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Intensity and Timing Options in Real Estate Developments

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Developers make decisions around timing and intensity simultaneously when exercising a development option. Built on the early real options models, we allow the demand shock and the cost functions to be dependent on the intensity of real estate development. Based on a set of input parameters, the numerical results show that demand uncertainty delays development activities, and the rental elasticity to density change has an inverse effect on the deferment option values. In a market where the intensity impact on rental income is small, development activities are likely to be curtailed when market volatility increases. More empirical tests could be conducted on whether more smaller-scale projects are triggered in down markets relative to up markets.

Keywords:

Real Options, Optimal timing, Optimal Intensity, Real Estate Development

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1. Introduction

Real estate development is a capital-intensive investment. Developers make simultaneous decisions on optimal timing and production capacity when developing vacant lands. Williams (1991) and Capozza and Li (1994) model optimal intensity and optimal timing as joint option triggering conditions in their real options models. In the models, developers simultaneously decide on when to start developing and how much to build on a land.

Real estate investment is lumpy; there are two technical constraints that make an investment decision irreversible once committed. First, the zoning rules impose a maximum permissible development intensity of a land parcel. Second, as real estate projects are built sequentially, it is costly to change the intensity decision on the land after the foundation and floor plate are built. Given the "lumpiness" nature of the investment, developers cannot just change the capacity as and when the need arises. Therefore, they have to decide on the optimal development intensity at the same time when the option to develop a land is exercised.

Phasing out and varying the intensity are not realistic in most stand-alone real estate development projects. However, there are exceptions for some of the more large-scale real estate development projects, such as the Canary Wharf development[1] in the United Kingdom (UK), and the Marina Bay Financial Centre (MBFC)[2] in Singapore. In these projects, the developers diversify their risks by developing the lands incrementally and sequentially over a long period of time. The two real estate projects fit well into the capacity choice model of Pindyck (1988), which offers flexibility to allow developers to choose different intensities at each phase of the development, as long as the marginal return of the investment is positive.

Like Williams (1991) and Capozza and Li (1994), we model the intensity and the timing decisions as joint decisions in a real estate development process. The capital choice and development timing decisions are not separable. They are simultaneously and jointly solved in the real options models. This paper adds two variations to the early models of Williams (1991) and Capozza and Li (1994). First, we model the cash flow as a function of a stochastic demand

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1 The Canary Wharf development is a massive urban regeneration project that covers an area of 8.6 acres in the London Docklands. To date, approximately 6 million net square feet of office floor space have been constructed.

2 The MBFC is a proposed landmark development in the Marina South downtown of Singapore. The project is built on a site of 3.55 hectares that is connected to an adjoining subterranean space of 1.8 hectares. The project consists of 438,000 square meters (sqm) of integrated commercial space and at least 60% of the space is used as office space.

shock.[3] Second, we allow the demand and the cost functions to be linked via the same development intensity variable.

Our numerical analyses show that the intensity option premiums increase the waiting option premiums, which further delays real estate developments in a volatile market. In a market with an oversupply phenomenon, developers are likely to defer mega projects that carry significantly higher development risks until the market conditions improve. However, developers view small-scale projects more favorably in a volatile market, and are more likely to exercise the development option earlier if project payoffs commensurate with the market risks.

The paper is organized into six sections. Section 2 reviews the relevant real options literature. Section 3 specifies the structures with necessary assumptions for the optimal timing and capacity choice options model. Section 4 derives the theoretical model and its analytical solutions. Based on a set of parameter inputs, Section 5 conducts the numerical analyses and shows the comparative statics for the proposed optimal timing and development intensity. Section 6 concludes the findings.

2. Literature Review

The typical optimal timing model of McDonald and Siegel (1986) has been extended to evaluate the behavior of investors in different investment asset classes. Titman (1985) employs a discrete-time model to explain the significance of the waiting to develop option for parking lots located in an exclusive residential neighborhood in the United States (US). Williams (1991) develops an optimal timing model for continuous development with stochastic project cost and stochastic cash flow processes. Clarke and Reed (1988) and Sing (2000) examine optimal timing options found in vacant development lands. These models all point to the same conclusion that uncertainty in the future prices of underlying property increases the value of waiting as an option, and thus delays the land development process.

