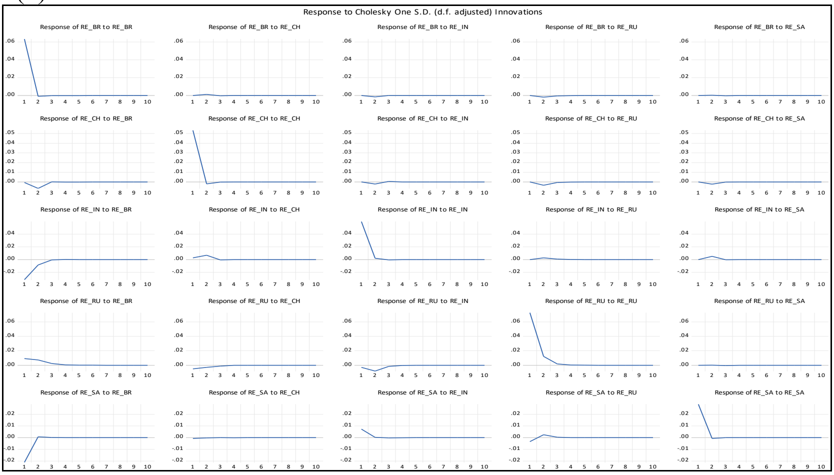


Table 1 VAR (1,1): Out-Sample Period (2007-2017)

Panel A: Bonds

Parameter	Brazil	China	India	Russia	South Africa
Brazil	0.1833	-0.0202	0.2881	-0.0201	0.0332
	(4.4095)	(-0.2429)	(1.5718)	(-0.24301)	(0.7153)
China	0.0077	1.7959	-0.8782	-1.4175	0.2652
	(0.0038)	(-0.4477)	(-0.0993)	(-0.3533)	(0.1185)
India	-0.0004	0.0050	-0.4140	0.0049	0.0114
	(-0.0440)	(0.2812)	(-10.5584)	(0.2801)	(1.1494)
Russia	-0.0226	1.4237	0.8546	1.0447	-0.2573
	(-0.0113)	(0.3549)	(0.0966)	(0.2605)	(-0.1150)
South Africa	0.1055	-0.0280	0.0383	-0.0280	-0.0003
	(2.7839)	(-0.3701)	(0.2292)	(-0.3697)	(-1.9232)
F-Statistic	5.2479	17.9005	23.0227	17.9331	1.1701
Akaike information criterion	-5.8292)	-4.4438	-2.8618	-4.4433	-5.6106
Schwarz criterion	-5.7830	-4.3973	-2.8157	-4.3907	-5.5643

(Continued…)

(Table 1 Continued)

Panel B: Commodities					
Parameter	Brazil	China	India	Russia	South Africa
Brazil	-0.4092	-0.0125	-0.0063	0.0339	-0.0089
	(-10.5168)	(-2.6652)	(-0.6138)	(0.7887)	(-0.5621)
China	0.3609	0.2844	0.0436	-0.5516	-0.2129
	(0.9718)	(6.3448)	(0.4442)	(-1.3461)	(-1.4044)
India	0.0028	0.0159	0.0670	0.1208	-0.0092
	(0.0162)	(0.7539)	(1.4443)	(0.6232)	(-0.1288)
Russia	-0.0701	0.0059	0.0048	-0.2919	0.0033
	(-1.9894)	(1.3848)	(0.5099)	(-7.5058)	(0.2270)
South Africa	-0.0943	-0.0078	0.0627	0.1935	-0.0114
	(-0.8626)	(-0.5876)	(2.1706)	(1.6049)	(-0.2545)
F-Statistic	22.7883	11.7035	2.3085	12.2434	0.7016
Akaike information criterion	-0.8059	-5.0351	-3.4670	-0.6091	-2.5984
Schwarz criterion	-0.7597	-4.9888	-3.4208	-0.5629	-2.5522

Panel C: Equities					
Parameter	Brazil	China	India	Russia	South Africa
Brazil	-0.1469	-7.4597	1.2495	2.5543	7.1695
	(-2.0726)	(-1.3644)	(0.2433)	(0.3575)	(1.2132)
China	0.0000	0.0178	0.1028	-0.0552	-0.0616
	(-0.0920)	(0.4235)	(2.5982)	(-1.0039)	(-1.3525)
India	-0.0002	-0.0531	-0.0257	0.1499	0.0724
	(-0.3129)	(-0.8988)	(-0.4625)	(1.9421)	(1.1336)
Russia	-0.0008	-0.0213	-0.0082	-0.0094	0.0520
	(-2.0001)	(-0.6589)	(-0.2684)	(-0.2233)	(1.4883)
South Africa	-0.0004	0.0826	0.1263	0.0982	-0.0959
	(-0.4955)	(1.2342)	(2.0106)	(1.1239)	(-1.3261)
F-Statistic	1.9912	2.5000	2.7793	2.7791	2.7064
Akaike information criterion	-12.2983	-3.6080	-3.7334	-3.0728	-3.4525
Schwarz criterion	-12.2520	-3.5618	-3.6871	-3.0266	-3.4062

(Continued…)

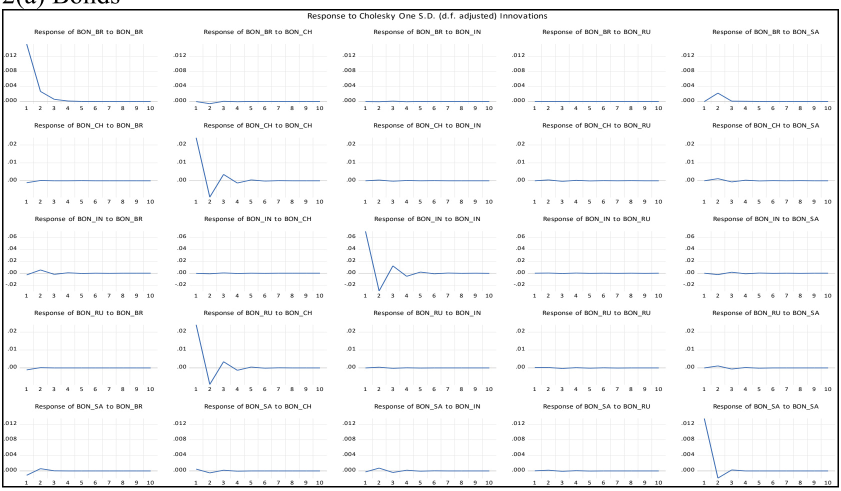
(Table 1 Continued)

Panel D: Real Estate					
Parameter	Brazil	China	India	Russia	South Africa
Brazil	-0.0197	-0.1362	-0.0720	0.0276	0.0031
	(-0.3642)	(-2.9988)	(-1.2489)	(0.4412)	(0.1009)
China	0.0236	-0.0399	0.1344	-0.0325	-0.0011
	(0.4708)	(-0.9456)	(2.5099)	(-0.5584)	(-0.0391)
India	-0.0259	-0.0286	0.0162	-0.1256	0.0097
	(-0.5738)	(-0.7487)	(0.3345)	(-2.3889)	(0.3717)
Russia	-0.0229	-0.0504	0.0466	0.1679	0.0332
	(-0.6359)	(-1.6582)	(1.2075)	(4.0062)	(1.6033)
South Africa	0.0107	-0.0780	0.1783	0.0010	-0.0233
	(0.1145)	(-0.9927)	(1.7871)	(0.0093)	(-0.0449)
F-Statistic	0.2134)	2.6753	3.8118	5.8949	0.6335
Akaike information criterion	-2.6706	-3.0149	-2.5382	-2.3731	-3.7809
Schwarz criterion	-2.6244	-2.9687	-2.4919	-2.3269	-3.7347

Notes: In each cell, the first number is the coefficient and the number in brackets is the t-test. All variables highlighted in grey are statistically significant for the VAR values as they are at least 2 irrespective of being negative or positive. The VAR results should be read in conjunction with the Cholesky decomposition as illustrated in Figure 1.

Figure 2 In-Sample Cholesky Decomposition

2(a) Bonds



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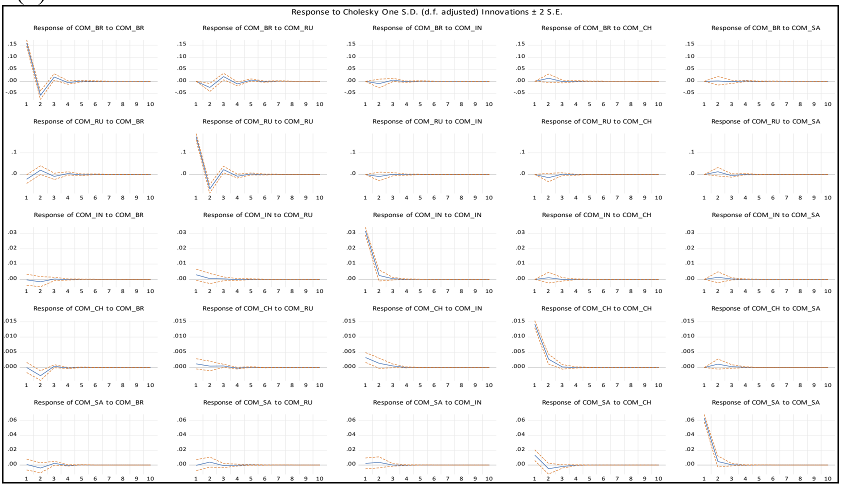
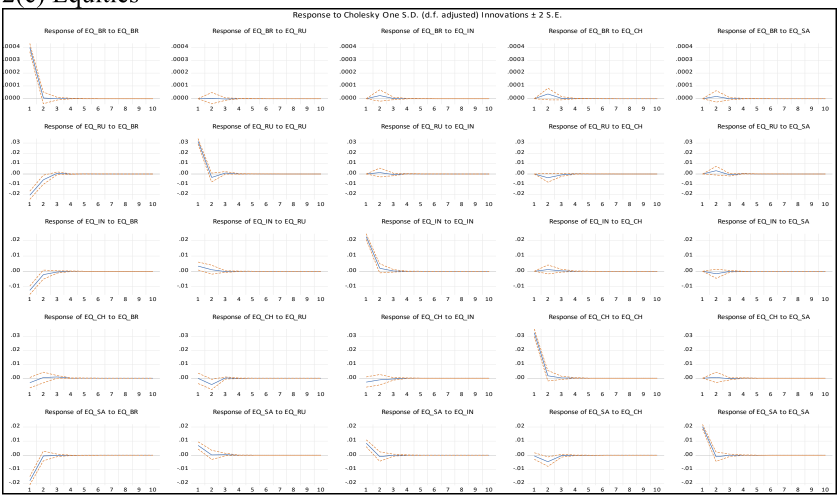
(Figure 2 Continued)

2(b) Commodities



2(c) Equities



(Continued…)

(Figure 2 Continued)

2(d) Real Estate

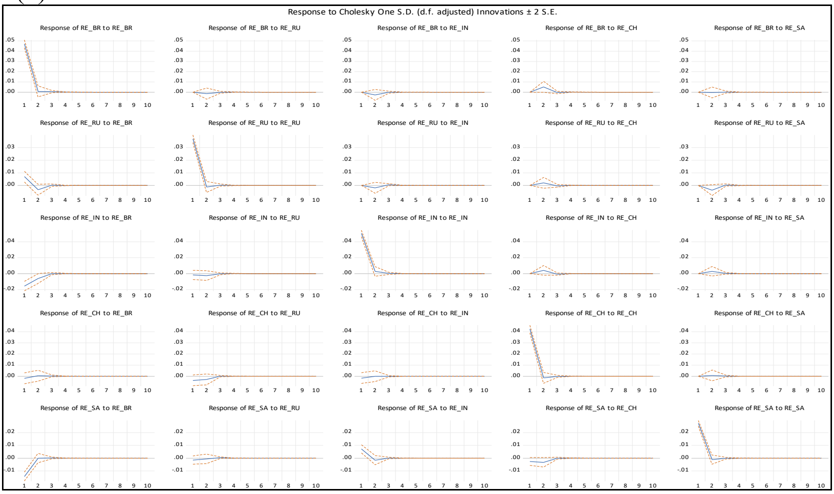


Table 2 VAR (1,1): In-Sample Period (2012-2017)

Panel E: Bonds

Parameter	Brazil	China	India	Russia	South Africa
Brazil	0.1876	-0.0120	0.2688	-0.0121	0.0278
	(3.8632)	(-0.1567)	(1.1956)	(-0.1578)	(0.6435)
China	-0.0817	-2.0308	-1.6449	-1.5335	-0.7866
	(-0.0304)	(0.4779)	(-0.1322)	(-0.3608)	(-0.3289)
India	0.0001	0.0056	-0.4205	0.0056	0.0099
	(0.0114)	(0.3399)	(-8.7141)	(0.3386)	(1.0653)
Russia	0.0553	1.6457	1.6068	1.1477	0.7689
	(0.0259)	(0.3874)	(0.1292)	(0.2701)	(0.3216)
South Africa	0.1664	0.08445	-0.1679	0.0837	-0.1354
	(3.0025)	(0.9643)	(-0.6544)	(0.9552)	(-2.7462)
F-Statistic	4.5645	14.0761	15.7002	14.1098	2.0675
Akaike information criterion	-5.5172	-4.6011	-2.4522	-4.6006	5.7506
Schwarz criterion	-5.4585	-4.5423	-2.3935	-4.5419	-5.6919

(Continued…)

(Table 2 Continued)

