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The Pricing of Construction Loans

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In this study, we use a simple 2-period game theoretic model to determine a mutually acceptable interest rate for a construction loan. This mutually acceptable interest rate is the rate that makes a developer indifferent between using 100% equity financing and a construction loan. It is also the highest interest rate that a developer is willing to pay and a bank is willing to lend. The three risk factors identified in the model are the loss, leverage and first-phase loan ratios. Our analytical and numerical analyses indicate that the derived mutually acceptable interest rate has desirable properties as the rate increases with an increase in the three identified risk factors.

Keywords

Construction Loan, Real Option, Risk Shifting, Interest Rate

1. Introduction

Construction loans are generally viewed to be riskier than residential or commercial mortgages due to the uncertainty on whether the construction will be completed according to the specifications and schedules. Surprisingly, however, real estate researchers have given much more attention to the default risk of mortgages than to that of construction loans. In addition, the existence of the secondary market imposes a uniform standard on the underwriting of mortgages. This is why we observe that interest rates on mortgages do not differ much if the mortgage underwriting conforms to certain established standards. On the other hand, the terms of each construction loan can significantly differ, as each construction project has its unique characteristics. The unique characteristics of each construction project and the differing terms in construction loan contracts suggest that the risk-return (interest rate) trade-off of a construction loan should be analyzed at the project level.

It is fair to say that there is limited research with a main focus on construction loans. Ambrose and Peek (2008) argue that credit markets are critical to the market positions of developers, especially for private developers. They report that, during the 1988 to 1993 period, many banks with a deteriorated financial condition reduced their lending to the construction industry and there was a sustained decline in the market share of large private homebuilders. Consequently, there was an increase in the market share of public homebuilders who had better access to external funds. Chan (1999) demonstrates that credit availability has played a significant role in residential constructions. Credit availability impacts housing supply because it affects the cost of construction loans as well as the ability of builders to respond to a favorable market condition. In other words, early studies focus on the impacts of credit (construction loans) availability on the construction sector and the market structure or market supply condition of a region.

Another line of the literature focuses on the relationship between construction sectors and economic growth. We find many studies that address this topic. For example, Jackman (2010) and Alhowaish (2015) study the causal relationships between construction and growth in developing countries. However, we can only identify one early study in the real estate literature that specifically examines construction loan risk. Lusht and Leidenberger (1979) report that residential construction lending risk is driven by multiple factors, which include the unavailability of materials, inflationary cost overruns, the property development experience of the borrower, and the lending experience of the lender. However, while the study identifies the risk factors of construction loans, it does not link the risks discussed to the pricing decision (the interest rate charged) on construction loans.

This paper therefore aims to provide a basic model framework that establishes a linkage between the risks identified in construction loans and the interest rate charged by lenders. In this model, we identify three factors to represent the risk. The first risk factor is the loss ratio. The lender might face an entirely different situation when a construction loan borrower defaults versus when a mortgage borrower defaults. When a mortgage borrower defaults, the collateral is a completed property that can be used or produce income. Hence, the mortgage lender can dispose the property without much difficulty. However, when the borrower of a construction loan defaults, the lender needs to deal with an unfinished building, of which the value will be difficult to realize (by selling or renting). In other words, depending on the type of property and the stage of construction, the lender will face varying degrees of difficulty to recover the outstanding construction loan balance from the sale of an unfinished property. This is why some lenders request the developer to be personally liable for the construction loan or provide other properties as additional collateral. Given this, we know that the construction loan interest rate charged by the lender should be a function of the expected loss ratio when a borrower defaults.

The second and third risk factors that we include in the model are the first-phase loan ratio and the leverage ratio, respectively. Intuitively, we know that the higher the loan-to-value ratio of a project, the more likely it is for a borrower to default. In addition, the loss of the lender due to a default is also greater with a higher leverage ratio. Unlike a mortgage contract (which pays a lump sum amount when the contract is signed), a construction loan is released in phases in accordance with a construction schedule and the actual progress. Given this, the first-phase loan ratio has a similar impact on the default probability as the leverage ratio. A high leverage ratio with a low initial-phase loan ratio will have a much lower impact on the default decision than when the initial-phase loan ratio is also high. Similarly, a high initial-phase loan ratio will have a significant impact on the interest rate when a high leverage ratio is also used.

We develop a simple 2-period game theoretic model to determine a mutually acceptable interest rate for a construction loan. The mutually acceptable interest rate in our model is the rate that makes a developer indifferent between using 100% equity financing and a construction loan. In other words, it is the highest interest rate that a bank can charge and a developer is willing to accept. If the interest rate proposed by the lender is higher than the mutually acceptable interest rate derived by the model, the developer will be better off with 100% equity financing for the construction project.

The model that we use follows closely the models developed by Chan, Fang and Yang (2008) and Chan, Wang and Yang (2012) for analyzing the presale decisions of developers (in which a developer can abandon a construction project that is presold to buyers). We derive a closed-form solution for the construction loan interest rate and analyze its relationships with the three risk factors. Consistent with intuition and the results reported by Chan, Fang and Yang (2014), we find that the mutually acceptable interest rate and the probability of an abandonment decision increase with an increase in the loss, leverage and first-phase loan ratios. Our numerical results also show that the impact of any one of the three risk factors on the construction loan interest rate or default probability increases when the magnitude of the other two risk factors increases.

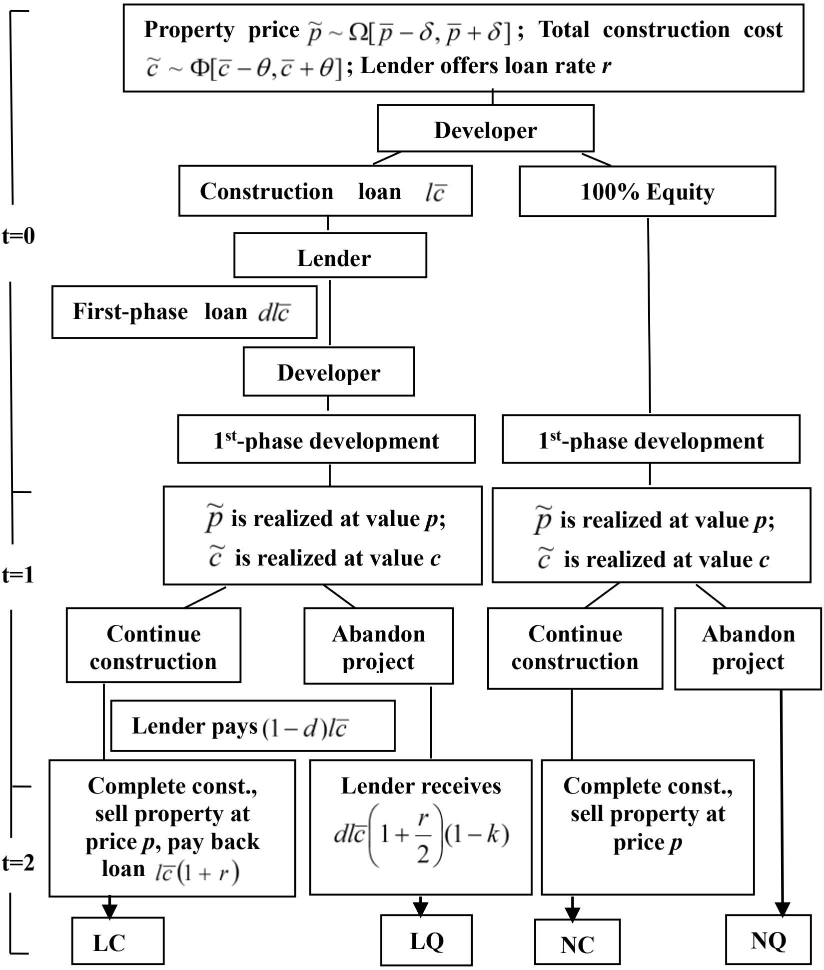
Section 2 discusses our model setup. We use a backward induction method to solve for a mutually acceptable construction loan interest rate. In this section, we also discuss the impacts of the loss, leverage and first-phase loan ratios on the mutually acceptable interest rate and the probability of a default. Section 3 uses a numerical analysis to explore results that we cannot obtain from the analytical analyses that we report in Section 2. We conclude and suggest areas for future research in Section 4.