Capozza and Helsley (1989, 1990) extend urban land pricing structures by evaluating the option to convert agricultural lands into urban lands. By assuming that household income and land rent are stochastic, the conversion of agricultural land is a first hitting time process; they find that the rental uncertainty delays the conversion process, and reduces the equilibrium city size. The delay in conversion is associated with increases in the growth premium of agricultural lands located at the boundaries of urban land. By adding a spatial

[3] Intuition of how an industrial-wide demand shock is used to represent equilibrium price setting behavior in a competitive market framework is given in Chapter 8 of Dixit and Pindyck (1994).

and a temporal risk structure to an early urban land pricing model, Capozza and Sick (1994) show that the prices of agricultural land pending conversion to urban land increase with growth rate of rent in the urban areas and unsystematic risk, but decrease with risk aversion.

Pindyck (1988) is the first to consider capacity choice in investment decisions. In his model where firms are able to continuously and incrementally expand capacity, he shows that firms hold less capacity in a volatile market. In the optimal development timing model in Williams (1991), the capacity choice decision involves choosing an optimal development density that will jointly trigger a real estate development project. He solves jointly and simultaneously the optimal timing and the optimal development intensity decisions in his model. Capozza and Li (1994) extend the option of density choice to model the conversion of vacant land to urban land. In their optimal stopping framework, timing and density decisions in the conversion of vacant land are made simultaneously by developers. They find that uncertainty of the density of housing or commercial development increases the hurdle rents, and delays decisions around the development.

Early real options models assume that there is a single monopolistic firm, which makes decisions that are not affected by the entry or exit of other firms in the market. When strategic interaction is considered, modeling the timing and intensity options becomes more complex. Grenadier (1996) develops a model for optimal timing with strategic interaction in a duopoly case to explain for the development cascade and overbuilding phenomena in the real estate market. Extending the model to an equilibrium market structure, Grenadier (2002) shows that options to wait become less feasible with increased competition among developers. Without interaction, developers exercise their development options earlier than predicted through the standard real options model.

3. Model Specifications

In the proposed model, we assume that a developer has already acquired a vacant developable land. S/he faces the decisions of "*when*" and "*how much*" to develop the land. Unlike the standard decision rule of a "*net present value (NPV) greater than or equal to zero*", the standard real options models show that developers will only implement development options when the net profits are high enough to compensate them for giving up their waiting options (McDonald and Siegel, 1986; Clarke and Reed, 1998; Sing 2000). Williams (1991) and Capozza and Li (1994) show that the optimal timing decision is not independent, but interacts with the intensity decision in determining the option premiums. In our proposed model, development intensity is defined endogenously in the demand and the cost functions, where the curvature of these two functions are dependent on the intensity variable. The development intensity has significant effects on the development timing options.

At the individual project level, real estate investments are lumpy and indivisible. However, the flexible choice model in Pindyck (1988), where marginal investments are expanded sequentially, is limited. Following Williams (1991) and Capozza and Li (1994), we model the intensity option as a joint decision with an optimal development timing option.

4. The Model

Assuming that the developer faces the following inverse demand function:

$$R = YD(q) \tag{1}$$

where R is the project cash flow and $D(q)$ is a market demand curve that concaves up with respect to the development intensity, (q), where $[D'(q) > 0, and\ D''(q) < 0]$. Y is an exogenous economic shock that is assumed to follow a geometric Brownian motion:

$$\frac{dY}{Y} = \mu dt + \sigma dw \tag{2}$$

where μ is the expected drift rate of Y, σ is the volatility of Y[4], and dw is the incremental change in the standard Wiener process.