Panel F: Commodities					
Parameter	Brazil	China	India	Russia	South Africa
Brazil	-0.3855	-0.0166	-0.0097	0.0746	(-1.0249)
	(-7.4290)	(3.4736)	(-0.9339)	(1.2989)	
China	0.8517	0.1761	0.0542	-1.2171	(-1.6324)
	(1.3579)	(3.0423)	(0.4329)	(-1.7521)	
India	-0.3983	0.0242	0.0728	-0.1864	(1.2772)
	(-1.3512)	(0.8910)	(1.2363)	(-0.5711)	
Russia	-0.1453	0.0010	0.0015	-0.3677	(1.1867)
	(-3.0541)	(0.2295)	(0.1561)	(-6.9804)	
South Africa	0.0280	0.0177	0.0216	0.1967	(1.2906)
	(0.1981)	(1.3538)	(0.7648)	(1.2551)	
F-Statistic	12.6282	5.3503	0.7579	11.8936	0.0651
Akaike information criterion	-0.8229	-5.5888	-4.0445	-0.6187	-2.6074
Schwarz criterion	-0.7506	-5.5165	-3.9722	-0.5464	-2.5350
Panel G: Equities					
Parameter	Brazil	China	India	Russia	South Africa
Brazil	0.0795	-6.1630	-3.1423	-15.9395	-4.7373
	(1.0222)	(-0.9579)	(-0.6267)	(-2.1685)	(-0.8339)
China	0.0011	0.0537	0.0342	-0.1100	-0.1395
	(1.6271)	(0.9391)	(0.7611)	(-1.6849)	(-2.7636)
India	0.0009	-0.0566	0.1208	-0.0125	-0.0386
	(0.8208)	(-0.6371)	(1.7324)	(-0.1229)	(-0.4921)
Russia	-0.0002	-0.1442	0.0363	-0.1418	0.0223
	(-0.2543)	(-2.3803)	(0.7626)	(-2.0488)	(0.4165)
South Africa	0.0009	0.0287	-0.0790	0.1526	-0.0512
	(0.7747)	(0.3102)	(-1.0876)	(1.4447)	(-0.6272)
F-Statistic	0.8648	1.5195	1.2508	3.1221	1.6589
Akaike information criterion	-12.7912	-3.9589	-4.4415	-3.6926	-4.2078
Schwarz criterion	-12.7188	-3.8867	-4.3692	-3.6203	-4.1355

(Continued…)

(Table 2 Continued)

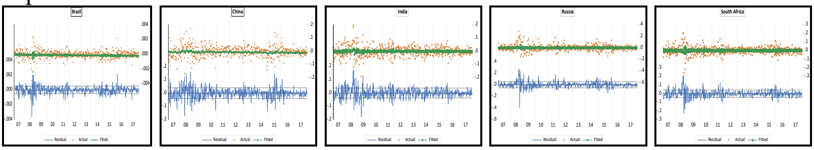
Panel H: Real Estate					
Parameter	Brazil	China	India	Russia	South Africa
Brazil	0.0041	0.0284	-0.0729	-0.1126	-0.0207
	(0.0635)	(0.4825)	(-0.9954)	(-2.1609)	(-0.4699)
China	0.1213	-0.0306	0.1037	0.0393	-0.0794
	(1.8983)	(-0.5308)	(1.4455)	(0.7707)	(-1.8378)
India	-0.0482	-0.0046	0.0426	-0.0174	-0.0282
	(-0.8873)	(-0.0943)	(0.6990)	(-0.4013)	(-0.7678)
Russia	-0.0309	-0.0771	-0.0512	-0.0362	-0.0266
	(-0.4209)	(-1.1644)	(-0.6213)	(-0.6179)	(-0.5363)
South Africa	-0.0128	0.0293	0.1061	-0.1355	-0.0415
	(-0.1304)	(0.3308)	(0.9652)	(-1.7336)	(-0.6271)
F-Statistic	1.0215	0.3552	0.0529	1.5103	0.8958
Akaike information criterion	-3.2510	-3.4551	-3.0200	-3.7027	-4.0345
Schwarz criterion	-3.1787	-3.3828	-2.9477	-3.6305	-3.9622

Notes: In each cell, the first number is the coefficient and the number in brackets is the t-test. All variables highlighted in grey are statistically significant for the VAR values as they are at least 2 irrespective of being negative or positive. The VAR results should be read in conjunction with the Cholesky decomposition as illustrated in Figure 2.

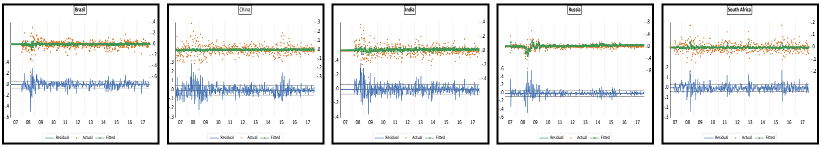
For every index type in every row of Figure 3, the first country is Brazil followed by China and then India, and subsequently Russia. The last country is always South Africa (SA). For equity indices, all five countries experienced main shocks in the 2007-2008 period, as illustrated by the residuals. For all of the indices of the BRICS nations, volatility spillovers are sensitive to regime changes and/or switching effects. The latter point probably confirms that one possible reason for regime changes might be structural break points. Interestingly, liquid indices stay longer in one regime than illiquid indices for most of the BRICS nations. For more on these findings, see Kola and Sebehela (2020). The mentioned pattern goes against a generally held view on stock market reactions: first, currencies, then commodities; thereafter, equities, followed by bonds and finally, listed real estate. One possible reason for the contradicting finding might be that illiquid indices offer more value and hedging advantages during changing of regimes of indices. The latter point is left for future research.

Figure 3 Filtered Regime Probabilities-Out Sample: 2007-2017

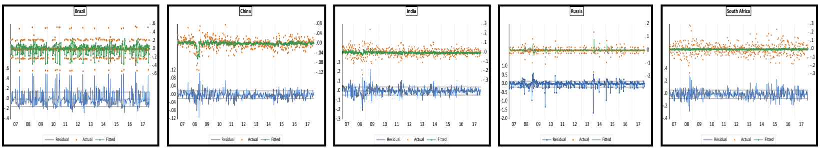
Equities



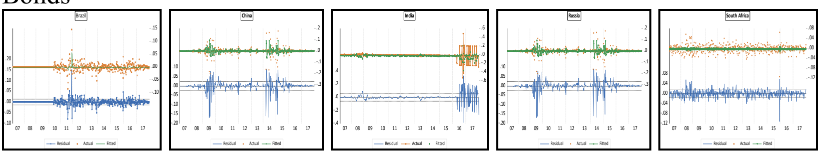
Real Estate



Commodities



Bonds



5.2 Structural Break Points

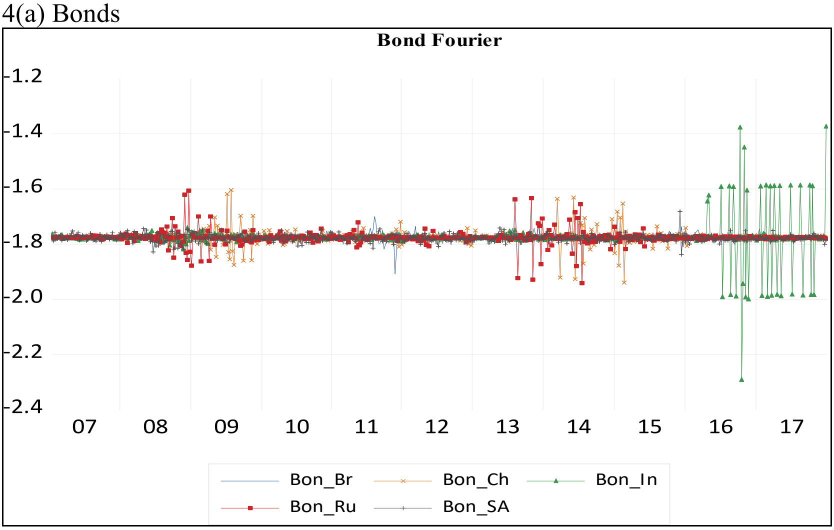
For structural breaks, the breaks based on returns are calculated based on the out-and-in sample sizes, while volatility breaks are based only on the in-sample size; see Figure 3. This is because volatilities are calculated based on returns. Moreover, a reasonable period of a data set is necessary. Thus, the period is 6 years from 2007 to 2012 on a weekly basis. This provides 312 data points for calculating the volatilities. In principle, volatilities are for the in-sample period. However, volatilities are not considered for the period of 2007 to 2011 because they should be calculated based on returns prior to the 2007 period. However, that will be make those volatilities incomparable.

For each plot, Bon denotes bonds, Com is commodity, Eq is equity and RE is real estate. Furthermore, Br stands for Brazil, Ch for China, In for India, Ru for Russia and SA for South Africa. Evident break points imply that structural break points as per integral transforms can be easily picked up. Hidden points in this case mean that those structural points are not easily picked up; however, integral transforms illustrate them, albeit not so easily. Broadly, there is an interconnection between the structural break points irrespective whether those points are on an upward and/or downward trajectory. Interestingly, this can be inferred that there are technical recessions and expansions that are invisible to market commentators and policy makers. This might imply that some policies and/or strategies incorporated a poor understanding of unfolding in stock markets. Results for every formula based on each index are synthesised in the respective tables below. The salient points in the tables are summarised versions of the graphs.

Given that Fourier transforms generate derivatives, this implies that Fourier transforms of bonds, commodities, equities and listed real estate symbolise casual effects, as shown in Figure 4. During the period of 2007-2017, the Brazilian bond market had small proceeds and short maturity according to market commentators. Moreover, during that time, less than half of the bonds were investment-grade and at least 50% were not traded at all. Those listed trades should cause movements in bonds. Evident structural break points were in 2008, 2011, 2013 and 2017 while hidden points were at different points between 2007 to 2017. There are more hidden break structural points than evident ones. This might be because Brazil had good relations with many Western countries before the election of President Jair Bolsonaro in 2019. The listed real estate market of Brazil has the same pattern as that of the bonds in terms of break point sequence. Nevertheless, the evident break points equal to the hidden break points. It can be inferred from Okunev et al. (2000) that real estate has stable patterns. Among the macroeconomic factors that contribute to structural break points (i.e. evident and hidden), sovereign borrowing is included according to the OECD Sovereign Borrowing Outlook 2017.

Figure 4 [1] Fourier Transform

4(a) Bonds



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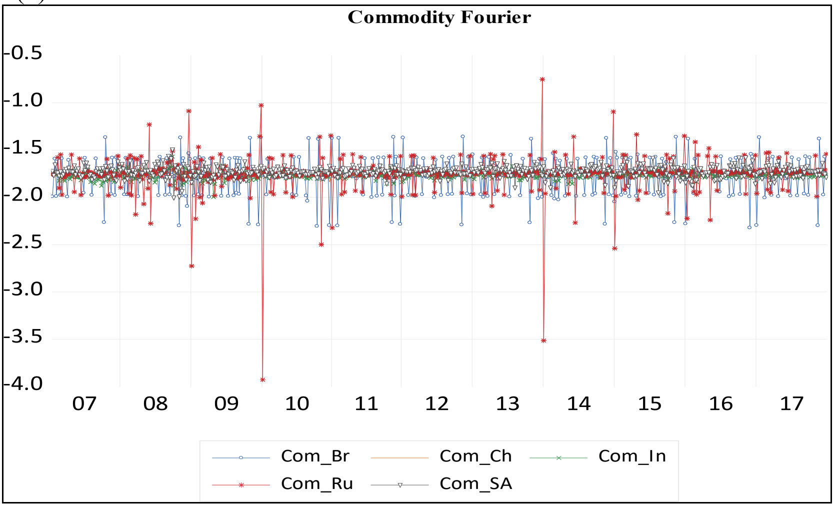
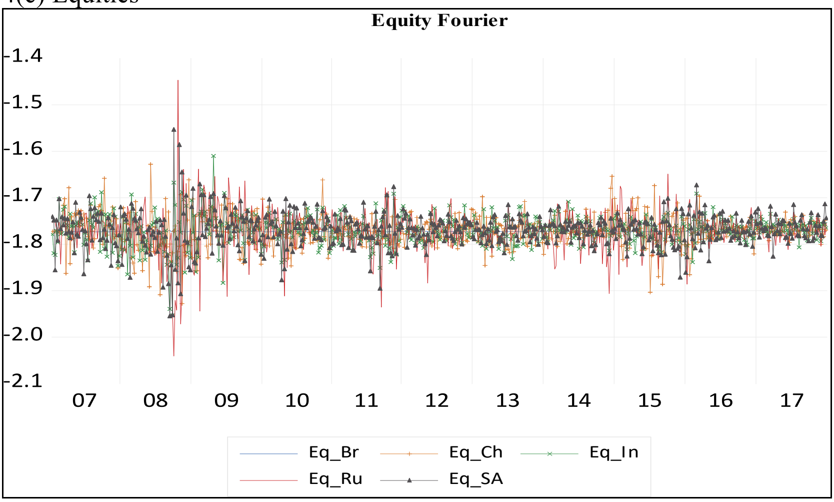
(Figure 4 Continued)

4(b) Commodities



4(c) Equities



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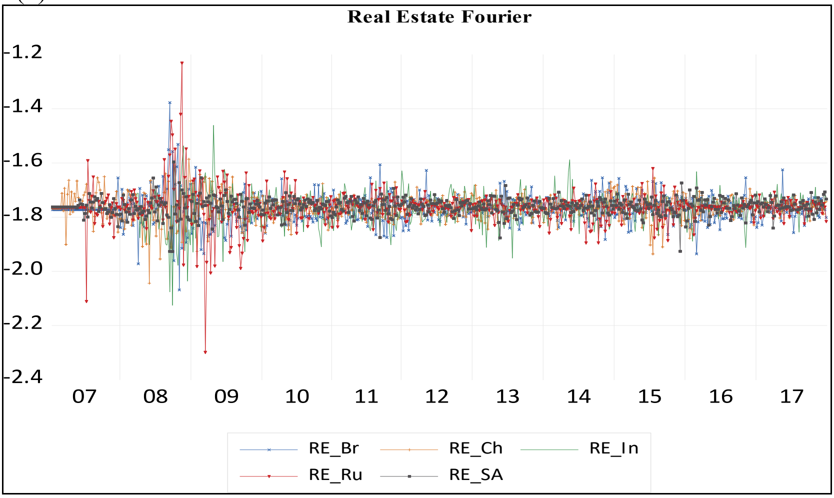
(Figure 4 Continued)

4(d) Real Estate



According to market commentators, Russia experienced a recession during 2008/2009. Given that its economy is highly dependent on commodities prices, the effects of the global subprime crisis were evident in Russia. The Russian state divested in thousands of Russian firms. During that time, both the bond market and Russian ruble collapsed. During 2011, market commentators state that Russia held the lowest $\frac{debt}{gross\ domestic\ product\ (GDP)}$ among most emerging markets. In 2016, Russia started to issue short-term debt paper that matures in a month. In 2008, 2011, 2015 and 2016, structural break points were evident. However, during 2007, 2009, 2010, 2012, and 2015 and including periods of evident break points, there were hidden structural break points. It seems that evident and hidden structural break points are interrelated; to the best of our knowledge, this is the first article to show interrelated structural break points.