2. Model

In our model, a developer builds a property in two phases. The two phases are specified as $t \in [0,1]$ and $t \in [1,2]$. At the beginning, or when $t = 0$, the developer launches a first-phase construction and decides on the financing method. The developer will select either 100% equity financing or a combination of equity and a construction loan. We assume that there are two uncertainties that the developer needs to deal with at $t = 0$: the construction cost $\tilde{c}$ (which is known with a probability distribution function Φ $[\bar{c}-\theta, \bar{c}+\theta]$) and the future market price of the property $\tilde{p}$ (which is known with a probability distribution function Ω $[\bar{p}-\delta, \bar{p}+\delta]$). We define $\bar{c}$ as the expected construction cost and $\bar{p}$ as the expected future market price. The dispersions of the construction cost and the future price are defined as θ and δ, respectively. We assume that the realized values of the construction cost c and future property price p cannot be observed until at time $t = 1$.

At $t = 0$, if the developer decides to finance a construction with a construction loan, we assume that the amount of the construction loan is exogenously determined at l% of the expected construction cost, or $l\bar{c}$. (In other words, we do not model the optimal leverage related issues.) To simplify the mathematical presentation, we set the risk-free rate to zero (or set the expected inflation rate and real return to zero). However, the borrower will pay a risk-adjusted interest rate on the construction loan that varies with the loan characteristics.

Figure 1 Decision Tree for Developer and Lender



Note: Game covers three time stages: $t=0$, $t=1$ and $t=2$. Market price p and total construction cost c are unknown until $t=1$. However, at $t=0$, their distributions $\tilde{p} \sim \Omega[\bar{p}-\delta, \bar{p}+\delta]$ and $\tilde{c} \sim \Phi[\bar{c}-\theta, \bar{c}+\theta]$ are known to public. At $t=0$, lender offers two-period loan at rate r and developer decides to finance construction with either equity or combination of loan $l\bar{c}$ and equity. Lender will price loan to make developer indifferent between using a loan and 100% equity. Lender will provide first-phase loan payment $dl\bar{c}$ if loan is accepted by developer. Developer launches 1st-phase development at $t=0$. At $t=1$, upon observing the realized p and c, developer decides to either continue or abandon construction. At $t=2$, if property is completed, developer will sell it to market at price p and pays loan amount and interest to lender. Game ends at one of the following terminal nodes: LC (leverage and completion), LQ (leverage and abandonment), NC (equity financing and completion), and NQ (equity financing and abandonment).

Figure 1 shows the decision tree of our model under the two financing methods. At $t=0$, knowing the probability distributions of $\tilde{c}$ and $\tilde{p}$, the lender will offer a construction loan with a selected interest rate to the developer. The construction loan amount is $l\bar{c}$ and the two-period interest rate is r. Given the package offered by the lender, the developer will decide to either accept the loan and finance the rest of the construction cost with equity or reject the loan and finance the construction with 100% equity. To simplify the model, we assume that the first-phase construction cost is h percent of the total construction cost $\bar{c}$. If the developer decides to use the construction loan, the lender immediately releases a first-phase construction loan $dl\bar{c}$, where d is the percentage of the first-phase construction loan in the total expected construction cost $l\bar{c}$. Regardless of whether a construction loan financing method or a 100% equity financing method is used, the developer will launch the first-phase construction at $t=0$ and pay the first-phase construction cost $h\bar{c}$. We assume that $h \geq dl$ so that the amount of the first-phase construction cost will be no less than the amount of the first-phase construction loan.

At $t=1$, the actual construction cost c and the property price p are realized and known to the two players. Observing the realized values, the developer will decide on whether to continue the construction (based on the information regarding the second-phase construction costs, second-phase construction loan, future property price and agreed upon construction loan interest rate) or abandon the project and default on the construction loan. If the developer decides to use 100% equity financing, the only decision that s/he needs to make is whether to abandon the project at $t=1$. If the developer decides to abandon the project, s/he incurs a loss of the sunk first-phase construction cost. The developer will pay the remaining construction cost $c-h\bar{c}$ if s/he decides to continue the project. Under this route, the construction will be completed at $t=2$ and the developer will sell the property to the market at the realized market price p.

Under the construction loan financing route, at $t=1$, if the developer chooses to continue the construction, s/he will incur a second-phase construction cost $c-h\bar{c}$ and borrow a second-phase construction loan $(1-d)l\bar{c}$. The project will be completed at $t=2$. The developer will sell the property to the market at price p and pay off the construction loan with interest that totals $l\bar{c}(1+r)$. We define r as the interest rate on the construction loan for the entire construction period (or the period from $t=0$ to $t=2$).

If the developer chooses to abandon the project, s/he will not receive the second-phase construction loan nor pay the second-phase construction cost. Under this circumstance, the developer will pay the lender a portion (could be zero) of the first-phase construction loan (with interest), or $dl\bar{c}(1+\frac{r}{2})(1-k)$. The parameter $k \in [0,1]$ measures the loss ratio of the lender. When $k=1$, the lender will not be able to recover anything after the developer abandons the project. When $k=0$, the lender recovers all of the principal and interest of the first-phase construction loan from the developer. In this model, $k \in [0,1]$ is exogenously determined. Note that we use $\frac{r}{2}$ as the interest rate for the first-phase construction loan as we assume that the length of the first-phase construction period is half that of the total project period. (Our analysis is not sensitive to the selection of the length of the first period.) The information status and cash flows at each decision point are illustrated in Figure 2.

Given all the parameter values, for each exogenously determined leverage ratio l, the lender will offer the highest corresponding interest rate that the developer is willing to accept for the project. In other words, the lender will set an interest rate r that makes the developer indifferent between the two financing methods (using 100% equity or a construction loan). Under this framework, for a given leverage ratio l, there will be a corresponding mutually acceptable interest rate r that will be offered by the lender. However, since the leverage ratio l is exogenously given, our model will not be able to solve for the optimal leverage ratio (and, therefore, the optimal interest rate of the construction loan) for the lender and the developer.