The project costs of the development are deterministic and have a convex function with respect to the development intensity, where $[\ C'(q) > 0, and\ C''(q) > 0]$, as follows:

$$C = C(q) \tag{3}$$

As in the model in Williams (1991), vacant land ownership is analogous to a call option, which gives a developer the right to claim cash flow generated from property built thereon when exercised. Therefore, the value of a development option, $[V]$, can be defined as a function of the stochastic shock variable:

$$V = V(Y) \tag{4}$$

Based on Itô's lemma, the incremental change in the value of the development option over a short interval of time, *dt*, is extended as follows:

$$dV = \left(\frac{1}{2}\sigma^2 Y^2 \frac{\partial^2 V}{\partial Y^2} + \mu Y \frac{\partial V}{\partial Y}\right) dt + \sigma \frac{\partial V}{\partial Y} dw \tag{5}$$

Since firm-specific idiosyncratic risk can be fully diversified, we simplify Equation (5) to the following ordinary differential equation (ODE):

4 The volatility variable, σ, in Equation (2), is a constant idiosyncratic risk that is firm specific, which is not related to the systematic market risk in Capozza and Sick (1994).

$$\frac{1}{2}\sigma^2 Y^2 \frac{\partial^2 V}{\partial Y^2} + \mu Y \frac{\partial V}{\partial Y} - rY = 0 \tag{6}$$

We assume that there is a trigger value of $[Y^*]$, at which the development will commence as long as the market shock exceeds Y^*. The optimal time at which the development occurs is represented by $T = \inf[T \geq 0, Y(T) = Y^*]$. At time T, the value of the development option is equal to the net discounted future cash flow of the project, which is given as follows:

$$V(Y^*, q) = \frac{D(q)q}{r - \mu} e^{-(r-\mu)\tau} Y^* - C(q) \tag{7}$$

where τ is the time to build the variable, which is assumed to be constant over time.

When the development option is triggered at time T, the optimal development density, q^*, can be solved simultaneously such that the land value as defined in Equation (7) is maximized. The optimal intensity is determined by taking the first order derivation of Equation (7), and equating it to zero, $[\partial V / \partial q = 0]$.

To solve for the optimal development intensity, we omit the curvature of the demand and cost equations by using the following simplified functional forms:[5]

$$D(q) = a - bq \tag{8}$$

$$C(q) = c + dq \tag{9}$$

Based on Equations (6) to (9), the optimal development intensity of the subject land, where $[q \geq 0]$, is defined as follows:

$$q^* = \frac{a}{2b} - \frac{d}{2KbY^*} \tag{10}$$

where $K = \dfrac{e^{-(r-\mu)\tau}}{r - \mu}$

The value of development option at time T in Equation (9) is redefined as:

$$\begin{Bmatrix} V(Y^*) = \dfrac{a^2 K Y^*}{4b} + \dfrac{d^2}{4bKY^*} - c - \dfrac{ad}{2b}; & \text{for } (q^* \geq 0) \\ V(Y^*) = 0; & \text{otherwise} \end{Bmatrix} \tag{11}$$

The two-part value functions for land with embedded development options, $V(Y^*)$, given Y^*, in Equation (11), are shown in Figure 1.

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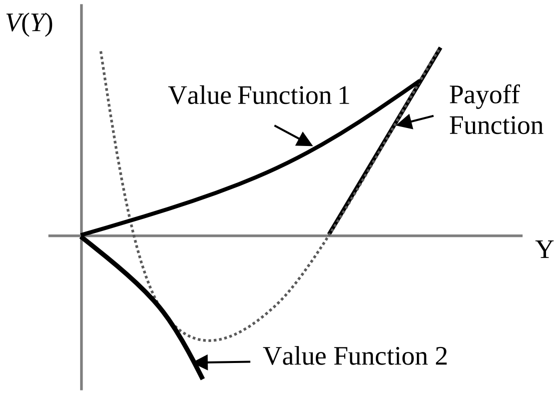
5 A complex analytical solution for the development timing and intensity options that take into account the effects of the economics of scale are derived in the Appendix.

