Mortgage Market Induced Booms and Busts in the Housing Market in a Modified DiPasquale-Wheaton Model

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In this paper, we offer a nontechnical pedagogical tool that demonstrates the interrelationship between the mortgage market and the housing market. While the importance of the mortgages in the housing market is well established in the literature, an instructive model is still needed. We present a novel and straightforward extension of the DiPasquale and Wheaton (DPW) model, with a price-rent (PR) ratio that incorporates the loan-to-value (LTV) ratio. In our modified DPW model, the LTV adjusted PR ratio allows for an explicit analysis of the opposing effects of lending gain and risk pricing on the user costs of housing. Moreover, the model highlights how changes in the mortgage market may contribute to booms and busts in the markets for both owner-occupied and rental housing.

Keywords
Price-rent ratio, Loan-to-value ratio, User cost, Housing market equilibrium.

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1. Introduction

Real estate constitutes as an important and large economic component of the national economy in most developed countries. In studies on real estate economics, the macro-economic aspects of the sector are therefore of great relevance and interest. How will changes in the macro economy spill over into the market for housing? And how will changes in the housing market alter the prices of real estate assets and the activity in the construction sectors? These and more, are relevant questions that can be discussed and analysed by means of the seminal model by DiPasquale and Wheaton (1992). This model describes the functioning of the real estate system and identifies the links of two important sectors of the economy, namely the property and asset markets. Indeed, the DiPasquale-Wheaton (DPW) model constitutes as a seminal pedagogical tool for analysing the long-run equilibrium of the real estate system. By means of simple demand and supply analysis, the model offers an intuitive introduction to the main components in the real estate markets. While this model is typically used to study commercial real estate, it is also applied to owner-occupied real estate (DiPasquale and Wheaton 1992). Leung and Wang (2007) and Lisi (2015) are among those who analyse housing markets by directly using the DPW framework. This approach abstracts away from several real housing market features, such as user cost of housing and the role of housing finance.

The US sub-prime crisis demonstrates the important link between housing and mortgage markets, as argued by, for instance, Duca et al. (2010), Koetter and Poghosyan (2010) and Goodhart and Hoffman (2008). Lower mortgage rates made funding cheaper and, as housing demand increased, house price growth picked up. As a gap between house price growth and mortgage rates developed, loan-to-value (LTV) ratios rose and risk taking among both mortgagors and mortgagees increased. As market sentiment turned and house prices started to fall, the increased risk exposure negatively affected both mortgagors and mortgagees. The US house price collapse between 2007 and 2009 affected rental markets as well as the number of housing-starts and revealed the links between the different parts of the housing market system.

This paper presents a straightforward extension of the DPW model by using a conventional approach for analysing housing markets. The price-rent (PR)-ratio links the cost of rental housing to the cost of owner-occupied housing and is considered an equilibrium condition for housing markets. Within the DPW framework, the PR-ratio illustrates how the market for owner-occupied housing is related to the rental housing market as well as the housing construction industry. By allowing for both direct- and indirect effects, this framework is useful for analysing housing market dynamics in both the short and long run.

The theoretical framework of the standard DPW model does not fully include the interrelation between housing financing and housing markets. We thus enrich the model by allowing for an adjusted PR-ratio that takes the financing
structure of housing investments into account. The LTV adjusted PR-ratio includes how housing finance affects the user cost of owner-occupied housing and thus the tenure decision of households (owning or renting), and consequently, the housing market equilibrium. An increase in the LTV-ratio may decrease or increase the user cost of owner-occupied housing. This depends on whether the lending gains or risk pricing dominates the user cost effect of a larger LTV-ratio. Nonetheless, a shock to the LTV-ratio is transmitted from the market for owner-occupied housing to the rental market and housing construction industry, i.e., there is interdependence within the housing market system.

When the lending gain dominates the effect on user cost, a larger LTV-ratio results in higher house prices and increased housing construction, which reduce rent, as the housing supply increases over time. When risk pricing dominates, a larger LTV-ratio increases user cost, house prices fall which leads to a reduction in housing construction; hence, rent increases as supply contracts. The model thus shows how mortgage markets might produce booms or busts in housing markets, which affect the rental market, housing construction industry, and price of owner-occupied housing, where both booms and busts entail periods of overshooting.