2.1 Abandonment Decision of Developer

Using a backward induction method, we first solve for the abandonment decision rules of the developer under the two alternative financing methods. Lemma 1 reports the results.

Lemma 1 *At $t=1$, the developer will continue the construction if the construction cost c and the market price for the property p satisfy*

$$c \le \hat{c} = p + h\bar{c} \tag{1}$$

when using a 100% equity financing method, and satisfy

$$c \le \hat{c}' = p - \bar{c}\left[l(r+d) - h - dl(1-k)\left(1+\frac{r}{2}\right)\right] \tag{2}$$

when using a construction loan financing method. In other words, $\hat{c}$ and $\hat{c}'$ are the construction costs that make the developer indifferent between commitment and abandonment under 100% equity and construction loan financing, respectively.

Proof. See Appendix A.

Figure 2 Information Available and Cash Flows at Each Decision Point

	t=0	t=1	t=2
Information status:	$\tilde{p} \sim \Omega[\bar{p}-\delta, \bar{p}+\delta]$, $\tilde{c} \sim \Phi[\bar{c}-\theta, \bar{c}+\theta]$	$\tilde{p} \rightarrow p$ and $\tilde{c} \rightarrow c$	
1. Loan			
Developer's cash flows when:			
Developer continues	$dl\bar{c} - h\bar{c}$	$(1-d)l\bar{c} - (c - h\bar{c})$	$p - (1+r)l\bar{c}$
Developer abandons	$dl\bar{c} - h\bar{c}$		$-dl\bar{c}\left(1+\frac{r}{2}\right)(1-k)$
Lender's cash flows when:			
Developer continues	$-dl\bar{c}$	$-(1-d)l\bar{c}$	$(1+r)l\bar{c}$
Developer abandons	$-dl\bar{c}$		$dl\bar{c}\left(1+\frac{r}{2}\right)(1-k)$
2. Equity Financing			
Developer's cash flows when:			
Developer continues	$-h\bar{c}$	$-(c-h\bar{c})$	p
Developer abandons	$-h\bar{c}$		

Note: At $t=0$, market price p and total construction cost c are unknown to public. However, all players know distributions of $\tilde{p} \sim \Omega[\bar{p}-\delta, \bar{p}+\delta]$ and $\tilde{c} \sim \Phi[\bar{c}-\theta, \bar{c}+\theta]$. At $t=1$, actual values of price and construction cost are realized (as p and c). At $t=0$, developer chooses to finance construction either with 100% equity or loan. Developer launches 1st-phase development by paying first-phase construction cost $h\bar{c}$. If developer chooses to borrow, s/he will receive a first-phase loan amount $dl\bar{c}$ from lender. At $t=1$, after observing realized p and c, developer decides to abandon or continue project. If developer decides to continue project, s/he will pay second-phase construction cost $c-h\bar{c}$. If developer uses construction loan, lender will release second-phrase construction loan $(1-d)l\bar{c}$. At $t=2$, developer will receive p from sale of property and pay off the construction loan with interest $(1+r)l\bar{c}$ if construction loan is used. If developer chooses to abandon project when construction loan is used, s/he will pay back lender a portion of the first-phase construction loan (with interest), or $dl\bar{c}(1+r/2)(1-k)$.

From Lemma 1, we know that a developer will continue the project if $c \le \hat{c} = p + h\bar{c}$ when a 100% equity financing method is used. The developer will continue the project if $c \le \hat{c}' = p - \bar{c}\left[l(r+d) - h - dl(1-k)\left(1+\frac{r}{2}\right)\right]$ when a construction loan financing method is used. From Equations (1) and (2), we find that $\hat{c}' < \hat{c}$, or

$$\hat{c}' - \hat{c} = -l\bar{c}\left[\frac{r}{2}(2-d) + dk\left(1+\frac{r}{2}\right)\right] < 0. \tag{3}$$

Equation (3) indicates that it is more likely for the developer to abandon a project when there is a construction loan than when a 100% equity financing method is used. In other words, the developer will have a higher expected abandonment probability when a construction project is leveraged than when it is unleveraged. This resembles the risk shifting effect of financial leverage addressed in the capital structure literature.

To make the model realistic and simplify the mathematical derivation, we make two assumptions. First, regardless of the financing method used, we assume that the developer will have the chance to abandon the project when the realized market price is the average market price $\bar{p}$. This means that

$$\hat{c}\,|_{p=\bar{p}} = \bar{p} + h\bar{c} < \bar{c} + \theta, \tag{4}$$

and

$$\hat{c}'\,|_{p=\bar{p}} = \bar{p} - \bar{c}\left[l(r+d) - h - dl(1-k)\left(1+\frac{r}{2}\right)\right] < \bar{c} + \theta. \tag{5}$$

From Equation (3), we know that if Equation (4) holds, Equation (5) must also hold. Given this,

$$\bar{p} < (1-h)\bar{c} + \theta. \tag{6}$$

This assumption guarantees that, when the construction cost is sufficiently high, the abandonment option can still be "in-the-money" with the expected future market price $\bar{p}$.

Second, regardless of the financing method used, we assume that the developer will have the chance to continue the project even if the realized future market price is at the average price $\bar{p}$. This means that

$$\hat{c}\,|_{p=\bar{p}} = \bar{p} + h\bar{c} > \bar{c} - \theta \tag{7}$$

and

$$\hat{c}'\,|_{p=\bar{p}} = \bar{p} - \bar{c}\left[l(r+d) - h - dl(1-k)\left(1+\frac{r}{2}\right)\right] > \bar{c} - \theta. \tag{8}$$

From Equation (3), we know that if Equation (8) holds, Equation (7) must also hold. This means that

$$\bar{p} > \bar{c} - \theta + \bar{c}\left[l(r+d) - h - dl(1-k)\left(1+\frac{r}{2}\right)\right]. \tag{9}$$

This assumption guarantees that, when the construction cost is sufficiently low, the abandonment option can still be "out-of-the-money" with the expected future market price $\bar{p}$. From Equation (9), we can find the upper boundary of the construction loan interest rate r, or

$$r < r^M = \frac{\bar{p} - (1-h+dlk)\bar{c} + \theta}{l\bar{c}\left[1 - \frac{d}{2}(1-k)\right]}. \tag{10}$$

2.2 Expected Profit Functions

We now discuss the expected profit of the developer from the project and the probability of a developer abandoning the project. If a developer uses 100% equity, the expected profit from the project is

$$E(\pi) = \int_{\bar{p}-\delta}^{\bar{p}+\delta} \int_{\bar{c}-\theta}^{\hat{c}} \left(p_p - \tilde{c}\right) d\Phi(\tilde{c})\, d\Omega(\tilde{p}) + \int_{\bar{p}-\delta}^{\bar{p}+\delta} \int_{\hat{c}}^{\bar{c}+\theta} \left(-h\tilde{c}\right) d\Phi(\tilde{c})\, d\Omega(\tilde{p}). \tag{11}$$