During 2007 to 2017, it seems the Russian interest rates, budget deficit, GDP growth, inflation, exchange rate movements, interbank interest rates, repo rate and political environment primarily affected the Russian economy. It is interesting that Fourier transform can capture structural break points caused by different macroeconomic variables. The listed real estate index for Russia illustrates similar structural break points as those of the bond index. Numerous previous studies have illustrated that listed real estate mimics the bond market due to similarities in their traits: (i) convexity, (ii) sensitivity to the same term structures and (iii) yield patterns.

The Indian bond market has traded at low yields during the period of 2007-2017 and hovered around 5%. Akram and Das (2019) illustrate that Indian bonds are influenced by interest rates and monetary policy. Moreover, they find that government policy is influenced by government fiscal variables (Akram and Das, 2019; page 168). Just like Brazil, there are more hidden than evident structural break points. This is probably due to the influence of political parties on the economy.

For the real estate market, there are more evident than hidden structural break points. Both can be spotted throughout 2007-2017. India uses nominal values in doing property valuations (Abidoye and Chan 2017). It is well known that nominal values do not necessarily reflect real values in property markets. The latter statement might explain for some of the structural break points (evident and hidden). Note that for the Indian bonds, there is one evident break point and the rest are hidden points. For Indian real estate, there are numerous evident and hidden points. However, evident break points outnumber hidden break points.

For the Chinese bond market, there are more evident than hidden structural break points. China is known to have purchased the largest quantity of bond assets and some of those are denominated in U.S. dollars. This might lead to the influence of China in the global bond market. In terms of the 2008/2009 global financial crisis, its impact on China was not as severe as on the Western countries.

According to market commentators, China has become a major contributor to global trade and product integration in the global bond market. The Chinese real estate market is integral to the financial system of China. Therefore, their real estate market partly explains why break points (i.e. evident and hidden) of bonds and real estate are similar. At the heart of Chinese property is the housing market, which is influenced by the government. South Africa is largely a net importer of skills and services because of its history. Therefore, its bond yields are relatively high when compared to other emerging countries. Moreover, market commentators tend to advocate government intervention which should lead to everyone benefiting from the economic growth.

The equity markets of the BRICS have been volatile throughout 2007-2017. For South Africa, its equities move in tandem with the European markets. Given that Europe felt the 2008/2009 subprime crisis effect, so did South Africa, even though the effects were minimal in South Africa. Brazil, Russia, China and India are among the major consumers of equity products. Furthermore, some of the major corporates from those countries have dual listings in the developed countries. For all of the BRICS countries except for Brazil, hidden structural break points outnumber evident break points. Overall, there are more structural break points shown in equities than those in bonds and/or real estate indices. The commodities market shows some interesting patterns. There are virtually no evident structural break points for the commodities consumers (i.e. India and China), while both evident and hidden points are there for the commodities producers (i.e. Brazil, Russia and South Africa). Brazil and Russia have more evident than hidden points, while South Africa has more hidden than evident break points. This might be due to Russia and Brazil having a more sizable economy than South Africa. Table 3 presents the structural break dates based on the Fourier transform. Moreover, commodities including oil and gas are highly influenced by government policies.

The inverse Fourier transform gives rise to linear equations, which implies that the results of inverse Fourier transforms show the impact of each variable in a given situation. The results of the inverse Fourier transform diagrams and structural break dates are shown in Figure 5 and Table 4, respectively. The inverse Fourier transform for bond indices of the BRICS countries shows the opposite of the Fourier transforms for their bond indices. First, there are more evident than hidden structural break points, except for Brazil and South Africa as both countries are based in the Southern hemisphere.

Secondly, for Russia, India and China, evident break points exceed hidden break points. It is the opposite for Brazil and South Africa. The real estate indices for the five countries exhibit some patterns for the bonds. Similarly, the evident break points outnumber the hidden points for Brazil and China. Thus, inverse Fourier transforms for bonds show similar patterns to the inverse Fourier transform for real estate. The equities of the BRICS show that India, China and South Africa have more hidden than evident break points. Brazil has no evident break points and Russia has more evident than hidden points. Thus, the results of the equities show mixed patterns. This could be due the fact that equities are the largest asset classes, at least in the BRICS countries.

For commodities countries, India, China and South Africa have no evident break points but have hidden break points. For South Africa, this is surprising given that the country is a major contributor to the commodities market. One notable thing about South Africa is that despite its massive contribution to commodities globally, the country is not involved in setting commodities prices. Moreover, South Africa does not have a seat on any organisation that sets prices of any commodity in the world. South Africa should become involved in setting commodities prices going forward. Given that the indices used in the inverse Fourier transform are the same ones used in the Fourier transform, this study assumes that the earlier stated macroeconomic variables have a casual effect on inverse Fourier transforms. One should note the following in Laplace and inverse Laplace transforms. Index returns use Fourier transform while index volatilities use Laplace transform. Naturally, stock prices change, leading to changes in returns and then volatilities. To put it simply, changes in stock returns should supersede changes in stock volatilities. By extension, most of the impact should be felt in returns than volatility. Laplace transforms will either confirm or disconfirm this logical thinking.

Table 3 **Fourier Transform**

Parameter	**Country**	**Brazil**	**Russia**	**India**	**China**	**South Africa**
Bonds	**Evident Break Point**	23rd Jun 2008, 29th Sep 2008, 17th Nov 2008, 25th Jul 2011, 15th Aug 2011, 3rd Oct 2011, 28th Nov 2011, 10th Jun 2013, 7th Oct 2013, 7th Dec 2015, 14th Dec 2015, 14th Mar 2016, 22nd Aug 2016, 7th Nov 2016 and 27th Mar 2017	30th Apr 2007, 6th Aug 2007, 1st Oct 2007, 4th Feb 2008, 30th Jun 2008, 27th Oct 2008, 22nd Dec 2008, 27th Dec 2010, 16th Jul 2012, 19th Jan 2015, 7th Dec 2015, 14th Dec 2015, 2nd May 2016, 4th Jul 2016, 15th Aug 2016, 5th Sep 2016, 10th Oct 2016, 31st Oct 2016, 14th Nov 2016, 14th Nov 2016, 23rd Jan 2017, 20th Feb 2017, 3rd Apr 2017, 1st May 2017, 26th Jun 2017, 14th Aug 2017, 2nd Oct 2017 and 25th Dec 2017	28th Nov 2011	10th Mar 2008, 19th May 2008, 23rd Jun 2008, 27th Oct 2008, 22nd Dec 2008, 4th May 2009, 11th May 2009, 6th Jul 2009, 20th Jul 2009, 14th Sep 2009, 28th Sep 2009, 9th Nov 2009, 16th Nov 2009, 20th Jun 2011, 5th Dec 2011, 19th Dec 2011, 26th Nov 2011, 12th Mar 2012, 10th Dec 2012, 21st Jan 2013, 17th Mar 2014, 31st Mar 2014, 9th Jun 2014, 16th Jun 2014, 4th Aug 2014, 1st Sep 2014, 6th Oct 2014, 22nd Dec 2014, 5th Jan 2015, 26th Jan 2015, 23rd Feb 2015, 20th Jul 2015, 10th Aug 2015, 5th Oct 2015, 4th Jan 2016 and 11th Jan 2016	23rd Jun 2008, 22nd Sep 2008, 17th Nov 2008, 12th Sep 2009, 17th Jun 2013, 9th Sep 2013, 12th Jan 2015, 2nd Mar 2015, 7th Dec 2015, 14th Dec 2015, 22nd Aug 2016, 7th Nov 2016 and 27th Mar 2017

(Continued…)

(Table 3 Continued)

Parameter	Country	Brazil	Russia	India	China	South Africa
Bonds	**Hidden Break Point**	14th May 2007, 6th Aug 2007, 19th Nov 2007, 28th Apr 2008, 16th Dec 2008, 15th Jun 2009, 26th Oct 2009, 26th Apr 2010, 26th Jul 2010, 25th Oct 2010, 28th Mar 2011, 12th Mar 2012, 27th Aug 2012, 19th Nov 2012, 26th May 2014, 26th Jan 2015, 14th Sep 2015, 13th Jun 2016, 3rd Jul 2017, 30th Oct 2017 and 18th Dec 2017	19th May 2008, 23rd Dec 2008, 15th Jun 2009 and 7th Mar 2016	26th Feb 2007, 28th May 2007, 17th Sep 2007, 14th Jan 2008, 11th Jan 2010, 11th Jul 2011, 27th Feb 2012, 21st May 2012, 10th Aug 2012, 1st Jul 2013, 25th Nov 2013, 22nd Sep 2013, 22nd Sep 2014, 4th May 2015, 31st Aug 2015, 11th Jan 2016 and 19th Dec 2016	21st May 2007, 20th Aug 2007, 12th Nov 2007, 17th Jan 2011, 28th Mar 2011, 26th Apr 2010, 30th May 2016, 29th Aug 2016 and 25th Dec 2017	29th Jan 2007, 19th Mar 2007, 30th Apr 2007, 9th Jul 2007, 30th Jul 2007, 24th Sep 2007, 31st Mar 2008, 19th May 2008, 8th Jun 2009, 29th Sep 2009, 12th Apr 2010, 26th Jul 2010, 26th Sep 2011, 2nd Jul 2012, 11th Nov 2013, 2nd Jun 2014, 24th Aug 2015, 20th Jun 2016, 10th Jul 2017, 16th Oct 2017 and 18th Dec 2017

(Continued…)

(Table 3 Continued)

Parameter	Country	Brazil	Russia	India	China	South Africa
Equities	**Evident Break Point**	17th Dec 2007, 14th Jan 2008, 21st Jan 2008, 18th Feb 2008, 14th Apr 2008, 19th May 2008, 21st Jul 2008, 27th Oct 2008, 26th Jan 2009, 23rd Feb 2009, 29th Jun 2009, 15th Oct 2009, 19th Apr 2010, 4th Oct 2010, 25th Oct 2010, 17th Jan 2011, 25th Jul 2011, 12th Sep 2011, 7th Nov 2011, 5th Mar 2012, 2nd Apr 2012, 7th May 2012, 26th Jan 2012, 27th May 2013, 4th Nov 2013, 20th Jan 2014, 8/9/2014, 26th Jan 2015, 16th Mar 2015, 29th Feb 2016 and 16th May 2016	16th Jul 2007, 13th Aug 2007, 29th Oct 2007, 3rd Mar 2008, 18th Aug 2008, 22nd Sep 2008, 17th Nov 2008, 8th Dec 2008, 16th Mar 2009, 13th Apr 2009, 18th May 2009, 15th Jun 2009, 6th Jul 2009, 12th Oct 2009, 1st Feb 2010, 30th Aug 2010, 1st Aug 2011, 24th Dec 2012, 18th Mar 2013, 29th Apr 2013, 16th Dec 2013, 3rd Feb 2014, 3rd Mar 2014, 9th Jun 2014, 11th Aug 2014, 29th Sep 2014, 13th Oct 2014, 17th Nov 2014, 20th Jul 2015, 14th Sep 2015, 9th Nov 2015, 19th Dec 2016, 6th Feb 2017, 22nd May 2017 and 26th Jun 2017	22nd Jan 2007, 19th Feb 2007, 20th Aug 2007, 26th Nov 2007, 3rd Mar 2008, 14th Apr 2008, 12th May 2008, 10th Jun 2008, 30th Jun 2008, 27th Apr 2009, 22nd Jun 2009, 21st Nov 2011, 9th Jan 2012, 19th Nov 2012, 4th Mar 2013, 15th Apr 2013, 29th Jul 2013, 23rd Sep 2013, 16th Dec 2013, 12th May 2014, 2nd Jun 2014, 21st Jul 2014, 20th Oct 2014, 18th May 2015, 14th Sep 2015, 14th Mar /2016 and 9th Oct 2017	9th Apr 2007, 21st May 2007, 3rd Sep 2007, 8th Oct 2007, 11th Feb 2008, 2nd Jun 2008, 7th Jul 2008, 28th Jul 2008, 22nd Dec 2008, 7th Jun 2010, 7th Jun 2010, 1st Nov 2010, 15th Nov 2010, 28th Nov 2011, 19th Mar 2012, 21st Jan 2013, 6th May 2013, 15th Dec 2014, 23rd Feb 2015, 22nd Jun 2015, 20th Jul 2015, 7th Sep 2015 and 14th Mar 2016	25th Jun 2007, 23rd Jul 2007, 14th Dec 2007, 28th Jan 2008, 11th Aug 2008, 6th Oct 2008, 29th Sep 2008, 3rd Nov 2008, 17th Nov 2008, 12th Jan 2009, 9th Mar 2009, 22nd Jun 2009, 13th Jul 2009, 14th Sep 2009, 14th Dec 2009, 18th Dec 2010, 20th Jun 2011, 30th Jan 2012, 27th Aug 2012, 8th Oct 2012, 8th Apr 2013, 10th Feb 2014, 21st Apr 2014, 8th Dec 2014, 26th Jan 2015, 16th Mar 2015, 17th Aug 2015, 5th Oct 2015, 22nd Feb 2016, 13th Mar 2017, 10th Jul 2017 and 14th Aug 2017

(Continued…)

(Table 3 Continued)