Figure 1 Optimal Payoff Function for Development Options, V(Y*)



After determining the optimal development intensity, Equation (11) is used as the value matching condition in solving the optimal timing of development. The smooth pasting condition that ensures the development occurs at time T is given as follows:

$$V'(Y^*) = \frac{a^2K}{4b} - \frac{d^2}{4bKY^{*2}} \tag{12}$$

An absorbing boundary condition for the natural value of the development is defined as [Y=0]. At this point, the developer will never exercise the development option. The value of the option is nil and defined as follows:

$$V(0) = 0 \tag{13}$$

The ODE (6) is a homogenous second order differential equation, which has the following general solution form:

$$V(Y) = B_1Y^{\beta_1} + B_2Y^{\beta_2} \tag{14}$$

where β_1 and β_2 can be solved by using the following quadratic equation:

$$\frac{1}{2}\sigma^2\beta(\beta-1) + \mu\beta - r = 0 \tag{15}$$

We obtain:

$$\beta_1 = \frac{-(\mu - 1/2\,\sigma^2) + \sqrt{(\mu - 1/2\,\sigma^2)^2 + 2r\sigma^2}}{\sigma^2} > 0 \tag{16a}$$

$$\beta_2 = \frac{-(\mu - 1/2\,\sigma^2) - \sqrt{(\mu - 1/2\,\sigma^2)^2 + 2r\sigma^2}}{\sigma^2} < 0 \tag{16b}$$

Based on the value absorbing constraint, $[B_2 = 0]$, B_1 in Equation (14) can be solved subject to the upper constraint in Equation (11), where $[q^* \geq 0]$. The following system of equations is derived:

$$B_1 Y^{*\beta_1} = \frac{a^2KY^*}{4b} + \frac{d^2}{4bKY^*} - c - \frac{ad}{2b} \tag{17a}$$

$$B_1\beta_1 Y^{*\beta_1 - 1} = \frac{a^2K}{4b} - \frac{d^2}{4bKY^{*2}} \tag{17b}$$

The analytical solution for B_1 and Y^* is given below:

$$B_1 = \frac{a^2KY^{*(1-\beta_1)}}{4b} + \frac{d^2}{4bKY^{*(1+\beta_1)}} - \left(c + \frac{ad}{2b}\right) Y^{*(-\beta_1)} \tag{18}$$

There are two groups of solutions for Y^*, which correspond to Functions 1 and 2 in Figure 1 respectively:

$$Y^* = \frac{(ad + 2bc)\beta_1 + \sqrt{(ad + 2bc)^2\beta_1^2 - a^2d^2(\beta_1^2 - 1)}}{a^2K(\beta_1 - 1)} \tag{19}$$

$$Y^* = \frac{(ad + 2bc)\beta_1 - \sqrt{(ad + 2bc)^2\beta_1^2 - a^2d^2(\beta_1^2 - 1)}}{a^2K(\beta_1 - 1)} \tag{20}$$

Subject to the minimum development intensity constraint, $[q^* \geq 0]$, the solution in Equation (20) is dropped from the option payoff function. Therefore, based on Equations (18) and (19), the value of the timing and intensity of the development option for the subject land is calculated as follows:

$$\begin{Bmatrix} V(Y) = B_1 Y^{\beta_1} & \text{for } Y < Y^* \\ V(Y) = \dfrac{a^2KY}{4b} + \dfrac{d^2}{4bKY} - c - \dfrac{ad}{2b} & \text{for } Y \geq Y^* \end{Bmatrix} \tag{21}$$

5. Numerical Analyses

Using Equation (21), numerical analyses are conducted through reasonable assumptions for the input parameters as summarized in Table 1. The effects of changes in the volatility of demand shock, development intensity and other input parameters on the trigger value, optimal intensity and value of the development option are evaluated.

Table 1 Input Assumptions for Numerical Analyses

Input Parameter	Base Value
Instantaneous drift of economic shock	0.06
Demand shock volatility	$\sigma = 0.2$
Fixed cost	$c = \$5 \times 10^6$
Variable cost (psm)	$d = \$2000$
Fixed demand parameter	$a = \$5000$
Inverse demand parameter (psm)	$b = \$0.5$
Risk free interest rate	$r = 10\%$
Time-to-build (year)	$\tau = 3$

5.1. Demand Shock Volatility

Figure 2 shows a positive relationship between the demand shock volatility and trigger value, which is represented by $[\partial Y^*/\partial \sigma > 0]$. When the future market environment is uncertain, a higher value is required to trigger a development option. This is translated into higher payoffs for the developer to offset the development costs, and at the same time, compensate him/her for giving up the waiting option. The results imply that there are fewer development activities in a highly volatile economic environment. A developer is less inclined to start a development until there is less market uncertainty.