The first double-integral term in Equation (11) is the expected payoff when the developer continues the project. The second double-integral term is the expected payoff if the developer abandons the project. The probability for the developer to abandon the project is

$$Pr(\text{ABD}) = \int_{\bar{p}-\delta}^{\bar{p}+\delta} \int_{\hat{c}}^{\hat{c}+\theta} d\Phi(\tilde{c})\, d\Omega(\tilde{p}) = \frac{(1-h)\bar{c} + \theta - \bar{p}}{2\theta}, \tag{12}$$

When a construction loan is used, the expected profit of the developer is

$$\begin{aligned} E(\pi') = & \int_{\bar{p}-\delta}^{\bar{p}+\delta} \int_{\bar{c}-\theta}^{\hat{c}'} \left(p_p - \tilde{c} - rl\bar{c}\right) d\Phi(\tilde{c})\, d\Omega(\tilde{p}) \\ & + \int_{\bar{p}-\delta}^{\bar{p}+\delta} \int_{\hat{c}'}^{\bar{c}+\theta} \left(dl\bar{c} - h\tilde{c} - dl\bar{c}\left(1 + \frac{r}{2}\right)(1-k) \right) d\Phi(\tilde{c})\, d\Omega(\tilde{p}). \end{aligned} \tag{13}$$

Similar to Equation (11), the first double-integral term in Equation (13) is the expected payoff if the developer continues the construction. The second double-integral term is the expected profit if the developer abandons the project. Under this construction loan financing method, the probability that the developer abandons the project is

$$Pr(\text{ABD})' = \int_{\bar{p}-\delta}^{\bar{p}+\delta} \int_{\hat{c}'}^{\bar{c}+\theta} d\Phi(\tilde{c})\, d\Omega(\tilde{p}) = \frac{(1-h+dkl)\bar{c} + \theta - \bar{p} + \bar{c}lr\left[1 - \frac{d}{2}(1-k)\right]}{2\theta}. \tag{14}$$

2.3 Mutually Acceptable Interest Rate

We are now in the position to discuss the mutually acceptable interest rate for both the lender and the developer when a construction loan is used. Simply put, the lender will set the construction loan interest rate r at a level such that the developer is indifferent between using a construction loan financing method and a 100% equity financing method. We obtain

$$r^* \in \{r \mid E(\pi) = E(\pi'(r))\}, \tag{15}$$

subject to

$$r^* < r^M = \frac{\bar{p} - (1 - h + dlk)\bar{c} + \theta}{l\bar{c}\left[1 - \frac{d}{2}(1-k)\right]},$$

where $E(\pi)$ and $E(\pi'(r))$ are as specified in Equations (11) and (13), respectively. This means that when the construction loan interest rate is set at r^*, the expected payoffs of the developer with or without a construction loan will be the same. Under this framework, the unconstrained interest rate selected by both the lender and the developer is

$$r^* = \frac{2}{A}\left(B - \sqrt{B^2 - C}\right), \tag{16}$$

where

$$A = \bar{c}l\left[2 - d(1-k)\right]^2 > 0, \tag{17}$$

$$\begin{aligned} B &= \theta\left[2 + d(1-k)\right] + \left[\bar{p} - \bar{c}\left(1 - h + dkl\right)\right]\left[2 - d(1-k)\right] \\ &= \left\{\bar{p} - \left[\bar{c} - \theta + \bar{c}\left[l(r+d) - h - dl\left(1 - k)(1 + \frac{r}{2}\right)\right]\right]\right\}\left[2 - d(1-k)\right] \\ &\quad + \frac{Ar}{2} + 2\theta d(1-k) \\ &> 0, \end{aligned} \tag{18}$$

$$C = dkA\left\{2\left[(1-h)\bar{c} + \theta - \bar{p}\right] + \bar{c}dkl\right\} > 0. \tag{19}$$

From Equation (9), we know that Equation (18) > 0. From Equation (6), we know that Equation (19) > 0. In addition, the selected interest rate r^* cannot exceed the boundary condition specified in Equation (10). To ensure that the condition $r^* < r^M$ holds, we need

$$\frac{2}{A}\left(B - \sqrt{B^2 - C}\right) < \frac{\bar{p} - (1 - h + dlk)\bar{c} + \theta}{l\bar{c}\left[1 - \frac{d}{2}(1-k)\right]}. \tag{20}$$

This constraint will ensure that the lender will not select a construction loan interest rate r^* that is too high to induce a high default probability. With this additional constraint, a mutually acceptable loan interest rate r^* is the one specified by Equation (16) subject to Equation (20).

2.4 Relationship with Other Parameters

With this mutually acceptable interest rate r^* in mind, we now analyze its relationship with the loss ratio k, first-phase construction loan ratio d, and leverage ratio l. We report this result in Proposition 1. We also analyze the relationships between the probability of abandonment decision $\Pr(ABD)'$ and the loss ratio k, first-phase construction loan ratio d, and leverage ratio l. We report the results in Proposition 2.

Proposition 1 *The mutually acceptable interest rate r^* is increasing in the loss ratio k, and increasing in the first-phase loan ratio d if $k > \frac{1}{1+\frac{2}{r}}$. The interest rate r^* is also increasing in the leverage ratio l if $\Omega > 0$, where*

$$\Omega = 4\theta\left[d\left(k(1+\frac{r}{2})-\frac{r}{2}\right)-r\right] \\ +4\left[d\left(k(1+\frac{r}{2})-\frac{r}{2}\right)+r\right]\left\{\bar{c}\left[(1-h+dkl+lr)-\frac{dlr}{2}(1-k)\right]-\bar{p}\right\}. \tag{21}$$

For a given interest rate r,

$$\frac{d\Omega}{dl} > 0, \tag{22}$$

$$\frac{d\Omega}{dd} > 0 \text{ if } k > \frac{1}{1+\frac{2}{r}}, \tag{23}$$

and

$$\frac{d\Omega}{dk} > 0. \tag{24}$$

Proof. See Appendix B.

The result is quite intuitive. Proposition 1 shows that the lender and the developer will agree on a higher interest rate when the first-phase construction loan ratio is larger. This makes sense. The higher the first-phase loan ratio, the greater the likelihood for a developer to default on the construction loan. Lai, Wang and Zhou (2004), Chan, Wang and Yang (2012) and Chan, Fang and Yang (2014) provide the intuition for this observation. Proposition 1 also indicates that when the loss ratio k is large, the developer and the lender will accept a high construction loan interest rate. This high interest rate is used to compensate the lender in case of a default, as the loss of the lender is greater when the loss ratio is higher.

Proposition 1 also indicates that a higher leverage ratio l means a higher construction loan interest rate if the magnitude of the first-phase loan and the loss ratios is high enough (see Equations (21), (22), (23) and (24)). This is true because an increase in the leverage ratio has little effect on the payoff of the lender when the first-phase loan and loss ratios are both zero. We now examine the effects of k, d and l on the probability of an abandonment decision $\Pr(ABD)'$. Proposition 2 reports the results.