Parameter	Country	Brazil	Russia	India	China	South Africa
Equities	**Hidden Break Point**	29th Oct 2007, 1st Mar 2009, 21st Mar 2011, 6th Jun 2011, 9th Jan 2012, 6th Feb 2012, 9th Jan 2012, 6th Feb 2012, 14th Jul 2014, 28th Jul 2014, 8th Dec 2014, 27th Apr 2015, 8th Aug 2016, 2nd Jan 2017, 30th Jan 2017, 25th Sep 2017 and 27th Nov 2017	12th May 2008, 21st Jul 2008, 12th Jan 2009, 16th Feb 2009, 24th Aug 2009, 26th Jul 2010, 27th Sep 2010, 27th Dec 2010, 7th Mar 2011, 2nd Mar 2011, 27th Jun 2011, 22nd Aug 2011, 12th Sep 2011, 3rd Oct 2011, 31st Oct 2011, 5th Dec 2011, 30th Jan 2012, 30th Apr 2012, 24th Sep 2012, 8th Jul 2013, 4th Nov 2013, 2nd Dec 2013, 15th Dec 2014, 9th Feb 2015, 23rd Mar 2015, 18th Jul 2015, 4th Sep 2017 and 11th Dec 2017	14th May 2007, 19th Jan 2009, 14th Sep 2009, 25th Jan 2010, 26th Apr 2010, 13th Feb 2012, 26th Mar 2012, 21st May 2012, 30th Jul 2012, 24th Sep 2012, 3rd Dec 2012, 20th May 2013, 13th Jan 2014, 21st Jul 2014, 23rd Feb 2015, 11th Jul 2016, 26th Sep 2016, 20th Feb 2017 and 22nd May 2017	21st Jan 2008, 6th Apr 2009, 20th Jul 2009, 21st Sep 2009, 23rd Nov 2009, 11th Jan 2010, 20th Feb 2012, 13th Aug 2012, 3rd Sep 2012, 22nd Oct 2012, 4th Nov 2013, 10th Feb 2014, 17th Mar 2014, 6th Apr 2015, 23rd Nov 2015, 4th Nov 2016, 23rd May 2016, 29th Aug 2016, 1st May 2017 and 2nd Oct 2017	22nd Jan 2007, 12th Feb 2007, 30th Apr 2007, 9th Jul 2007, 10th Sep 2007, 8th Oct 2007, 12th Nov 2007, 28th Jul 2008, 16th Mar 2009, 18th Jan 2010, 5th Apr 2010, 13th Sep 2010, 27th Dec 2010, 9th May 2011, 15th Aug 2011, 31st Oct 2011, 16th Jan 2012, 13th Feb 2012, 9th Apr 2012, 30th Apr 2012, 14th Jan 2013, 30th Jun 2014, 28th Jul 2014, 13th Oct 2014, 4th Jun 2015, 13th Jul 2015, 13th Jul 2015, 28th Mar 2016, 8th Aug 2016, 7th Nov 2016, 12th Dec 2016, 9th Jan 2017, 10th Apr 2017, 29th May 2017, 9th Oct 2017 and 4th Dec 2017

(Continued…)

(Table 3 Continued)

Parameter	Country	Brazil	Russia	India	China
Commodity	**Evident Break Point**	17th Dec 2007, 14th Jan 2008, 21st Jan 2008, 18th Feb 2008, 14th Apr 2008, 19th May 2008, 21st Jul 2008, 27th Oct 2008, 26th Jan 2009, 23rd Feb 2009, 29th Jun 2009, 15th Oct 2009, 19th Apr 2010, 4th Oct 2010, 25th Oct 2010, 17th Jan 2011, 25th Jul 2011, 12th Sep 2011, 7th Nov 2011, 5th Mar 2012, 2nd Apr 2012, 7th May 2012, 26th Jan 2012, 27th May 2013, 4th Nov 2013, 20th Jan 2014, 8/9/2014, 26th Jan 2015, 16th Mar 2015, 29th Feb 2016 and 16th May 2016	5th Mar 2007, 23rd Apr 2007, 4th Jun 2007, 22nd Oct 2007, 25th Feb 2008, 21st Apr 2008, 2nd Jun 2008, 22nd Dec 2008, 9th Feb 2009, 26th Oct 2009, 28th Dec 2009, 1st Mar 2010, 26th Apr 2010, 7th Jun 2010, 1st Nov 2010, 27th Dec 2010, 7th Mar 2011, 25th Apr 2011, 6th Jun 2011, 31st Oct 2011, 26th Dec 2011, 27th Feb 2012, 11th Jun 2012, 29th Oct 2012, 10th Dec 2012, 11th Mar 2013, 11th Apr 2013, 30th Nov 2013, 6th Jan 2014, 27th Jan 2014, 27th Jan 2014, 28th Apr 2014, 16th Jun 2014, 29/12/2014, 16th Feb 2015, 27th Apr 2015, 8th Jun 2015, 28th Sep 2015, 5th Oct 2015, 9th Nov 2015, 14th Mar 2016, 2nd May 2016, 6th Jun 2016, 7th Nov 2016, 20th Feb 2017, 24th Apr 2017, 15th May 2017, 5th Jun 2017, 12th Jun 2017, 30th Oct 2017, 6th Nov 2017 and 25th Dec 2007	None	28th Nov 2011

(Continued…)

(Table 3 Continued)

Parameter	Country	Brazil	Russia	India	China
Commodity	**Hidden Break Point**	2nd Nov 2009, 29th Mar 2010, 25th Apr 2011, 10th Dec 2012, 25th May 2015 and 7th Mar 2016	1st Sep 2008, 26th Sep 2009, 20th Apr 2009, 1st Jun 2009, 7th May 2012, 8th Sep 2014, 3rd Nov 2014 and 11th Sep 2017	19th Mar 2007, 18th Jun 2007, 1st Oct 2007, 7th Jul 2008, 22nd Jun 2009, 25th Jan 2010, 19th Jul 2010, 8th Nov 2010, 31st Jan 2011, 18th Jul 2011, 3rd Oct 2011, 18th Mar 2013, 9th Sep 2013, 25th Nov 2013, 2nd Jun 2014, 13th Jul 2015, 8th Feb 2016, 4th Jul 2016, 19th Sep 2016 and 18th Dec 2017	26th Feb 2007, 28th May 2007, 17th Sep 2007, 14th Jan 2008, 11th Jan 2010, 11th Jul 2011, 27th Feb 2012, 21st May 2012, 10th Aug 2012, 1st Jul 2013, 25th Nov 2013, 22nd Sep 2013, 22nd Sep 2014, 4th May 2015, 31st Aug 2015, 11th Jan 2016 and 19th Dec 2016

(Continued…)

(Table 3 Continued)

Parameter	Country	Brazil	Russia	India	China
Real Estate	**Evident Break Point**	17th Dec 2007, 14th Jan 2008, 21st Jan 2008, 18th Feb 2008, 14th Apr 2008, 19th May 2008, 21st Jul 2008, 27th Oct 2008, 26th Jan 2009, 23rd Feb 2009, 29th Jun 2009, 15th Oct 2009, 19th Apr 2010, 4th Oct 2010, 25th Oct 2010, 17th Jan 2011, 25th Jul 2011, 12th Sep 2011, 7th Nov 2011, 5th Mar 2012, 2nd Apr 2012, 7th May 2012, 26th Jan 2012, 27th May 2013, 4th Nov 2013, 20th Jan 2014, 8th Sep 2014, 26th Jan 2015, 16th Mar 2015, 29th Feb 2016 and 16th May 2016	16th Jul 2007, 13th Aug 2007, 29th Oct 2007, 3rd Mar 2008, 18th Aug 2008, 22nd Sep 2008, 17th Nov 2008, 8th Dec 2008, 16th Mar 2009, 13th Apr 2009, 18th May 2009, 15th Jun 2009, 6th Jul 2009, 12th Oct 2009, 1st Feb 2010, 30th Aug 2010, 1st Aug 2011, 24th Dec 2012, 18th Mar 2013, 29th Apr 2013, 16th Nov 2013, 3rd Feb 2014, 3rd Mar 2014, 9th Jun 2014, 11th Aug 2014, 29th Sep 2014, 13th Oct 2014, 17th Nov 2014, 20th Jul 2015, 14th Sep 2015, 9th Nov 2015, 19th Dec 2016, 6th Feb 2017, 22nd May 2017 and 26th Jun 2017	14th Apr 2008, 19th May 2008, 2nd Jun 2008, 4th Aug 2008, 15th Sep 2008, 29th Sep 2008, 27th Oct 2008, 15th Dec 2008, 12th Jan 2009, 16th Mar 2009, 27th Apr 2009, 22nd Jun 2009, 3rd Aug 2009, 9th Nov 2009, 31st May 2010, 13th Sep 2010, 13th Nov 2010, 21st Feb 2011, 14th Mar 2011, 17th Aug 2011, 22nd Aug 2011, 2nd Jan 2012, 6th Feb 2012, 18th Nov 2012, 14th Jan 2013, 4th Mar 2013, 9th Sep 2013, 7th Oct 2013, 3rd Mar 2014, 19th May 2014, 27th Jun 2016, 7th Nov 2016, 8th Dec 2016, 29th Dec 2016, 30th Jan 2017, 17th Apr 2017, 14th Aug 2017 and 30th Oct 2017	9th Apr 2007, 21st May 2007, 3rd Sep 2007, 8th Oct 2007, 11th Feb 2008, 2nd Jun 2008, 7th Jul 2008, 28th Jul 2008, 22nd Feb 2008, 7th Jun 2010, 22nd Oct 2008, 7th Jun 2010, 1st Nov 2010, 15th Nov 2010, 28th Nov 2011, 19th Mar 2012, 21st Dec 2013, 6th May 2013, 15th Dec 2014, 23rd Feb 2015, 22nd Jun 2015, 20th Jul 2015, 7th Sep 2015 and 14th Mar 2016

(Continued…)

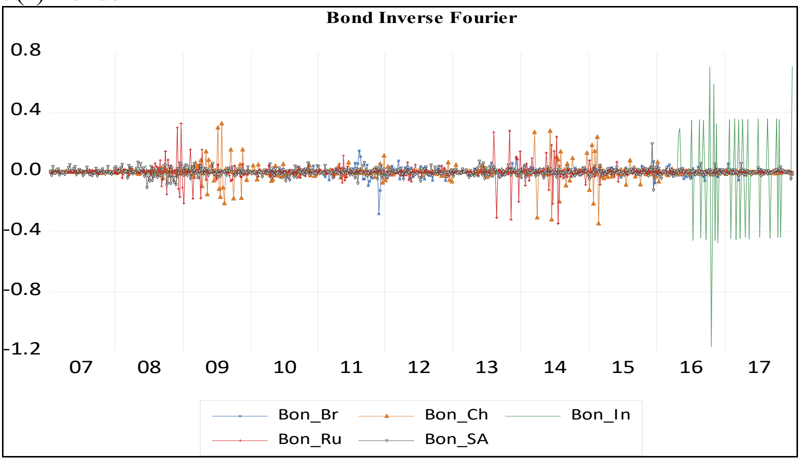
(Table 3 Continued)

Parameter	Country	Brazil	Russia	India	China
Real Estate	**Hidden Break Point**	29th Oct 2007, 1st Mar 2019, 21st Mar 2011, 6th Jun 2011, 9th Jan 2012, 6th Feb 2012, 9th Jan 2012, 6th Feb 2012, 14th Jul 2014, 28th Jul 2014, 8th Dec 2014, 27th Apr 2015, 8th Aug 2016, 2nd Jan 2017, 30th Jan 2017, 25th Sep 2017 and 27th Nov 2017	12th May 2008, 21st Jul 2008, 12th Jan 2009, 16th Feb 2009, 24th Aug 2009, 26th Jul 2010, 27th Sep 2010, 27th Dec 2010, 7th Mar 2011, 2nd May 2011, 27th Jun 2011, 22nd Aug 2011, 12th Sep 2011, 3rd Oct 2011, 31st Oct 2011, 5th Dec 2011, 30th Jan 2012, 30th Apr 2012, 24th Sep 2012, 8th Jul 2013, 4th Nov 2013, 2nd Dec 2013, 15th Dec 2014, 9th Feb 2015, 23rd Mar 2015, 18th Jun 2015, 4th Sep 2017 and 11th Nov 2017	7th Apr 2007, 14th Dec 2009, 15th Dec 2009, 22nd Mar 2010, 27th Jun 2011, 28th May 2012, 23rd Jul 2012, 13th May 2013, 29th Jul 2013, 16th Dec 2013, 31st Mar 2014, 14th Jul 2014, 12th Jan 2015, 6th Apr 2015, 5th Oct 2015, 5th Sep 2016, 2nd Jan 2017 and 3rd Jul 2017	21st Jan 2008, 6th Apr 2009, 20th Jul 2009, 21st Sep 2009, 23rd Nov 2009, 23rd Nov 2009, 11th Jan 2010, 20th Feb 2012, 13th Aug 2012, 3rd Sep 2012, 22nd Oct 2012, 4th Nov 2013, 10th Feb 2014, 17th Mar 2014, 6th Apr 2015, 23rd Nov 2015, 4th Nov 2016, 23rd May 2016, 29th Aug 2016, 1st May 2017 and 2nd Oct 2017

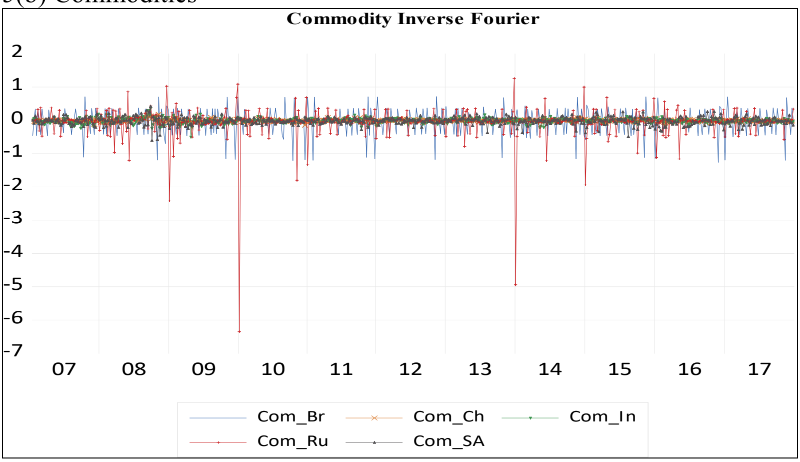
The volatility curves (i.e. Fourier transform) are slightly flatter when compared with the returns curves (i.e. Laplace transform). Furthermore, there are fewer structural break points in volatilities than break points illustrated by returns. One possible explanation is that some of the volatility effects might be that returns captured those shocks earlier. As stated earlier, movements in returns should supersede and give direction to volatility movements. In terms of movements, volatilities show an upward trajectory during 2007-2017 for the four indices of the BRICS countries. Revisiting the return curves, the returns fluctuate with time. This confirms the earlier statement that volatilities follow the preceding movements of returns.