Figure 2 Volatility Effect on Option Trigger Value

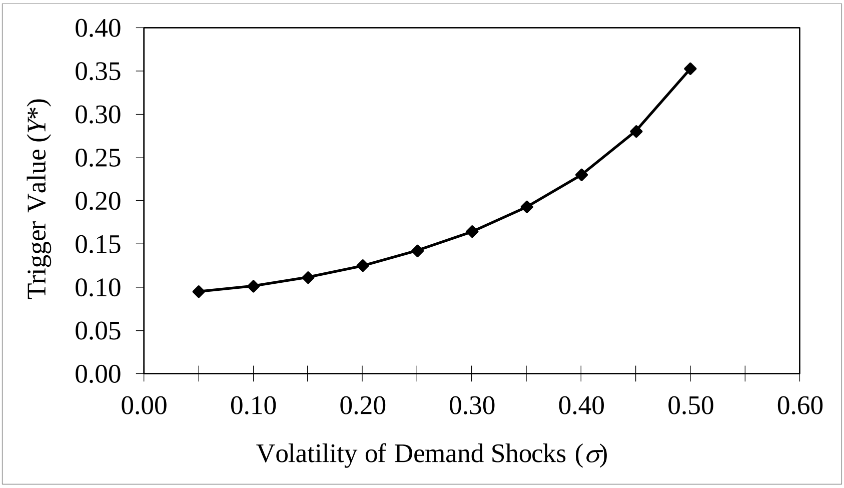


Figure 3 shows that higher market volatility and higher trigger value encourage developers to increase their development intensity, $[\partial q^*/\partial Y^* > 0]$. Consistent with the findings of Capozza and Li (1994), the result indicates that developers will delay a development until an optimal trigger is reached, and the development undertaken by the developer is likely to be more intensive.

Figure 3 Volatility Effect on Optimal Intensity

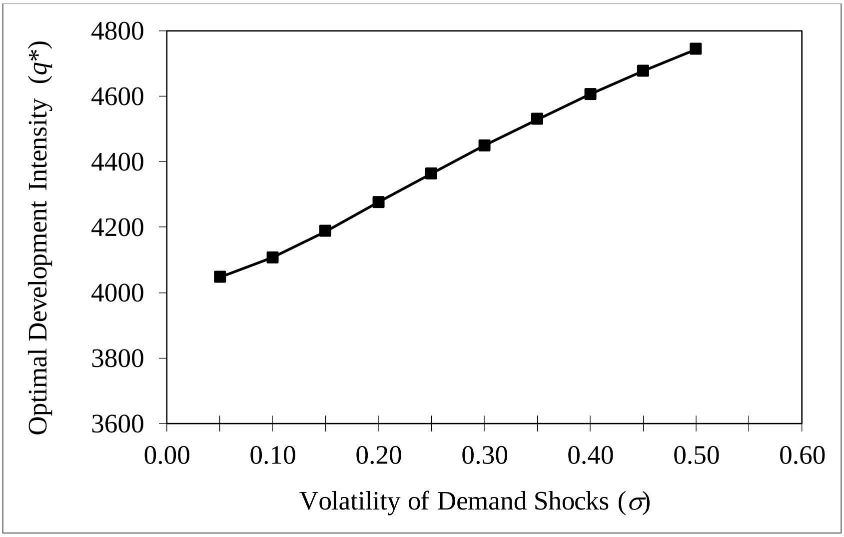
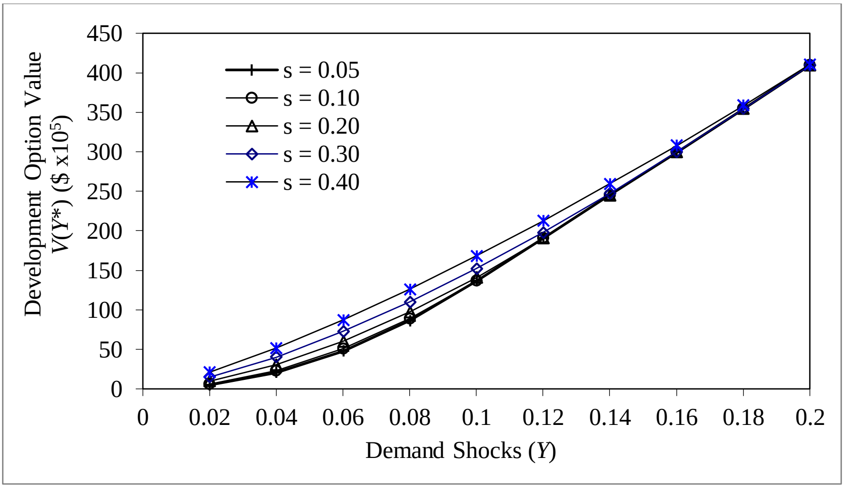