Proposition 2 *Under a mutually acceptable interest rate* r^*, *the probability of an abandonment decision* $\Pr(ABD)'$ *increases when the loss ratio* k *increases, or when the first-phase loan ratio* d *increases if* $k > \frac{1}{1+\frac{2}{r}}$. $\Pr(ABD)'$ *also increases when the leverage ratio* l *increases if* $\Omega > 0$.

Proof. See Appendix C.

Proposition 2 shows that the probability of an abandonment decision can also be an increasing function of k, d and l. Note that the results reported in Proposition 2 are derived when we fix the interest rate at the mutually acceptable interest rate level. We are also interested in analyzing if the impact of one risk factor (for example, first-phase loan ratio d) on the probability of an abandonment decision $\Pr(ABD)'$ can be affected by changes in the magnitude of the other two risk factors (in the example, loss ratio k and leverage ratio l). The analytical solution to this question is too complicated to provide clear intuitions. Given this, we decide to discuss the results by using numerical analysis.

3. Numerical Analysis

In this section, we provide numerical examples to demonstrate the analytical results derived in Propositions 1 and 2. We also analyze how the impact of one risk factor on the construction loan interest rate (and the abandonment risk) changes when the magnitude of the other risk factors changes. We make sure that the assigned parameter values guarantee that the conditions specified in Equations (6), (9) and (20) hold. The benchmark parameter value set includes 1) the first-phase construction cost ratio $h = 20\%$; 2) the expected construction cost $\bar{c} = 13.5$; 3) the dispersion of the construction cost $\theta = 7$; 4) the expected market price $\bar{p} = 14$; and 5) the dispersion of the market price $\delta = 9$. The qualitative conclusions from the results of our numerical analysis do not differ much if we select the parameter values within their reasonable ranges. Table 1 shows a few of the numerical examples under this benchmark parameter set.

Table 1 Numerical Examples with Benchmark Parameter Set

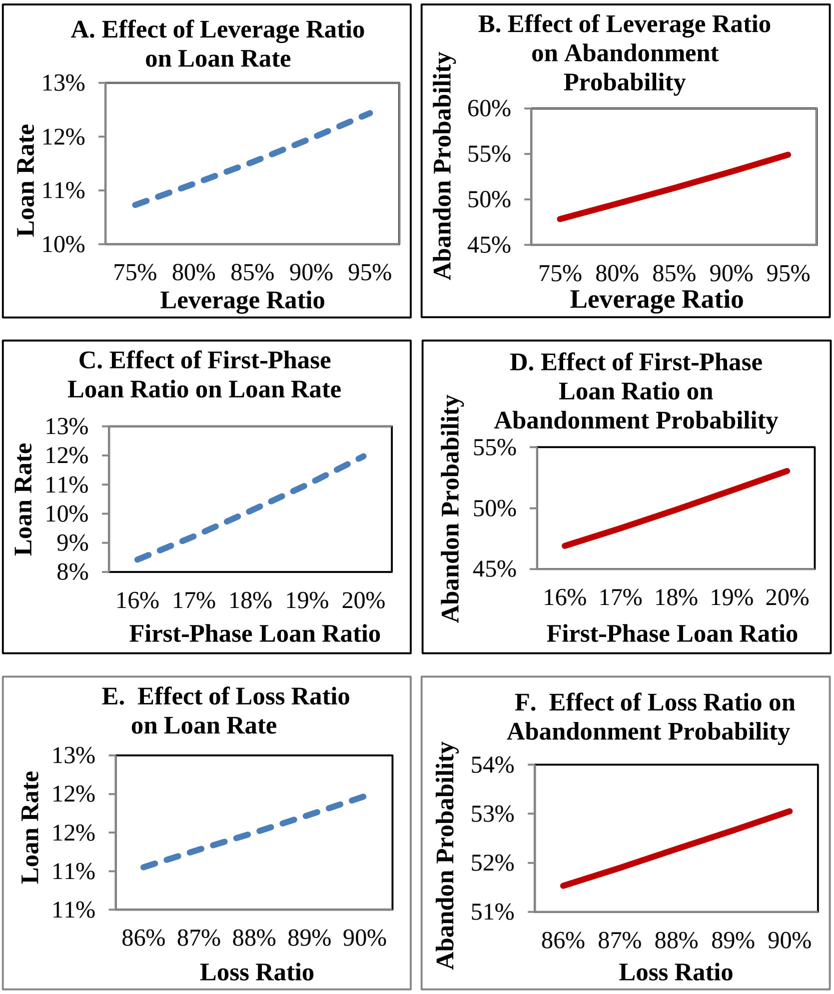
Selected k , l , d	$\Pr(ABD)'$	r^*	$E(\pi'(r))$
$k = 90\%, l = 95\%, d = 20\%$	54.91%	12.44%	1.98
$k = 89\%, l = 90\%, d = 20\%$	52.66%	11.73%	1.98
$k = 88\%, l = 85\%, d = 19\%$	49.14%	10.24%	1.98

Note: Table shows abandonment probability $\Pr(ABD)'$, loan interest rate r^* and expected profit of developer $E(\pi'(r))$ in each of the three numerical examples with benchmark parameters *k, l* and *d*, where *k* is the loss ratio, *l* is the leverage ratio and *d* is the first-phase loan ratio.

The results reported in Propositions 1 and 2 are illustrated in Figure 3 by using numerical analyses. We conduct comparative static analysis by spanning (within reasonable ranges) one of the three risk factors, k , d and l , while holding the other two risk factors constant. The six charts reported in Figure 3 indicate that when the construction loan interest rate and the abandonment decision are jointly determined, the former is positively affected by the leverage, first-phase loan and loss ratios. For Chart A titled "Effect of Leverage Ratio on Loan Rate" and Chart B titled "Effect of Leverage Ratio on Abandonment Probability", we specify that the first-phase loan ratio $d = 20\%$ and the loss ratio $k = 90\%$. We then span the leverage ratio l from 75% to 95%. In Chart C titled "Effect of First-Phase Loan Ratio on Loan Rate" and Chart D titled "Effect of First-Phase Loan Ratio on Abandonment Probability", we specify that the leverage ratio $l = 90\%$ and the loss ratio $k = 90\%$. We then span the first-phase loan ratio d from 16% to 20%. For Chart E titled "Effect of Loss Ratio on Loan Rate" and Chart F titled "Effect of Loss Ratio on Abandonment Probability", we specify that the leverage ratio $l = 90\%$ and the first-phase loan ratio $d = 20\%$. We then span the loss ratio k from 86% to 90%.