Figure 5 [2] Inverse Fourier Transform

5(a) Bonds



5(b) Commodities



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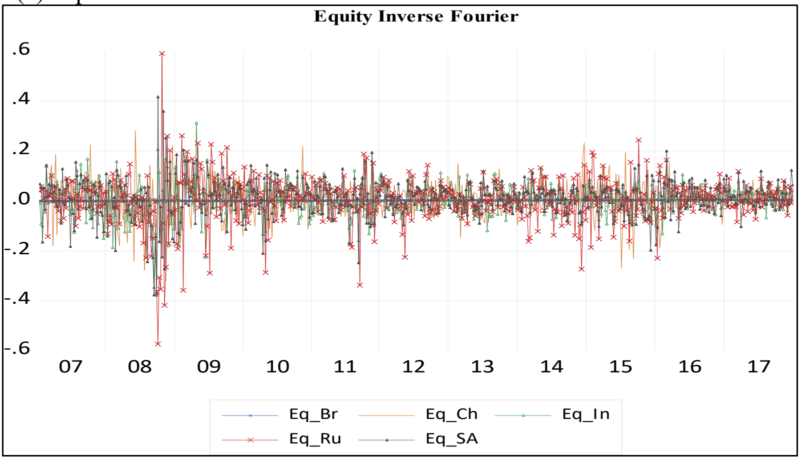
(Figure 5 Continued)

5(c) Equities



5(d) Real Estate

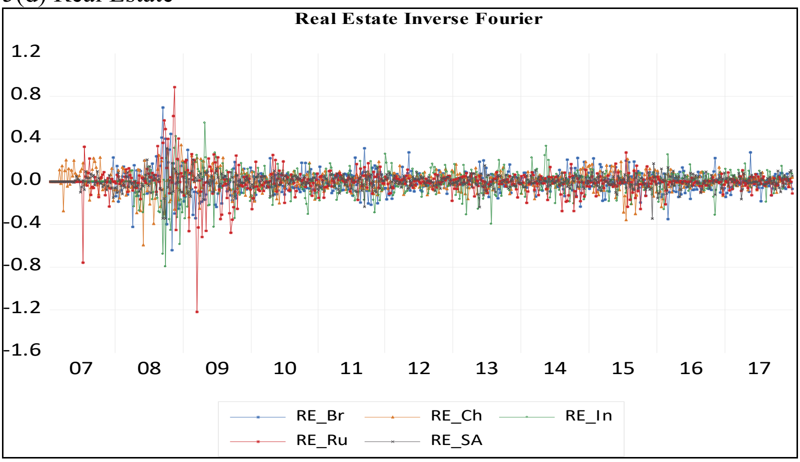


Table 4 Inverse Fourier Transform

Parameter	Country	Brazil	Russia	India	China	South Africa
Bonds	**Evident Break Point**	15th Aug 2011, 12th Sep 2011, 26th Sep 2011, 3rd Oct 2011 and 28th Nov 2011	6th Oct 2008, 1st Dec 2008, 15th Dec 2008, 22/12/2008, 9/2/2009, 13/4/2009, 18/5/2009, 6/7/2009, 7/9/2009, 23/5/2011, 12/8/2012, 4/12/2013, 9/12/2013, 30/12/2013, 19/5/2014, 16/6/2014 and 14/7/2014	2nd May 2016, 4th Jul 2016, 11th Jul 2016, 15th Aug 2016, 22nd Aug 2016, 5th Sep 2016, 19th Sep 2016, 10th Oct 2016, 31st Oct 2016, 23rd Jan 2017, 20th Feb 2017, 13th Mar 2017, 3rd Apr 2017, 1st May 2017, 26th Jun 2017, 14th Aug 2017, 2nd Oct 2017, 16th Oct 2017 and 25th Dec 2017	4th May 2009, 11th May 2009, 6th Jul 2009, 20th Jul 2009, 27th Jul 2009, 10th Aug 2009, 14th Sep 2009, 28th Sep 2009, 16th Nov 2009, 9th Nov 2011, 26th Dec 2011, 17th Mar 2014, 31st Mar 2014, 9th Jun 2014, 16th Jun 2014, 4th Aug 2014, 22nd Dec 2014, 5th Jan 2015, 19th Jan 2015, 26th Jan 2015, 16th Feb 2015 and 23rd Feb 2015	23rd Jun 2008, 13th Oct 2008 and 17th Nov 2008

(Continued…)

(Table 4 Continued)

Parameter	Country	Brazil	Russia	India	China	South Africa
Bonds	**Hidden Break Point**	15th Jan 2007, 6th Dec 2010, 7th Feb 2011, 7th Mar 2011, 25th Jul 2011, 15th Aug 2011, 12th Mar 2012, 19th Mar 2012, 9th Jul 2012, 30th Jul 2012, 10th Sep 2012, 15th Oct 2012, 19th Nov 2012, 20th Jul 2015, 12th Oct 2015, 7th Mar 2016 and 23rd May 2016	11th Feb 2008, 4th Aug 2008, 8th Sep 2008, 20th Sep 2010, 2nd May 2011, 16th May 2011, 14th May 2012, 11th Jun 2012, 29th Oct 2012, 27th Jan 2014, 3rd Mar 2014, 15th Dec 2014, 5th Jan 2015, 2rd Mar 2015, 1st Jan 2015 and 8th Jun 2015	15th Jan 2007, 18th Jun 2007, 7th Apr 2008, 16th Jun 2008, 27th Jul 2008, 25th Jan 2010 and 30th Jul 2012	15th Jun 2009, 2nd Nov 2009, 18th Jan 2010, 15th Feb 2010, 28th Jun 2010, 20th Jun 2011, 16th Jan 2012, 3rd Dec 2012, 24th Dec 2012, 21st Feb 2013, 23rd Mar 2015, 20th Jul 2015, 7th Sep 2015, 5th Oct 2015, 4th Jan 2016 and 11th Jan 2016	22nd Jan 2007, 30th Apr 2007, 28th May 2007, 19th Nov 2007, 11th Feb 2008, 5th May 2008, 15th Dec 2008, 12th Jul 2010, 30th Aug 2010, 8th Nov 2010, 8th Aug 2011, 9th Jul 2012, 7th Nov 2016, 27th Mar 2017, 23rd Oct 2017 and 18th Dec 2017

(Continued…)

(Table 4 Continued)

Parameter	Country	Brazil	Russia	India	China	South Africa
Equities	**Evident Break Point**	None	26th Feb 2007, 21st May 2007, 12th May 2008, 6th Oct 2008, 27th Oct 2008, 9th Feb 2008, 27th Apr 2009, 13th Jul 2009, 5th Oct 2009, 23rd Nov 2009, 11th Jan 2010, 1st Mar 2010, 29th Nov 2010, 28th Mar 2011, 16th May 2011, 27th Jun 2011, 10th Oct 2011, 20th Feb 2012, 4th Jun 2012, 10th Sep 2012, 17th Mar 2014, 5th May 2014, 1st Sep 2014, 27th Oct 2014, 2nd Feb 2015, 6th Apr 2015, 24th Aug 2015, 5th Oct 2015 and 25th Jan 2016	20th Aug 2007, 24th Sep 2007, 3rd Mar 2008, 30th Jun 2008, 27th Apr 2008, 25th May 2009, 19th Nov 2012, 15th Apr 2013, 29th Jul 2013, 9th Sep 2013, 2nd Dec 2013, 30th Jun 2014, 20th Apr 2015, 8th Feb 2016 and 23rd May 2016	19th Mar 2007, 9th Apr 2007, 8th Oct 2007, 9th Jun 2008, 25th Aug 2008, 22nd Dec 2008, 30th Mar 2009, 8th Jun 2009, 8th Mar 2009, 7th Jun 2010, 15th Nov 2010, 25th Feb 2013, 27th May 2013 and 20th Sep 2013	6th Oct 2008, 3rd Nov 2008, 12th Jan 2009, 13th Apr 2009, 17th Aug 2009, 14th Sep 2009, 21st Apr 2010, 10th May 2010, 5th Oct 2015, 25th Jan 2016 and 29th Feb 2016

(Continued…)

(Table 4 Continued)

Parameter	Country	Brazil	Russia	India	China	South Africa
Equities	**Hidden Break Point**	12th May 2008, 15th Sep 2008, 23rd Jan 2012, 7th May 2012, 27th May 2013 and 19th May 2014	20th Aug 2007, 8th Oct 2007, 10th Dec 2007, 11th Feb 2008, 16th Jun 2008, 11th Aug 2008, 10th May 2010, 19th Jul 2010, 19th Nov 2012, 29th Apr 2013, 16th Sep 2013, 23rd Jun 2014, 8th Jun 2015, 23rd May 2016, 5th Dec 2016, 13th Mar 2017, 10th Jul 2017, 6th Nov 2017 and 27th Nov 2017	19th Mar 2007, 16th Apr 2007, 26th Nov 2007, 30th Aug 2010, 15th Nov 2010, 2nd Feb 2011, 22nd Aug 2011, 10th Oct 2011, 7th Nov 2011, 9th Jan 2012, 16th Jul 2012, 10th Dec 2012, 4th Mar 2013, 3rd Mar 2014, 20th Feb 2017, 7th Aug 2017 and 6th Nov 2017	19th Nov 2007, 24th Mar 2008, 3rd Aug 2009, 23rd Dec 2009, 2nd Aug 2010, 27th Sep 2010, 13th Dec 2010, 2nd May 2011, 4th Jul 2011, 19th Dec 2011, 13th Dec 2012, 14th May 2012, 1st Oct 2012, 22nd Oct 2012, 24th Dec 2012, 6th Jan 2014, 22nd Sep 2014, 2nd Mar 2014, 4th May 2015, 22nd Jun 2015, 2nd Nov 2015, 14th Mar 2016, 19th Dec 2016 and 29th May 2017	14th May 2007, 23rd Jul 2007, 20th Aug 2007, 7th Jan 2008, 21st Apr 2008, 28th Jul 2008, 2nd Nov 2009, 25th Jan 2010, 21st Jun 2010, 30th Aug 2010, 24th Jan 2011, 14th Jan 2011, 23rd May 2011, 3rd Oct 2011, 21st Nov 2011, 12th Dec 2011, 9th Apr 2012, 16th Jul 2012, 22nd Apr 2013, 24th Jun 2013, 14th Oct 2013, 16th Dec 2013, 24th Mar 2014, 27th Oct 2014, 15th Dec 2014, 16th Mar 2015, 20th Apr 2015, 13th Jul 2015, 11th Jul 2016, 19th Sep 2016, 9th Jan 2017, 10th Apr 2017, 10th Jul 2017, 9th Oct 2017 and 18th Dec 2017

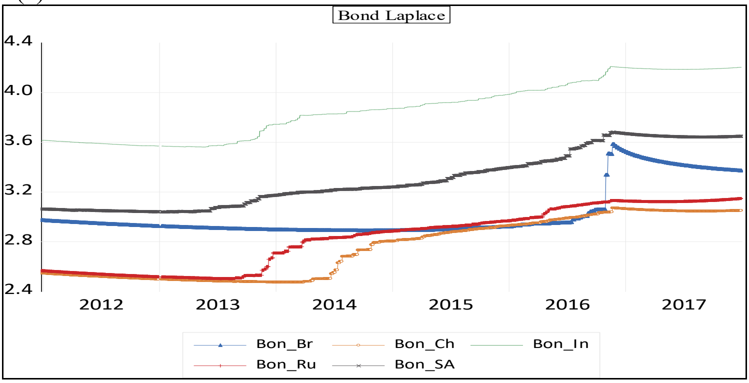
(Continued…)

(Table 4 Continued)

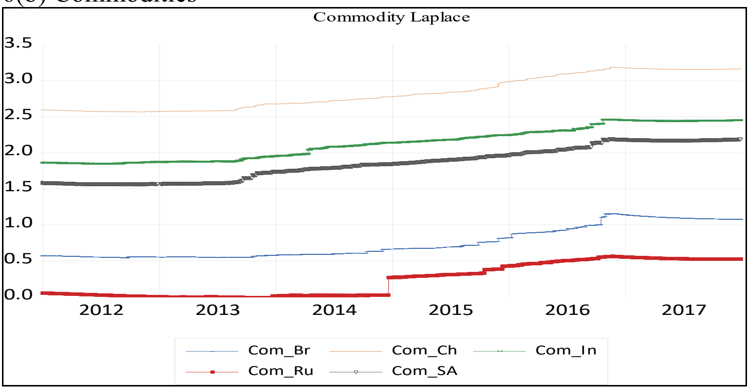
Parameter	Country	Brazil
Commodity	**Evident Break Point**	15th Jan 2007, 2nd Apr 2007, 25th Jun 2007, 6th Aug 2007, 8th Oct 2007, 3rd Dec 2007, 11th Feb 2008, 7th Apr 2008, 21st Jul 2008, 3rd Nov 2008, 15th Dec 2008, 6th Apr 2008, 8th Jun 2009, 13th Jul 2009, 10th Aug 2009, 26t Oct 2009, 14th Dec 2009, 1st Feb 2010, 30th Aug 2010, 18th Oct 2010, 7th Feb 2011, 1st Aug 2011, 29th Aug 2011, 26th Sep 2011, 14th Nov 2011, 26th Dec 2011, 6th Feb 2012, 16th Apr 2012, 18th Jun 2012, 23rd Jul 2012, 27th Aug 2012, 17th Sep 2012, 5th Nov 2012, 3rd Dec 2012, 18th Feb 2013, 25th Mar 2013, 15th Jul 2013, 9th Sep 2013, 28th Oct 2013, 9th Dec 2013, 3rd Mar 2014, 24th Mar 2014, 28th Apr 2014, 4th Aug 2014, 29th Sep 2014, 17th Nov 2014, 8th Dec 2014, 9th Feb 2015, 23rd Mar 2015, 27th Apr 2015, 9th Nov 2015, 4th Jan 2016, 27th Jun 2016, 15th Aug 2016, 5th Sep 2016, 10th Oct 2016, 2nd Jan 2017, 13th Feb 2017, 24th Apr 2017, 26th Jun 2017, 31st Jul 2017, 4th Sep 2017, 25th Sep 2017, 13th Nov 2017 and 11th Dec 2017
	Hidden Break Point	None
Real Estate	**Evident Break Point**	17th Dec 2007, 7th Apr 2008, 15th Sep 2008, 27th Oct 2008, 3rd Nov 2008, 26th Jan 2009, 23rd Feb 2009, 29th Jun 2009, 5th Oct 2009, 19tyh Apr 2010, 27th Sep 2010, 25th Oct 2010, 17th Jan 2011, 12th Sep 2011, 2nd Apr 2012, 7th May 2012, 27th May 2013, 8th Sep 2014, 3rd Nov 2014, 26th Jan 2015, 7th Nov 2016 and 15th May 2017
	Hidden Break Point	14th Jan 2008, 19th May 2008, 21st Jul 2008, 21st Jun 2010, 28th Mar 2011, 9th Jan 2012, 12th May 2012, 26th Nov 2012, 4th Nov 2013, 16th Dec 2013, 16th Mar 2015, 31st Aug 2015, 29th Feb 2016, 9th May 2016, 30th Jan 2017 and 23rd Oct 2017