Figure 4 shows the different option value curves in different volatile conditions. The intensity and timing option premiums collectively increase the option value for development lands. The results imply that the option of waiting to develop becomes more valuable when market volatility increases.

Figure 4 Value of Development Option

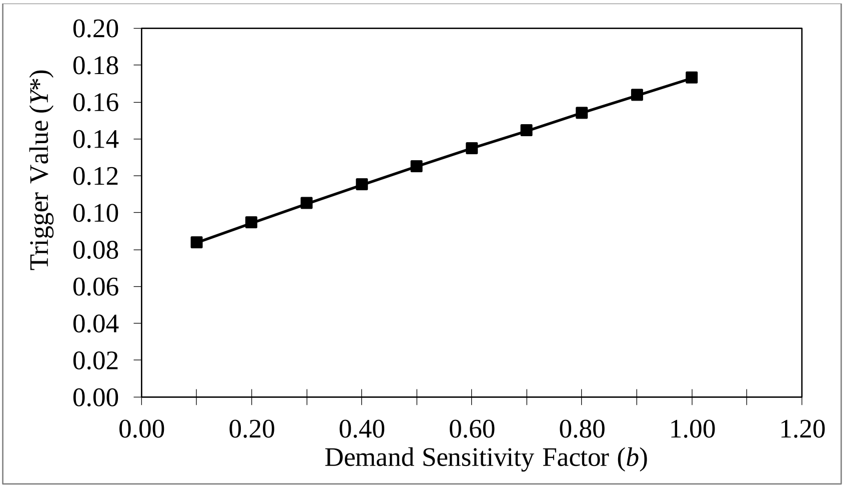


5.2. Changes in Development Intensity

The demand in Equation (8) is a decreasing function of development intensity, q. The parameter b in the equation measures the inverse multiplier effects of quantity on the market demand[6], which in turn, affect the rental cash flows of a project (Equation 1). By varying b over a reasonable range of values, the effects of the demand elasticity on the optimal timing and intensity decisions of a developer are numerically examined.

Figure 5 shows a positive relationship between the trigger value and b, where the slope of the line shows the elasticity of the development cost function on the development intensity. In a market where rental (demand) change is less sensitive to changes in development intensity, developers are likely to wait for more information before triggering development options on new projects. As a result, development activities are depressed in a highly volatile market.

Figure 5 Effects of Rental Sensitivity of Demand on Optimal Timing



In a market where rental (demand) change is highly sensitive to changes in development intensity, Figure 6 shows that increase in the value of b has a negative effect on the optimal development intensity. When b increases, developers reduce intensity in the development projects when the market is volatile.

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[6] Considering the demand function, $D(q)$, as an output factor and the development density, (q), as an input factor, b in Equation (8) can be interpreted as the marginal physical product (MPP) factor, which measures the sensitivity of the demand with respect to changes in one unit of density.