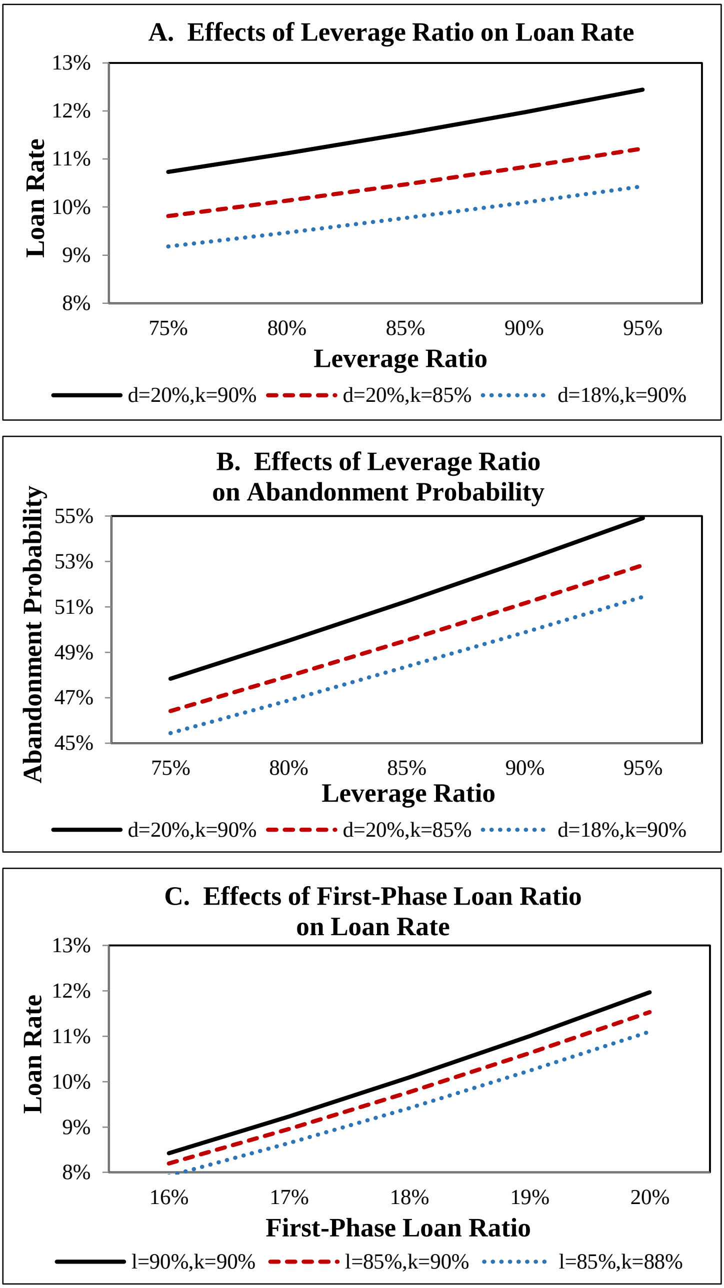
Figure 3 shows that, as predicted by Proposition 1, the construction loan interest rate that makes the developer indifferent between selecting a construction loan and using 100% equity is increasing in *l*, d and k . Figure 3 also shows that, as predicted by Proposition 2, the probability of an abandonment decision $\Pr(ABD)'$ is increasing in *l*, d and k . Note that in our numerical results reported in Figure 3, the expected payoff of the developer $E(\pi'(r))$ is fixed at 1.98. This is because the construction loan interest rate is set to make $E(\pi'(r)) = E(\pi)$ (see Equation (15)) and $E(\pi)$ is not affected by *l*, d or k .

Figure 3 First Set of Numerical Results



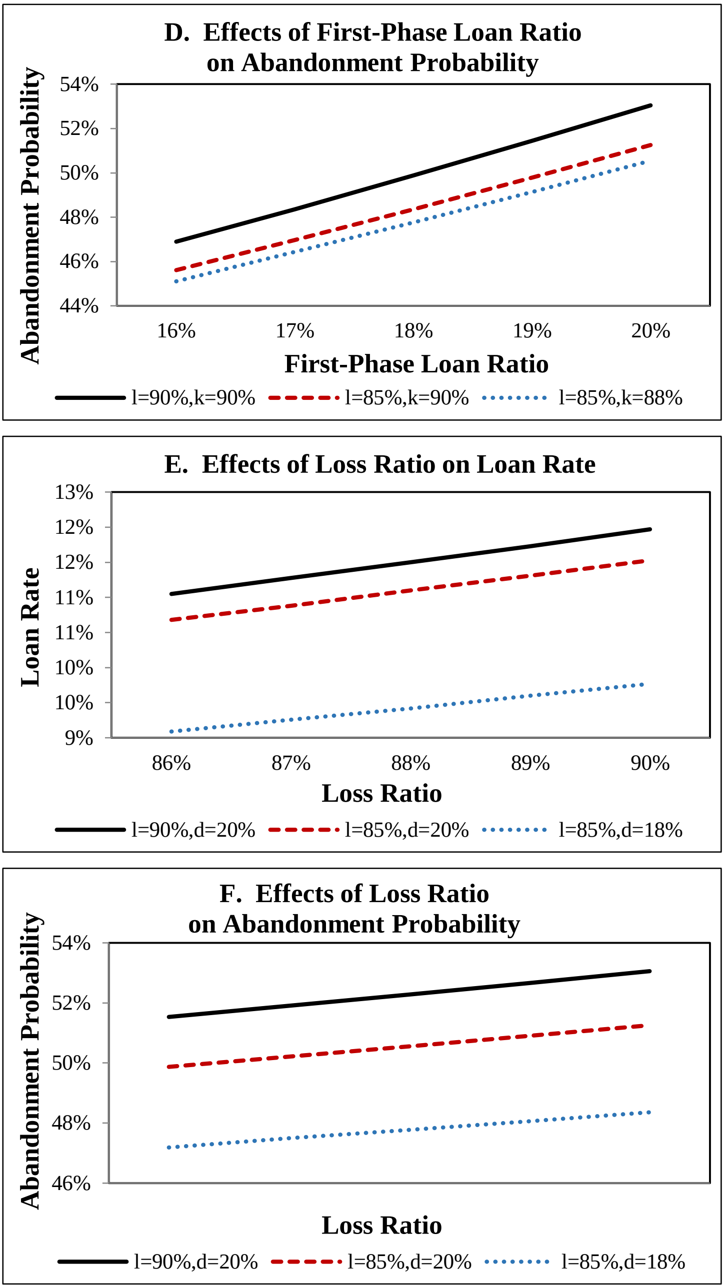
We now extend our numerical analysis presented in Figure 3 by analyzing if the effects of l, d or k on the mutually acceptable construction loan interest rate r^* and the probability of an abandonment $\Pr(ABD)'$ differ when the values of the other parameters vary. Specifically, we would like to see whether the impact of one risk factor (for example, the loss ratio k) on the loan rate (or probability of an abandonment) will systematically change when we change the magnitude of the other two risk factors (in this case, l and d). Figure 4 illustrates the results.

Figure 4 Second Set of Numerical Results



(Continued…)

(Figure 4 Continued)



Note: *k* is the loss ratio, *l* is the leverage ratio and *d* is the first-phase loan ratio.