Figure 6 [3] Laplace Transform

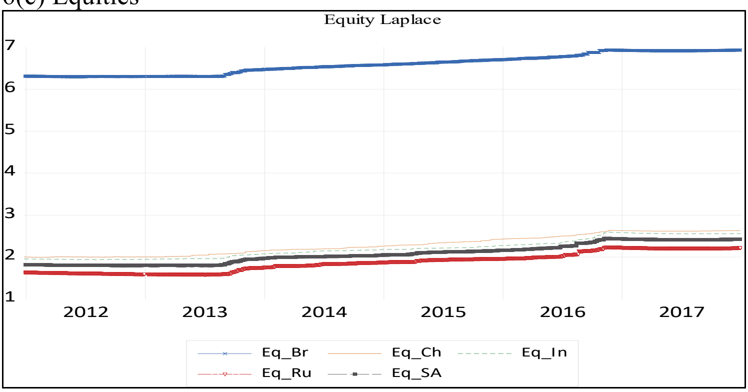
6(a) Bonds



6(b) Commodities



6(c) Equities



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(Figure 6 Continued)

6(d) Real Estate

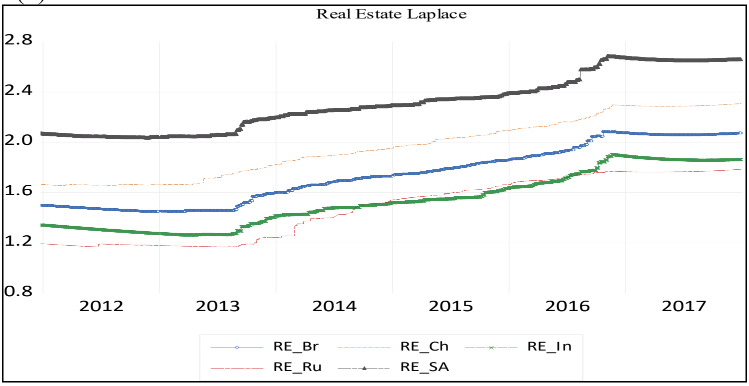


Figure 6 and Table 4 show the graphs and structural break dates based on the Laplace transform, respectively. For the Laplace transform of bonds, evident and hidden break points are almost equally spread out. For real estate indices, there are more evident than hidden break points. For the equities, the evident break points are outnumbered by the hidden break points as shown in Table 5. This has been illustrated and debated earlier. Nevertheless, the break points of commodities are almost equally spread between the evident and hidden points. In this section, it can be observed that if returns are correctly modelled then by extension, volatilities should be correctly captured. This partly explains why strategies like risk parity have not been as successful as traditional return strategies.

The inverse Laplace transform for the four indices of the BRICS countries shows that there are fewer break points of volatilities than points of returns. However, unlike Laplace transform, inverse Laplace transform shows more evident than hidden break points. Interestingly China and Russia have a break point on the same date (23rd November 2015) for bonds. According to Malle (2017), China and Russia made political and economic agreements for a number of fields during that time: '(i) energy, (ii) arms production, (iii) trade in national currencies, and (iv) strategic projects in transport and supporting infrastructure" (p.136).

Therefore, some structural break points are interrelated and caused by political, economic and corporate government agreements. It is doubtful whether these three elements were considered in the formation of the BRICS countries. Moreover, it is also ambiguous whether their implications for the financial markets were taken into account. Fundamentally, it seems that in liquid indices, evident exceed hidden break points while in illiquid indices, the patterns are indefinite.

Table 5 Laplace Transform

Parameter	Country	Brazil	Russia	India	China	South Africa
Bonds	**Evident Break Point**	15th Jul 2016, 28th Oct2016 and 28th Nov 2016	13th Nov 2013, 3th Apr 2014, 15th Apr 2016, 11th May 2016 and 18th Nov 2016	11th Oct 2013, 5th Dec 2013, 18th Aug 2014, 7th Oct 2016 and 11th Nov 2016	6th May 2014, 18th Jun 2014, 1st Aug 2014, 11st Sep 2014, 25th Sep 2014, 24th Oct 2014, 13th Nov 2014, 17th Nov 2016 and 29th Nov 2016	28th Jun 2013, 18th Nov 2013, 15th Jul 2016 and 24th Nov 2016
	Hidden Break Point	18th Apr 2016, 20th Sep 2016, 4th Nov 2016 and 18th Nov 2016	21st Nov 2013, 20th Dec 2013, 31st Jan 2014, 20th Jun 2016 and 23rd Nov 2016	30th Jul 2013, 28th Nov 2013, 26th Dec 2014, 26th Mar 2015, 22nd Nov 2016 and 18th Aug 2016	27th Feb 2014, 11th Jul 2014, 29th Aug 2014, 23rd Jan 2015 and 5th Apr 2016	18th Oct 2013 and 21st Nov 2016
Equities	**Evident Break Point**	23rd Aug 2013, 8th Nov 2013 and 10th Nov 2016	7th Mar 2013, 26th Jul 2013, 14th Nov 2013 and 22nd Nov 2016	28th Aug 2013, 19th Sep 2013, 10th Jun 2014, 13th Jul 2016, 21st Nov 2016 and 18th Jan 2017	15th Dec 2012, 8th Aug 2013, 4th Nov 2013, 27th Mar 2015, 6th Jul 2015, 17th Aug 2015, 3rd Nov 2016 and 16th Nov 2017	13th Jul 2013, 7th Nov 2013, 10th Apr 2015, 1st Jul 2016, 18th Aug 2016, 18th Nov 2016 and 19th May 2017
	Hidden Break Point	28th Mar 2012, 6th Jul 2012, 4th Oct 2012, 27th Dec 2012, 21st Sep 2016 and 9th Oct 2017	27th Jan 2012, 27th Mar 2012, 22nd Sep 2014 and 4th Dec 2015	13th Apr 2012, 31st Jan 2013, 13th Sep 2013 and 21st Apr 2015	28th Mar 2012, 22nd Oct 2012, 13th Sep 2013, 27th May 2015, 19th Jan 2017 and 6th Apr 2017	2nd Mar 2012, 11th May 2012, 1st Aug 2012, 24th Oct 2012, 19th Mar 2013, 26th Jul 2013, 31st Jan 2014, 25th Jul 2014 and 20th Jul 2016

(Continued…)

(Table 5 Continued)

Parameter	Country	Brazil	Russia	India	China	South Africa
Commodity	**Evident Break Point**	24th Oct 2013, 9th Oct 2014, 28th Nov 2014, 25th Sep 2015, 27th Nov 2015, 7th Dec 2015, 18th Jan 2016, 14th Oct 2016, 20th Oct 2016 and 7th Nov 2016	20th Dec 2013, 19th Dec 2014, 23rd Dec 2014, 16th Oct 2015, 19th Oct 2015, 16th Dec 2015 and 7th Dec 2016	26th Sep 2013, 14th Apr 2014, 21st Oct 2016 and 28th Oct 2016	2nd Sep 2013, 5th Dec 2013, 27th Nov 2015, 4th Dec 2015 and 28th Nov 2016	28th Jun 2013, 18th Nov 2013, 15th Jul 2016 and 24th Nov 2016
	Hidden Break Point	27th Sep 2012, 11th Jan 2013, 8th May 2013, 14th Oct 2014, 8th Dec 2014, 30th Sep 2015, 8th Jan 2016, 28th Oct 2016 and 25th Nov 2016	2nd Apr 2013, 10th Jun 2013, 26th Jul 2013, 22nd Oct 2013, 24th Oct 2014 and 20th May 2016	30th Jul 2013, 28th Nov 2013, 26th Dec 2014, 26th Mar 2015, 22nd Nov 2016 and 18th Aug 2016	26th Nov 2013, 25th Oct 2013 and 3rd Mar 2016	18th Oct 2013 and 21st Nov 2016

(Continued…)

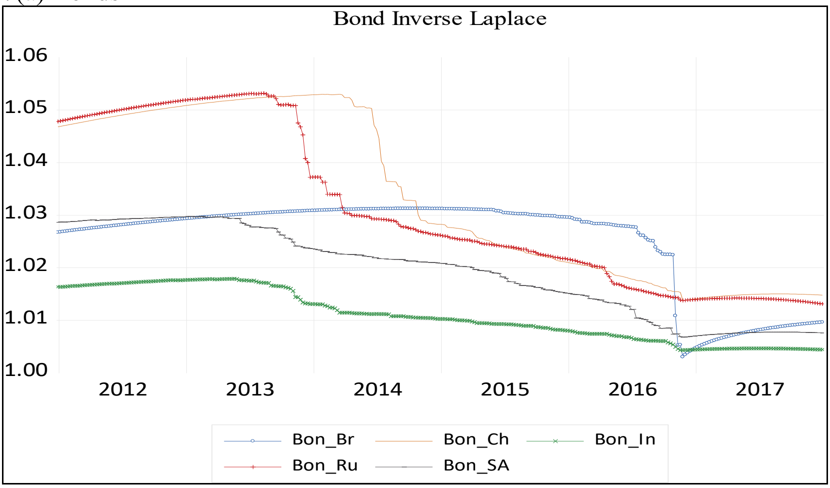
(Table 5 Continued)

Parameter	Country	Brazil	Russia	India	China	South Africa
Real Estate	**Evident Break Point**	25th Oct 2013, 16th Jun 2014, 6th May 2016, 25th Jul 2016, 27th Sep 2016 and 28th Oct 2016	22nd Jun 2012, 27th Nov 2013, 17th Jan 2014, 28th Feb 2014, 13th Mar 2014, 23rd Apr 2014, 3rd Jul 2014, 28th Aug 2014, 8th Oct 2014, 25th Dec 2014, 8th May 2015, 22nd Jun 2015, 20th Aug 2015, 4th Nov 2015, 12th Feb 2016, 18th Jul 2014, 8th Aug 2015, 22nd Jun 2015, 20th Aug 2015, 4th Nov 2015, 12nd Feb 2016, 18th Jul 2016 and 29th Sep 2016	27th Aug 2013, 20th Sep 2013, 7th Nov 2013, 6th Nov 2013, 16th Apr 2014, 15th May 2014, 5th Jun 2014, 19th Sep 2014, 17th Dec 2014, 16th Oct 2015, 27th Jan 2016, 17th Jun 2016, 4th Aug 2016, 29th Sep 2016 and 25th Nov 2016	17th May 2013, 17th Oct 2014, 20th Mar 2014, 26th May 2015, 16th Jun 2016 and 28th Nov 2016	7th Feb 2013, 5th Feb 2012, 27th Mar 2013, 29th Mar 2013, 26th Sep 2013, 18th Feb 2014, 19th Sep 2014, 3rd Apr 2013, 16th Sep 2015, 4th Dec 2015, 29th Dec 2015, 12th Apr 2016, 12th Aug 2016 and 8th Nov 2016
	Hidden Break Point	18th Jan 2013, 27th Mar 2013, 20th Feb 2014 and 11th Oct 2016	25th Jun 2013, 4th Apr 2014 and 17th Feb 2016	8th Apr 2013, 28th Jun 2013, 17th Apr 2013 and 31st Mar 2016	22nd Mar 2012, 15th Jul 2013, 12th Sep 2014 and 10th Jul 2016	25th May 2012, 7th Aug 2012, 21st Sep 2012, 10th Jun 2013, 13th Sep 2013, 10th Apr 2014 and 1st Jul 2016

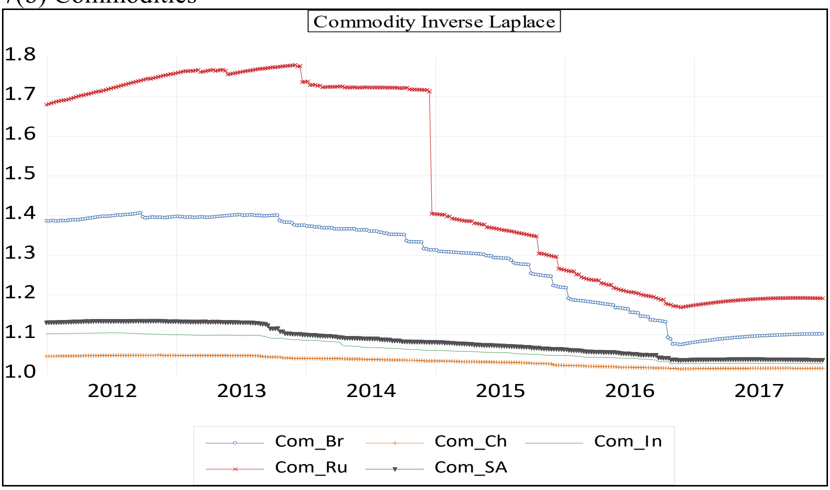
Figure 7 and Table 6 show the diagrams and structural break dates based on the use of the inverse Laplace transform, respectively.