Figure 6 Optimal Intensity of Different Price Sensitivity of Demand

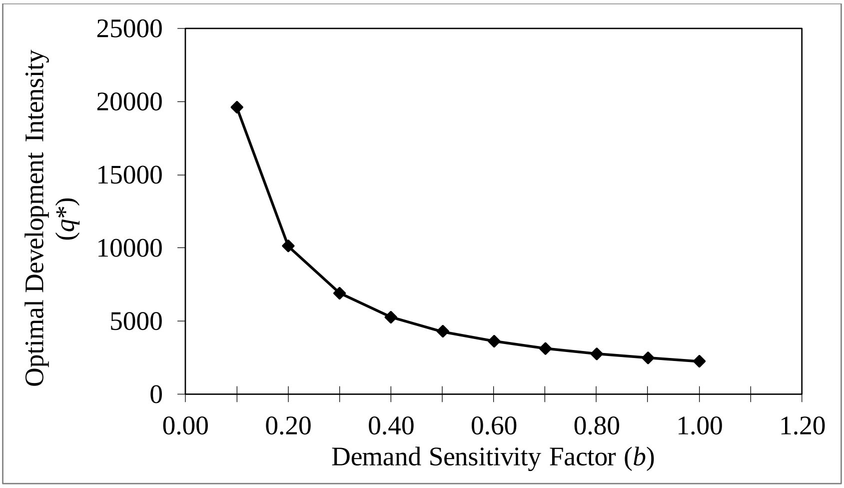
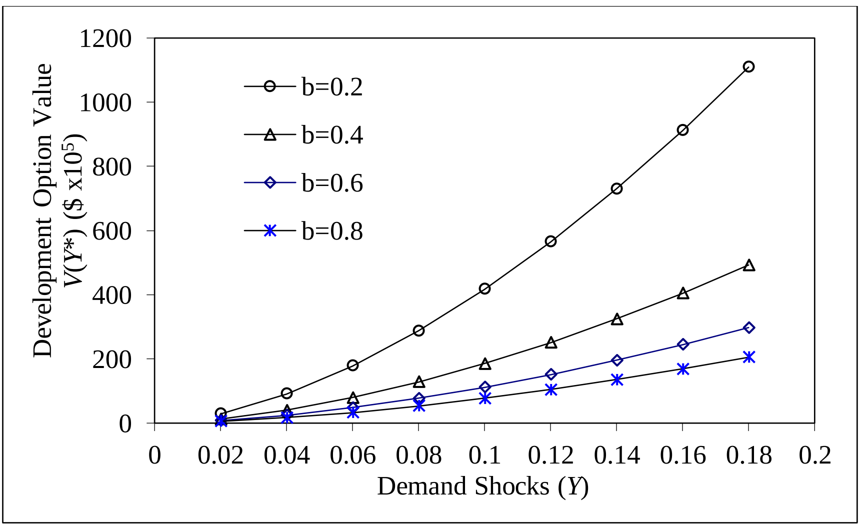


Figure 7 show a negative and convex relationship between *b* and the development option values. This implies that rental elasticity is positively correlated with the development option value. The waiting options become very valuable when the demand shock is high and the rental elasticity is low. These two factors curtail development activities, and thus lead to the slower growth of a city.

Figure 7 Development Option Value



5.3. Comparative Statics

The comparative statics in Table 2 summarize the relationships among the various input parameters and trigger value, optimal intensity and development option value.

Table 2 Comparative Statics

Input Parameter	Trigger Value (Y^*)	Optimal Intensity (q^*)	Value of the Development Option
Volatility of economic shock (σ)	+	+	+
Instantaneous Drift (μ)	-	-	+
Fixed Demand Parameter (a)	-	+	+
Demand Sensitivity parameter (b)	+	-	-
Fixed Cost (c)	+	+	-
Variable Cost (psm) (d)	+	-	-
Risk Free Interest Rate (r)	-	-	+
Time-to-Build (year) (τ)	+	+	-

6. Conclusion

Real estate investment is lumpy at the project level. It is not realistic to adjust the development intensity sequentially and incrementally after a real estate development process has been triggered. Built on the real options model of Pindyck (1988), Williams (1991), Capozza and Li (1994), this paper models the optimal intensity and optimal timing of a development as joint decisions. Developers have to make simultaneous decisions on the optimal intensity of a development when they exercise a development option. Our model defines cash flow as an inverse function of a demand shock variable. The development intensity variable is then built into the demand and the cost functions that drive the process of demand shift.