Overall, as all six charts in Figure 4 illustrate, we find that the impacts of one risk factor (for example, the leverage ratio l) on the loan rate and the probability of an abandonment decision are larger when the magnitude of the other two risk factors (in this example, the first-phase loan ratio d and the loss ratio k) is greater. This result is shown in Charts A and B of Figure 4. When we set $k = 90\%$, the positive effects of the leverage ratio l on the loan rate and on the probability of an abandonment decision are stronger when $d = 20\%$ than when the first phase loan ratio $d = 18\%$. Similarly, given $d = 20\%$, the positive effects of l on the loan rate and the probability of an abandonment decision are greater when $k = 90\%$ than when $k = 85\%$. Charts C and D (Charts E and F) in Figure 4 show similar patterns when d (loss ratio k) is used as the main risk factor. The impacts of the first phase loan ratio d (loss ratio k) on the loan rate and the probability of an abandonment decision are larger when the magnitude of leverage ratio l or loss ratio k (leverage ratio l or first-phase loan ratio d) is larger.

4. Conclusions

A review of the real estate literature indicates that the pricing of construction loans is under-researched. In addition, since construction loans have heterogeneous characteristics, their interest rates should significantly differ from each other based on the risk levels. Indeed, if a construction loan interest rate is independent of its risk, there will be a risk-shifting effect as developers can use risky development strategies (by borrowing as much as they can) and transfer the risk to the lender (by defaulting on the construction loan when it is best for the developer to do so).

In this paper, we develop a basic model for the pricing of construction loans based on the characteristics of the development. We pay particular attention to three risk factors: loss, leverage and first-phase loan ratios. Our basic model, while not detailed enough for practical usage yet, provides a framework for determining a construction loan interest rate based on the risk characteristics of the development. The results of our model are consistent with the intuition that increased development risk means higher construction loan interest rates.

There are three areas that we need to improve upon before this model can be used in day-to-day operations. First, we have not discussed how to estimate the loss ratio in this model. We just assume that, when a default happens, the bank can recover a portion of the loss from the developer or the to-be-completed properties. We have not discussed on how to estimate this ratio. It is possible that a developer can assume personal liability on the loan to reduce the risk. (For a recent discussion on how the attitude of developers on the investment affects the performance of the properties, see, for example, Sehgal, Upreti, Pandey and Bhatia (2015).) It is also possible that a developer can use a portfolio of properties as collateral for a group of construction loans. We think that the issues related to personal liability and the use of a portfolio of development properties as collateral for a group of construction loans might be fruitful areas for future research. (For a discussion on the benefit of using diversification strategies, see, for example, Cheok, Sing and Tsai, (2011) and Ciochetti, Lai and Shilling (2015).)

The second issue that might deserve our attention is the determination of an optimal loan-to-value ratio for a construction loan. In this paper, we provide a model to determine the interest rate of a construction loan when we know the loan-to-value ratio. In other words, the leverage ratio in our model is exogenously determined. However, both the lender and the developer might want to know the best leverage ratio that they should use. (For a discussion on optimal capital structure with the use of a real options approach, see, for example, Jou and Lee (2011).) To achieve this goal, we need to specify the objective functions of both the lender and the developer. Although it is not difficult to solve this issue once we know the objective functions of the two players, it might be difficult to come up with suitable objective functions for the developer and the lender. We recommend this topic for future research.

Finally, in this model, we assume that the developer and the lender will settle on an interest rate that makes the developer indifferent between using 100% equity financing and a construction loan. While this assumption is reasonable as a first step for the analysis, in reality, the lender will have to give the developer some benefits to motivate the developer to use a construction loan. Collins, Harrison, and Seiler (2015) model the negotiation of a borrower with a lender on a mortgage modification. Future researchers might be able to use a similar approach to include a negotiation process in the determination of the construction loan interest rate.

In this paper, we derive a mutually acceptable interest rate for a construction loan by treating the loss, leverage and first-phase loan ratios as exogenously determined. These constraints limit the use of the model at this point. If we can improve on the model by allowing these three factors to be endogenously determined, the model can then be used to derive the optimal (or equilibrium) construction loan interest rate. With these improvements, the model will become one that the real estate industry can use in its day-to-day operations for designing construction loan contracts.

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Appendix

A. Proof for Lemma 1

We solve the game by using a backward induction method. We first analyze the abandonment decision of the developer at $t=1$. At this point in time, both the developer and the lender know the value of the total construction cost c and the market price of the property p. With the information, the developer makes a decision to either abandon the project or continue the construction. When unlevered, if the developer continues the construction, the developer will incur a second-phase construction cost $c-h\bar{c}$ and sell the completed property at price p at $t=2$. If the developer abandons the project, the incremental payoff will be zero. To make the decision on abandonment, the developer compares the payoff from continuing the construction, $\Delta\pi|_{\text{continue}}$, with that from abandoning the project, $\Delta\pi|_{\text{abandon}}$, where

$$\Delta\pi|_{\text{continue}} = -(c-h\bar{c})+p, \tag{25}$$

$$\Delta\pi|_{\text{abandon}} = 0. \tag{26}$$

From Equations (25) and (26), we know that the necessary and sufficient condition for the unlevered developer to continue the construction is $\Delta\pi|_{\text{continue}} \geq \Delta\pi|_{\text{abandon}}$. This condition can be specified as $c \leq \hat{c} = p + h\bar{c}$ (which is Equation (1)), where $\hat{c}$ is the highest total construction cost under which an unlevered developer is willing to continue the construction.

When levered, if the developer continues the construction, the developer will receive a second-phase construction loan $(1-d)l\bar{c}$ and incur a second-phase construction cost $(c-h\bar{c})$. At $t=2$, the developer will sell the completed property at the market price p and pay back the construction loan at $l\bar{c}(1+r)$. If the developer abandons the project, the developer might need to pay the lender an amount equal to $dl\bar{c}(1+\frac{r}{2})(1-k)$. To make an abandonment decision, the developer compares the incremental payoff from continuing the construction, $\Delta\pi'|_{\text{continue}}$, with the payoff from abandoning the project, $\Delta\pi'|_{\text{abandon}}$, where

$$\Delta\pi'|_{\text{continue}} = -(c-h\bar{c})+(1-d)l\bar{c}+p-l\bar{c}(1+r), \tag{27}$$

$$\Delta\pi'|_{\text{abandon}} = -dl\bar{c}(1+\frac{r}{2})(1-k). \tag{28}$$

From Equations (27) and (28), we know that the necessary and sufficient condition for the levered developer to continue the construction is

$\Delta\pi'|_{\text{continue}} \geq \Delta\pi'|_{\text{abandon}}$. This can be specified as $c \leq \hat{c}' = p - \bar{c}\left[l(r+d) - h - dl(1-k)\left(1+\frac{r}{2}\right)\right]$ (which is Equation (2)), where $\hat{c}'$ is the highest total construction cost under which a levered developer is willing to continue the construction.

■ Q.E.D.