Figure 7 [4] Inverse Laplace Transform

7(a) Bonds



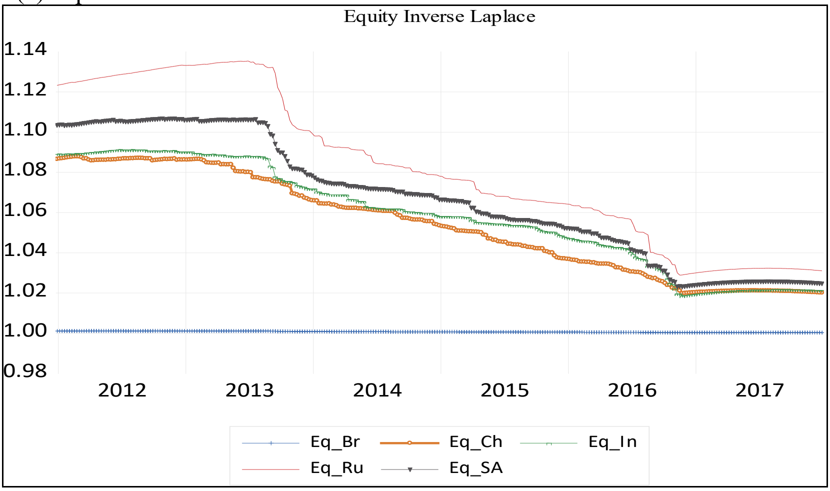
7(b) Commodities



(Continued…)

(Figure 7 Continued)

7(c) Equities



7(d) Real Estate

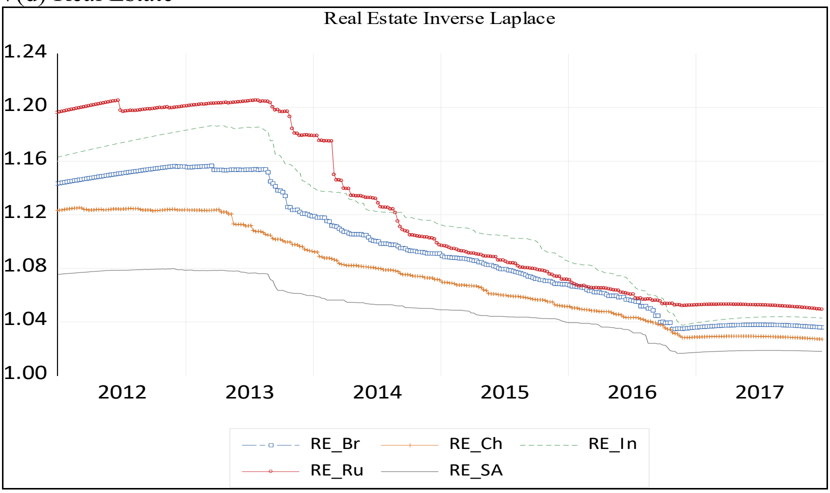


Table 6 Inverse Laplace Transform

Parameter	Country	Brazil	Russia	India	China	South Africa
Bonds	**Evident Break Point**	8th Jun 2015, 29th Jun 2015, 7th Sep 2015, 23rd Nov 2015, 15th Feb 2016, 11th Jul 2016, 1st Aug 2016, 29th Aug 2016, 19th Sep 2016, 24th Oct 2016, 14th Nov 2016 and 21st Nov 2016	19th Aug 2013, 2nd Sep 2013, 9th Sep 2013, 30th Sep 2013, 28th Oct 2013, 11th Nov 2013, 9th Dec 2013, 30th Dec 2013, 20th Jan 2014, 3rd Feb 2014, 10th Feb 2014, 17th Mar 2014, 31st Mar 2014, 21st Apr 2014, 9th Jun 2014, 25th Aug 2014, 13th Oct 2014, 7th Sep 2015, 28th Sep 2015, 29th Feb 2016, 11th Apr 2016, 16th Jan 2015, 19th Sep 2016, 14th Nov 2016 and 21st Nov 2016	16th Sep 2013, 7th Apr 2014, 12th Sep 2016 and 17th Oct 2016	17th Mar 2014, 14th Apr 2014, 5th May /2014, 26th May 2014, 16th Jun 2014, 23rd Jun 2014, 14th Jul 2014, 28th Jul 2014, 25th Aug 2014, 8th Sep 2014, 22nd Sep 2014, 20th Oct 2014, 10th Nov 2014, 1st Dec 2014, 19th Jan 2015, 30th Mar 2015, 27th Apr 2015, 18th May 2015, 7th Mar 2015, 11th Apr 2016, 9th May 2016, 27th Jun 2016, 19th Sep 2016 and 14th Nov 2016	18th Feb 2013, 15teh Apr 2013, 3rd Jun 2013, 8th Jul 2013, 12th Aug 2013, 16th Sep 2013, 28th Oct 2013, 18th Nov 2013, 30/03/2015, 15/06/2015, 20/07/2015, 29th Feb 2016, 11th Apr 2016, 30th May 2016, 20th Jun 2016, 4th Jul 2016, 11th Jul 2016, 8th Aug 2016, 19th Sep 2016, 17th Oct 2016, 31st Oct 2016, 7th Nov 2016, 21st Nov 2016 and 12th Jun 2017
	Hidden Break Point	24th Apr 2014, 28th Jul 2014, 29th Dec 2014, 23rd Mar 2015, 18st Jan 2016 and 14th Mar 2016	1st Jul 2013, 25th Nov 2013, 29th Sep 2014, 26th Jan 2015 and 23rd Nov 2015	27th Feb 2012, 10th Sep 2012, 18th Feb 2013 and 26th May 2014	7th Jul 2014, 13th Oct 2014, 17th Nov 2014 and 23rd Nov 2015- *Russia has the same hidden date point.*	9th Apr 2012, 10th Sep 2012, 17th Jun 2013, 5th Aug 2013, 7th Oct 2013, 19th May 2014 and 20th Apr 2015

(Continued…)

(Table 6 Continued)

Equities	**Evident Break Point**	None	18th Feb 2013, 1st Jul 2013, 12th Aug 2013, 2nd Sep 2013, 11th Nov 2013, 30th Dec 2013, 20th Jan 2014, 10th Dec 2014, 14th Apr 2014, 26th May 2014, 26th May 2014, 23rd Jun 2014, 1st Sep 2014, 15th Sep 2014, 13th Oct 2014, 1st Apr 2015, 6th Apr 2015, 20th Apr 2015, 20th Jul 2015, 21st Mar 2016, 11th Apr 2016, 27th Jun 2016, 18th Jul 2016, 15th Aug 2016, 15th Aug 2016, 22nd Aug 2016, 26th Sep 2016 and 14th Nov 2016	20th Feb 2012, 4th Mar 2012, 6th May 2013, 28th Oct 2013, 18th Aug 2014, 15th Apr 2014, 8th Dec 2014, 23rd Feb 2015, 27th Apr 2015, 25th May 2015, 19th Oct 2015, 7th Dec 2015, 18th Jul 2015 and 21st Nov 2016	20th Feb 2012, 4th Mar 2012, 6th May 2013, 28th Oct 2013, 18th Aug 2014, 15th Apr 2014, 8th Dec 2014, 23rd Feb 2015, 27th Apr 2015, 25th May 2015, 19th Oct 2015, 7th Dec 2015, 18th Jul 2015 and 21st Nov 2016	9th Apr 2012, 9th Jul 2012, 19th Aug 2013, 30th Sep 2013, 11th Nov 2013, 3rd Mar 2014, 7th Apr 2014, 7th Jul 2014, 11th Aug 2014, 22nd Dec 2014, 16th Mar 2015, 16th Nov 2015, 21st Mar 2016, 20th Jun 2016, 27th Jun 2016, 8th Aug 2016, 15th Aug 2016 and 19th Sep 2016
	Hidden Break Point	11th Jun 2012, 5th May 2014 and 8th Aug 2016	6th Feb 2012, 17th Dec 2012, 13th Oct 2016, 29th May 2017 and 18th Sep 2017	16th Apr 2012, 24th Sep 2012, 20th Jan 2014, 6th Jul 2015 and 2nd Oct 2017	16th Apr 2012, 24th Sep 2012, 20th Jan 2014, 6th Jul 2015 and 2nd Oct 2017	2nd Nov 2012, 2nd Sep 2013 and 17th Oct 2016

(Continued…)

(Table 6 Continued)

Parameter	Country	Brazil	Russia	India	China	South Africa
Commodity	**Evident Break Point**	12th Mar 2012, 17th Sep 2012, 1st Oct 2012, 1st Apr 2013, 8th Jul 2013, 14th Oct 2013, 6th Oct 2014, 20th Oct 2014, 24th Nov 2014, 1st Dec 2014, 12th Jan 2015, 18th May 2015, 15th Jun 2015, 3rd Aug 2015, 17th Aug 2015, 21st Sep 2015, 5th Oct 2015, 23rd Nov 2015, 30th Nov 2015, 4th Jan 2016, 11th Jan 2016, 23th May 2016, 29th Aug 2016, 10th Oct 2016, 17th Oct 2016 and 31st Oct 2016	4th Mar 2013, 18th Mar 2013, 22nd Apr 2013, 20th May 2013, 27th May 2013, 7th Oct 2013, 16th Dec 2013, 6th Jan 2014, 20th Jan 2014, 10th Feb 2014, 17/02/2014, 8th Dec 2014, 22nd Dec 2014, 20th Apr 2015, 25th May 2015, 12th Oct 2015, 19th Oct 2015, 7th Dec 2015, 21st Dec 2015, 1st Feb 2016 and 28th Nov 2016	16th Sep 2013, 7th Apr 2014, 12th Sep 2016 and 17th Oct 2016	26th Mar 2013, 28th Oct 2013, 10th Feb 2014, 1st Dec 2014, 15th Jun 2015, 30th Nov 2015 and 7th Oc t2016	29th Sep 2014, 24th Nov 2014, 1st Dec 2014, 15th Jun 2015, 14th May 2015, 27th Jul 2015, 28th Sep 2015, 16th Nov 2015, 30th Nov 2015, 28th Nov 2015, 18th Jan 2016, 4th Apr 2016, 1st Aug 2016, 29th Aug 2016, 10th Oct 2016 and 31st Oct 2016
	Hidden Break Point	11th Nov 2013, 9th Dec 2013, 26th May 2016, 25th Aug 2014 and 28th Nov 2016	4th Feb 2013, 15the Jul 2013, 14th Apr 2014, 29th Sep 2014, 4th Apr 2016, 23rd May 2016 and 3rd Oct 2016	27th Feb 2012, 10th Aug 2012, 18th Feb 2013 and 26th May 2014	13th May 2013, 9th Jun 2014, 25th Aug 2014 and 16th Feb 2015	19th Mar 2012, 10th Sep 2012, 12th Nov 2012, 1st Jul 2013, 26th Aug 2013 and 16th May 2016

(Continued…)

(Table 6 Continued)

Parameter	Country	Brazil	Russia	India	China	South Africa
Real Estate	**Evident Break Point**	18th Mar 2013, 19th Aug 2013, 9th Sep 2013, 30th Sep 2013, 21st Oct 2013, 9th Dec 2013, 3rd Feb 2014, 14th Apr 2014, 9th Jun 2014, 7th Jul 2014, 25th Aug 2014, 5th Jan 2015, 20th Apr 2015, 23rd Nov 2015, 15th Feb 2016, 18th Apr 2016, 18th Jul 2016, 22nd Aug 2016, 29th Sep 2016, 17th Oct 2016 and 24th Oct 2016	18th Feb 2012, 2nd Jul 2012, 02/09/2013, 23rd Sep 2013, 21st Oct 2013, 25th Nov 2013, 20th Jan 2014, 24th Feb 2014, 3rd Mar 2014, 7th Apr 2014, 14th Apr 2014, 28th Apr 2014, 26th May 2014, 3rd Jun 2014, 11th Aug 2014, 22nd Sep 2014, 29th Sep 2014, 6th Oct 2014, 8th Dec 2014, 5th Jan 2015, 15th Jun 2015, 10th Aug 2015, 24th Aug 2015, 2nd Nov 2015, 30th Nov 2015, 21st Dec 2015, 4th Nov 2016, 25th Jan 2016, 22nd Feb 2016, 23rd May 2016, 11th Jul 2016 and 19th Sep 2016	1st Apr 2013, 27th May 2013, 12th Aug 2013, 26th Aug 2013, 9th Sep 2013, 23rd Sep 2013, 21st Oct 2013, 25th Nov 2013, 30th Dec /2013, 27th Nov 2014, 7th Apr 2014, 21st Apr 2014, 15th May 2014, 26th May 2014, 9th Jun 2014, 15th Sep 2014, 8th Dec 2014, 5th Jan 2015, 30th Mar 2015, 13st Jul 2015, 28th Sep 2015, 26th Oct 2015, 30th Nov 2015, 21st Dec 2015, 25th Jan 2016, 14th Mar 2016, 13th Jun 2016, 25th Jul 2016, 8th Aug 2016, 5th Sep 2016, 26th Sep 2016, 24/10/2016 and 28th Nov 2016	24th Sep 2013, 15th Apr 2013, 15th Apr 2013, 13th May 2013, 27th May 2013, 1st Jul 2013, 8th Jul 2013, 22nd Jul 2013, 16th Sep 2013, 9th Dec 2013, 13th Jan 2014, 17th Mar 2014, 8th Sep 2014, 5th Jan 2014, 16th Feb 2015, 27th Apr 2015, 25th May 2015, 23rd Nov 2015, 13th Jun 2016, 25th Jul 2016, 26th Sep 2016, 24th Oct 2016 and 21st Nov 2016	26th Aug 2013, 3rd Sep 2013, 28th Oct 2013, 10th Feb 2014, 7th Apr 2014, 22nd Sep 2014, 30th Mar 2015, 7th Dec 2015, 4th Apr 2016, 4th Jul 2016, 8th Aug 2016, 22nd Aug 2016, 31st Oct 2016 and 7th Nov /2016