Using reasonable assumptions for the input parameters, we conduct numerical analyses on the joint optimal timing and intensity options model, and find that demand shock volatility has a significant and positive impact on the intensity premium, and option trigger and development option values. However, b in Equation (8), which is an inverse demand multiplier, is positively correlated with the option trigger values, but negatively correlated with development option and intensity option premiums. In a market where the rental changes are relatively less elastic to development intensity increases, development activities will likely be deferred when volatility is high. The scale of development, if undertaken, will likely be comparatively small.

The optimal timing option together with the intensity option significantly increase the value of waiting when the demand shock increases. Therefore, real estate development activities are curtailed, and city growth slows down in a highly volatile market. However, compared to the duopoly option model proposed by Grenadier (1996 and 2002), which predicts that competition drives development cascades amid an overbuilding phenomenon in the market, empirical tests could be conducted in the future to test the effects of intensity option premiums on development triggers for projects of different scales in times of down markets (Wong et al. 2019).

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Appendix

The problem of the effects of the economics of scale on optimal timing and optimal intensity can be examined by replacing the linear demand function in Equation (8) with a more generalized constant elasticity demand function that incorporates different returns to scale measures, γ. The demand function can be written as follows:

$$D(q) = bq^{-1/\gamma} \tag{A1}$$

where b is a positive constant parameter. The linear cost function as in Equation (9) remains unchanged.

The value of the option at time T can thus be rewritten as follows:

$$V(Y^*, q) = \frac{bq^{1-\frac{1}{\gamma}}}{r-\mu} e^{-(r-\mu)\tau} Y^* - c - dq \tag{A2}$$

By taking the first order derivation of Equation (A2) with respect to q, and equating it to zero, we can solve for the optimal development intensity q^* by rewriting the equation as follows:

$$bK\left(1 - \frac{1}{\gamma}\right) q^{-\frac{1}{\gamma}} Y^* = d \tag{A3}$$

$$q^* = \left(\frac{b\left(1 - \frac{1}{\gamma}\right) KY^*}{d}\right)^{\gamma} \tag{A4}$$

where

$$K = \frac{e^{-(r-\mu)\tau}}{r-\mu} \tag{A5}$$

By substituting q^* into the value function of the optimal development option, the following equation is derived:

$$V(Y^*) = MY^{*\gamma} - c \tag{A6}$$

where

$$M = \left[\left(\frac{b\left(1 - \frac{1}{\gamma}\right)}{d}\right)^{\gamma-1} - \frac{\left(b\left(1 - \frac{1}{\gamma}\right)\right)^{\gamma}}{d^{1-\gamma}}\right] K^{\gamma} \tag{A7}$$

The value of the optimal development option is derived from the ODE in Equation (6), and after being subjected to the value absorbing condition, the general solution form can be written as follows:

$$V(Y) = BY^{\beta 1} \quad \text{(A8)}$$

By matching the general solution to the value matching and the smooth pasting boundary conditions, the following system of equations is obtained:

$$BY^{*\beta_1} = MY^{*\gamma} - c \quad \text{(A9a)}$$

$$\beta_1 BY^{*(\beta_1 - 1)} = \gamma MY^{*(\gamma - 1)} \quad \text{(A9b)}$$

The analytical solutions for B and Y^* can be explicitly derived from Equations (A9a) and (A9b) as follows:

$$Y^* = \left[\frac{\frac{c}{\gamma}}{M\left(\frac{1}{\gamma} - \frac{1}{\beta_1}\right)} \right]^{\frac{1}{\gamma}} \quad \text{(A10)}$$

$$B = \left(\frac{M\gamma}{\beta_1}\right) Y^{*(\gamma - \beta_1)} \quad \text{(A11)}$$

By substituting B and Y* in Equations (A10) and A(11) into Equation (A3), the analytical solution for the optimal intensity, q^*, can also be obtained.

Discussion of the Analytical Solutions

The above analytical solution shows that [$Y^* > 0$], if and only if the condition [$\gamma > \beta_1$] is satisfied. This implies that when the elasticity coefficient, γ, is small relative to β_1, the waiting option has no value. The developer will start development immediately, because the trigger value Y^* approaches zero. Similarly, when the elasticity of the scale increases, a longer waiting time is expected as Y^* increases.