B. Proof for Proposition 1

From Equation (15), we define

$$f = E(\pi'(r^*)) - E(\pi(r^*)) = 0.$$

We know that the expected unlevered profit of the developer $E(\pi(r^*))$ is not affected by the parameters k, l and d. Using Equation (13) and taking a derivative of $E(\pi')$ with respect to k, d, l and r (assuming an exogenously determined r), we derive

$$\frac{\partial E(\pi')}{\partial k} = \frac{l\bar{c}d(1+\frac{r}{2})}{2\theta}\left\{\bar{c} + \theta - (\bar{p} + h\bar{c}) + \bar{c}l\left[r\left(1-\frac{d}{2}\right) + dk\left(1+\frac{r}{2}\right)\right]\right\} > 0, \tag{29}$$

$$\frac{\partial E(\pi')}{\partial d} = \frac{rl\bar{c}\left[k(1+\frac{2}{r})-1\right]}{4\theta}\left\{\bar{c} + \theta - (\bar{p} + h\bar{c}) + \bar{c}l\left[r\left(1-\frac{d}{2}\right) + dk\left(1+\frac{r}{2}\right)\right]\right\} > 0$$

$$\text{if } k > \frac{1}{1+\frac{2}{r}}; \tag{30}$$

$$\frac{\partial E(\pi')}{\partial l} = \frac{\bar{c}\Omega}{2\theta} > 0 \text{ if } \Omega > 0, \tag{31}$$

where $\Omega = \theta\left[d\left(k(1+\frac{r}{2}) - \frac{r}{2}\right) - r\right]$

$$+\left[d\left(k(1+\frac{r}{2}) - \frac{r}{2}\right) + r\right]\left\{\bar{c}\left[\left(1-h+dkl+lr\right) - \frac{dlr}{2}(1-k)\right] - \bar{p}\right\},$$

$$\frac{d\Omega}{dk} = d(1+\frac{r}{2})\left\{\left[\left(\bar{c}+\theta\right) - \left(\bar{p}+h\bar{c}\right)\right] + 2l\bar{c}\left[r\left(1-\frac{d}{2}\right) + dk\left(1+\frac{r}{2}\right)\right]\right\} > 0; \tag{32}$$

$$\frac{d\Omega}{dd} = \frac{1}{2}r\left[k(1+\frac{2}{r})-1\right]\left\{\left[\left(\bar{c}+\theta\right) - \left(\bar{p}+h\bar{c}\right)\right] + 2l\bar{c}\left[r\left(1-\frac{d}{2}\right) + dk\left(1+\frac{r}{2}\right)\right]\right\} > 0$$

$$\text{if } k > \frac{1}{1+\frac{2}{r}}; \tag{33}$$

$$\frac{d\Omega}{dl}=\bar{c}\left[d\left(k(1+\frac{r}{2})-\frac{r}{2}\right)+r\right]^2>0; \tag{34}$$

$$\frac{\partial E(\pi')}{\partial r}=-\frac{l\bar{c}}{2\theta}\{\left[1-\frac{d}{2}(1-k)\right]\left[\bar{p}-\left(\bar{c}-\theta+\bar{c}\left[l(r+d)-h-dl\left(1-k)(1+\frac{r}{2}\right)\right]\right)\right]$$
$$+2\theta d(1-k)\}<0. \tag{35}$$

It should be noted that Equations (29), (30), (32) and (33) hold because $\bar{c}+\theta-(\bar{p}+h\bar{c})>0$ (see Equation (6)). Equation (35) holds because $\bar{p}>\bar{c}-\theta+\bar{c}\left[l(r+d)-h-dl(1-k)\left(1+\frac{r}{2}\right)\right]$ (see Equation (9)). Furthermore, from Equations (29), (30), (31) and (35), we know $\frac{\partial E(\pi')}{\partial k}>0$, $\frac{\partial E(\pi')}{\partial d}>0$ if $k>\frac{1}{1+\frac{2}{r}}$, $\frac{\partial E(\pi')}{\partial l}>0$ if $\Omega>0$, and $\frac{\partial E(\pi')}{\partial r}<0$. With the results, we can derive

$$\frac{dr^*}{dk}=-\frac{\left(\frac{\partial f}{\partial k}\right)}{\left(\frac{\partial f}{\partial r^*}\right)}=-\frac{\left(\frac{\partial E(\pi')}{\partial k}\right)}{\left(\frac{\partial E(\pi')}{\partial r}\right)}>0, \tag{36}$$

$$\frac{dr^*}{dd}=-\frac{\left(\frac{\partial f}{\partial d}\right)}{\left(\frac{\partial f}{\partial r^*}\right)}=-\frac{\left(\frac{\partial E(\pi')}{\partial d}\right)}{\left(\frac{\partial E(\pi')}{\partial r}\right)}>0 \text{ if } k>\frac{1}{1+\frac{2}{r}}, \tag{37}$$

$$\frac{dr^*}{dl}=-\frac{\left(\frac{\partial f}{\partial l}\right)}{\left(\frac{\partial f}{\partial r^*}\right)}=-\frac{\left(\frac{\partial E(\pi')}{\partial l}\right)}{\left(\frac{\partial E(\pi')}{\partial r}\right)}>0 \text{ if } \Omega>0. \tag{38}$$

■ Q.E.D.

C. Proof for Proposition 2

Using Equation (14), we first take a derivative of $\Pr(ABD)'$ with respect to k, d, l and r (assuming r is exogenously determined). We obtain:

$$\frac{\partial \Pr(ABD)'}{\partial k}=\frac{l\bar{c}d(1+\frac{r}{2})}{2\theta}>0; \tag{39}$$

$$\frac{\partial \Pr(ABD)'}{\partial d}=\frac{l\bar{c}\left[k(1+\frac{r}{2})-\frac{r}{2}\right]}{2\theta}>0 \text{ if } k>\frac{1}{1+\frac{2}{r}}; \tag{40}$$

$$\frac{\partial \Pr(ABD)'}{\partial l}=\frac{\bar{c}\left[r(1-\frac{d}{2})+dk(1+\frac{r}{2})\right]}{2\theta}>0; \tag{41}$$

$$\frac{\partial \Pr(ABD)'}{\partial r} = \frac{l\bar{c}\left[1-\frac{d}{2}(1-k)\right]}{2\theta} > 0. \tag{42}$$

We now take a derivative of $\Pr(ABD)'$ with respect to k, d and l (with the endogenously solved r^*) by using Equation (14). Together with the results derived in Equations (36) to (42), we derive

$$\frac{d\Pr(ABD)'}{dk} = \frac{\partial \Pr(ABD)'}{\partial k} + \frac{\partial \Pr(ABD)'}{\partial r} \cdot \frac{dr^*}{dk} > 0; \tag{43}$$

$$\frac{d\Pr(ABD)'}{dd} = \frac{\partial \Pr(ABD)'}{\partial d} + \frac{\partial \Pr(ABD)'}{\partial r} \cdot \frac{dr^*}{dd} > 0 \text{ if } k > \frac{1}{1+\frac{2}{r}}\ ; \tag{44}$$

$$\frac{d\Pr(ABD)'}{dl} = \frac{\partial \Pr(ABD)'}{\partial l} + \frac{\partial \Pr(ABD)'}{\partial r} \cdot \frac{dr^*}{dl} > 0 \text{ if } \Omega > 0. \tag{45}$$

■ Q.E.D.