(Continued…)

(Table 6 Continued)

Parameter	Country	Brazil	Russia	India	China	South Africa
Real Estate	**Hidden Break Point**	3rd Dec 2012, 11th Aug 2014, 23rd May 2016 and 19th Jun 2017	23rd Apr 2012, 12th Nov 2012, 29th Jul 2013, 23rd Dec 2013, 11th May 2015, 22nd Aug 2016 and 16th Jan 2017	20th Aug 2012, 2nd Dec 2013, 3rd Mar 2014, 28th Jul 2014, 10th Nov 2014, 9th May 2016 and 7th Nov 2016	12th Mar 2013, 2nd Jul 2013, 28th Oct 2013, 24th Feb 2013, 21st Apr 2014, 20th Oct 2014, 28th Sep 2015, 2nd May 2016, 29th Aug 2016 and 26th Jun 2017	3rd Dec 2012, 11th Mar 2013, 6th May 2013, 17th Jun 2013, 23rd Dec 2013, 11th Aug 2014, 29th Dec 2014, 28th Sep 2015 and 3rd Apr 2017

5.3 Robustness Tests

In order to test the robustness of the results of the integral transforms, commonly used structural breaks tests are applied to verify the presence of break points: (i) augmented Dickey Fuller (ADF), (ii) ADF-generalized least squares (GLS) (ii) Phillips Perron (PP) and (iv) Zivot-Andrews (ZA) tests. The robustness test results are shown in Table 7. The traditional view of a unit root hypothesis is based on the theory that current shocks have a temporary effect and the long-run movements are not affected by such shocks. However, Perron (1989) challenges these findings and argues that the standard ADF tests are biased towards the non-rejection of the null hypothesis based on the notion that most time series are not characterised by unit roots but rather persistence arises only from large and infrequent shocks. The ADF-GLS test is also a modification of the ADF test, whereas in this test, it considers a series that features deterministic components in the form of a constant or linear trend. The testing procedure allows for de-trending a series to estimate the parameters of the series. This procedure assists in removing the means and linear trends for a series that is not far from the non-stationary region point.

The PP test can be viewed as a modified Dickey-Fuller (DF) unit root test that has been made robust to serial correlation in the error term by utilising the Newey-West (1987) heteroskedasticity and autocorrelation- consistent covariance matrix estimator. Under the PP test, the null hypothesis is that of a unit root being present, which is similar to the ADF test. The ZA is a proposed variation of the PP test that assumes the exact time of the breakpoint is unknown. The ZA break date is where the t-statistic is most significant; this is where the t-statistic from the ADF test of a unit root is at its minimum. This is the break point where there is strongest evidence against the null hypothesis of a unit root.

For the in-sample, the ADF results confirm the presence of structural break points during the period of 2012-2019, as the test values are statistically significant. That is, it is true for the four indices for every BRICS country. This is consistent with the findings of the integral transforms (i.e. Fourier and Laplace). In terms of the 2012-2017 period, all of the structural break points are confirmed by the ADF-GLS tests for the four indices, except for general equities and commodities indices for India and Russia, respectively. The reason for the consistent performance in Indian general equities might be due to the liberalisation of the market. According to Bekaert et al. (2003), the Indian general equities were liberalised in the late 1990s. For the Russian commodities markets, it might be the fact that Russia is the next largest exporter of commodities, especially oil and gas. The PP and ZA tests confirm the structural break points for the four indices of the BRICS nations during 2012-2017. The results of the same four indices for the in-sample are replicated by the out-sample period. The main difference is that the out-sample confirms the presence of structural break points as opposed to the in-sample data. This might

Table 7 Robustness Results

In Sample: 2012-2017				
Panel 1: General Equities				
Country	**ADF**	**ADF_GLS**	**PP**	**ZA**
Brazil	-17.27328 (0.0000)***	-3.2126 (0.0000)***	-17.39733 (0.0000)***	-18.1513 (0.000)***
Russia	-17.58079 (0.0000)***	-16.6755 (0.0000)***	-17.5814 (0.0000)***	-18.0856 (0.0006)***
India	-15.73895 (0.0000)***	-1.3263 (0.1860)	-15.6445 (0.0000)***	-16.1302 (0.0000)***
China	-16.65792 (0.0000)***	-5.1898 (0.0020)***	-16.8115 (0.0000)***	-17.5101 (0.000)***
South Africa	-12.82296 (0.0000)***	-3.7492 (0.0040)***	-17.8370 (0.0000)***	-18.1426 (0.0019)***
Panel 2: Real Estate				
Brazil	-17.30683 (0.0000)***	-2.9227 (0.0000)***	-17.4133 (0.0000)***	-18.0025 (0.0000)***
Russia	-18.27117 (0.0000)***	-14.3830 (0.0000)***	-18.2910 (0.0000)***	-18.7428 (0.0003)***
India	-16.1277 (0.0000)***	-17.7402 (0.0000)***	-16.1465 (0.000)***	-16.6302 (0.0001)***
China	-18.02238 (0.0000)***	-10.2775 (0.0000)***	-18.0307 (0.0000)***	-18.7896 (0.0000)***
South Africa	-18.07221 (0.0000)***	-17.8131 (0.0000)***	-18.0729 (0.0000)***	-18.2630 (0.0288)**
Panel 3: Commodities				
Brazil	-13.79419 (0.0000)***	-0.1055 (0.9160)	-65.1962 (0.0000)***	-26.3433 (0.1156)7
Russia	-11.89339 (0.0000)***	-0.4808 (0.6310)	-77.6376 (0.0000)***	-26.6164 (0.0004)***
India	-16.19014 (0.0000)***	-5.0916 (0.0000)***	-16.2187 (0.0000)***	-16.7072 (0.0000)***
China	-14.56573 (0.0000)***	-1.4849 (0.1390)***	-15.2546 (0.0000)***	-15.6712 (0.0000)***
South Africa	-16.6124 (0.0000)***	-5.4620 (0.0000)***	-16.6399 (0.0000)***	-17.0895 (0.0005)***
Panel 4: Bonds				
Brazil	-18.02346 (0.0000)***	-5.7488 (0.0000)***	-18.0466 (0.0000)***	-18.7128 (0.0000)***
Russia	-18.4531 (0.0000)***	-18.3137 (0.0000)***	-39.1954 (0.0000)***	-27.2352 (0.0000)***
India	-17.93421 (0.0000)***	-16.3294 (0.0000)***	-39.6338 (0.0000)***	-26.1177 (0.0004)***
China	-18.5369 (0.0000)***	-1.0673 (0.0000)***	-39.8675 (0.0000)***	-27.3225 (0.0000)***
South Africa	-19.7802 (0.0000)***	-38.3220 (0.0020)***	-19.8445 (0.0000)***	-19.7802 (0.0079)***

(Continued…)

(Table 7 Continued)

Out Sample: 2007-2017				
		Panel 5: General Equities		
Country	**ADF**	**ADF_GLS**	**PP**	**ZA**
Brazil	-26.1141	-4.7406	-26.0006	-27.1956
	(0.0000)***	(0.0010)***	(0.0000)***	(0.0963)*
Russia	-23.5397	-5.7182	-23.6378	-24.7129
	(0.0000)***	(0.0070)***	(0.0000)***	(0.0000)***
India	-14.5895	-7.2046	-22.4309	-23.2648
	(0.0000)***	(0.0020)***	(0.0000)***	(0.0000)***
China	-23.6047	-7.4981	-23.8296	-24.5763
	(0.0000)***	(0.0000)***	(0.0000)***	(0.0000)***
South Africa	-26.6127	-4.9341	-23.6127	-27.7004
	(0.0000)***	(0.0020)***	(0.0000)***	(0.0000)***
		Panel 6: Real Estate		
Brazil	-15.1422	-3.9855	-24.3789	-25.5267
	(0.0000)***	(0.0030)***	(0.0000)***	(0.0000)***
Russia	-19.7322	-6.9952	-21.0643	-21.5215
	(0.0000)***	(0.0000)***	(0.0000)***	(0.0000)***
India	-21.7249	-5.1461	-21.7161	-22.4564
	(0.0000)***	(0.0010)***	(0.0000)***	(0.0000)***
China	-24.5635	-10.1463	-24.5892	-25.3948
	(0.0000)***	(0.0000)***	(0.0000)***	(0.0050)***
South Africa	-24.2428	-17.8184	-24.2428	-24.9301
	(0.0000)***	(0.0000)***	(0.0000)***	(0.0000)***
		Panel 7: Commodities		
Brazil	-19.2138	-0.1696	-36.4268	-36.4268
	(0.0000)***	(0.8660)	(0.0001)***	(0.0719)*
Russia	-14.8599	-0.5089	-89.6084	-33.5459
	(0.0000)***	(0.6110)	(0.0001)***	(0.0017)***
India	-14.0478	-5.2404	-21.4659	-22.2974
	(0.0000)***	(0.0000)***	(0.000)***	(0.0000)***
China	-9.2169	-1.3740	-20.3033	-19.4907
	(0.0000)***	(0.1705)	(0.0000)***	(0.0000)***
South Africa	-24.4568	-5.5415	-24.4785	-24.7321
	(0.0000)***	(0.0000)***	(0.0000)***	(0.0000)***
		Panel 8: Bonds		
Brazil	-19.9030	-5.8396	-20.2053	-20.1958
	(0.0000)***	(0.0000)***	(0.0000)***	(0.0003)***
Russia	-27.0774	-5.1180	-45.5733	-35.3488
	(0.0000)***	(0.0000)***	(0.0001)***	(0.0000)***
India	-22.1265	-16.5746	-43.0290	-34.5019
	(0.0000)***	(0.0000)***	(0.0000)***	(0.2991)
China	-27.0955	-4.2682	-45.5372	-35.2522
	(0.0000)***	(0.0000)***	(0.0000)***	(0.0000)***
South Africa	-25.6287	-3.8638	-25.6144	-25.8981
	(0.0000)***	(0.0001)***	(0.0000)***	(0.0206)**

(Continued…)

(Table 7 Continued)

Notes: In every cell, the first variable is the test value and the one in the brackets is the p-value or significance level. ***, ** and * represent significance levels at 1%, 5% and 10%, respectively. Critical values for the ADF test -3.451, -2.870651, -2.5716 at 1%, 5% and 10% significance levels, respectively. Critical values for the ADF-GLS are -2.57, -2.89, and -3.48 at 1%, 5% and 10% significance levels, respectively. Critical values for the Phillip test -3.451, -2.870651, -2.5716 at 1%, 5% and 10% significance levels, respectively. Critical values for the ZA test are -5.57, -5.08, and -4.82 at 1%, 5% and 10% significance levels, respectively. ADF stands for augmented Dickey Fuller, (ii) ADF-GLS is ADF test modified by Elliott, Rothenberg, Stock (ERS) in 1992, (iii) PP is Phillips Perron 1988 test and (iv) ZA is for Zivot-Andrews 1992 test.

be probably due to the fact that structural break points tend to pick up over a longer rather than a shorter period of time (Holmes 2011). For the ZA tests, Bekaert et al. (2003) also explore the plotted structural break points. The salient points from the graphs are as follows starting with the in-sample period: (i) the bonds show that there is at least one structural break point up to five. Russia has two and South Africa has five break points. The bond market for South Africa is very volatile because many capital projects are financed through this market. Secondly, the commodities market structural break points range from three to six points. Thirdly, equities have one to two structural break points. Fourth, real estate structural break points range from one to nine only for South Africa. The salient point for the ZA analysis for the out-sample is that there are more structural break points during 2007-2017 than 2012-2017. This is consistent with the notion that a longer period results in increases in other structural break points. Fundamentally, for the years that were picked earlier as periods of structural breaks by integral transform, the ZA graphs confirm them as periods of structural breaks.

6. Conclusion

This study shows the following. First, integral transforms capture more structural break points than uni-or-multivariate models. In integral transforms, structural break points can be anything from tens to twenties in number while in uni-and-multivariate models, there tends to be few structural break points. Secondly, integral transforms illustrate systematic structural break points pattern(s). Thus, break points in returns lead break points in volatilities. In addition, returns tend to have more structural break points than volatilities. Thirdly, economic, political and government agreements provide a linkage between transatlantic structural break points. Lastly, in bonds, commodities, equities and real estate, evident structural break points lead hidden break points.

The implications of this study are as follows. First, it is commendable to use integral transforms in illustrating structural break points because integrals capture more break points including hidden structural break points. Similar to Enders and Holt (2012), it is more appropriate when different structural break points techniques are used together to detect structural break points. In Enders and Holt (2012), the Fourier transform and Bai-Perron test are used together and the results are quite revealing about structural breaks. Furthermore, in Enders and Hold (2012), the continuous and discrete nature of structural break points is captured by Fourier transform and the Bai-Perron test, respectively. Thus, appropriate detection of structural break points should include multiplicity in terms of movements and patterns. Secondly, in capturing structural break points, one should follow systematic patterns. Based on the nature of indices, some illiquid and others liquid, there is most likely going to be one technique that is appropriate for capturing structural break points. Thus, the model used to capture structural break points should take into account the (il)liquidity of indices. Thirdly, agreements between countries have implications for the financial markets. It can be inferred from Wong and Reddy (2018) that uniformity of measurements and interpretation of assets minimises inherent risk. Wong and Reddy (2018) opine that gearing levels of Australian REITs (hereafter, A-REITs) are relatively higher when compared to their global counterparts; therefore, A-REITs are very sensitive to short-and-long interest rate movements. On the other hand, Brooks and Tsolacos (2001) do not find significant sensitive effects of interest rates on the UK REITs. Thus, even though the standardisation of listed real estate funds into REITs is becoming a global phenomenon, different REIT associations still need to come to agreement on different REIT parameters including the level of gearing. Lastly, there are also hidden break points, which are normally led by evident structural break points. Evident structural break points do not necessarily occur and/or appear in insolation. There is always the possibility that evident structural break points are followed by hidden structural break points. The rule of thumb would be, when one measures measured evident structural break points, similarly hidden structural break points should be measured in that time series.

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