While the literature on financing aspects of the housing markets is rich, there are few papers on how this aspect is directly integrated into the DPW model. Thus, our contribution is to offer a non-technical and pedagogical model that explains how changes in the mortgage market might influence the short and long run equilibriums in the market for owner-occupied and rental housing, as well as the construction of new housing. Our ambition is thus to enrich the standard DPW model and still preserve its pedagogical simplicity and compactness.

The rest of the paper is structured as follows. The next section elaborates on the relationship between the financing structure of owner-occupied housing investments and the PR-ratio, particularly how the user cost depends on the LTV-ratio and how the relation between the LTV-ratio and user cost affects the PR-ratio. The third section briefly presents the DPW model. The fourth section analyses the role of the LTV-ratio in the housing market equilibrium, including both that of owner-occupied and rental housing and how changes in the LTV-ratio may create boom and bust scenarios. The last part concludes.

2. Price-rent-ratio, loan-to-value and housing market equilibrium

In our model, there are only two types of tenures available to households: rental and owner-occupied housing. Moreover, we assume that housing services offered in the rental and the owner-occupied housing markets are perfect
substitutes. While owner-occupied housing may indeed also serve as an investment object and a collateral for mortgages (Sommervoll et al. 2010), we place focus on the choice of housing services offered in the rental and owner housing markets.\(^1\) Hence, our focus is on the consumption motive of households, rather than investment motive of investors (landlords).

In the rental housing market, households pay an explicit market rent for housing services. For owner-occupied housing, the market value of housing services is not observable, but a rental equivalent price that reflects the user cost of housing: \(u\) may be derived. The user cost of owner-occupied housing conventionally includes both direct and indirect costs, such as mortgage interest payments, property tax, costs of maintenance and repair (i.e., depreciation costs), transaction costs and foregone interest on own capital (equity). In addition, user costs include a risk premium associated with future changes in the market price for owner-occupied vs rental housing, as well as capital gain/loss when selling the house. User costs are obviously dependent on the purchase price of the house. Moreover, by allowing for different sources of funding (equity and mortgage funding) which entail different costs, we can study how the funding structure of housing investments impacts the user cost. In this way, the housing market can be shown to be linked to the mortgage market through the user cost.

Assuming perfect substitutability between rental and owner-occupied housing, then the housing market equilibrium should equate the cost of renting and the user cost of owning. Several papers use the PR ratio to analyse the housing market equilibrium, see, for instance, Ayuso and Restoy (2006), Gallin (2008), Campbell et al. (2009), and Hill and Syed (2016). The PR-ratio equates the cost of housing tenure across the rental and owner-occupied housing markets.

Himmelberg et al. (2005) give the housing market equilibrium condition as

\[ R = uP \]

where \(R\) is the rental price, \(P\) is the purchase price of owner-occupied housing, and \(u\) is the per-dollar user-cost. It follows from Eq. (1) that if \(R > uP\), then owning is more favourable than renting, and there will be an upward pressure on \(P\) and downward pressure on \(R\) until a new market equilibrium is reached. If, on the other hand, \(R < uP\), then owning is less favourable and there will be a downward pressure on \(P\) and upward pressure on \(R\).

\(^1\) We thus ignore the fact that owner-occupied housing may also offer additional non-monetary benefits as opposed to rental housing (e.g., emotional benefits, social standing, etc.).
The equilibrium condition between rental and owner-occupied housing is rewritten as the more familiar PR-ratio:

\[
\frac{P}{R} = \frac{1}{u}
\]

which states that the price to rent ratio equals the inverse of the per-dollar user cost of housing. Hence, this constitutes as a non-arbitrage condition.

The process that underlies the non-arbitrage condition abstracts away from several real housing market features. In our long-run equilibrium model issues related to transaction cost, demographics, uncertainty, and liquidity are subdued, thus allowing the non-arbitrage condition to be at the centre of attention. Indeed, we highlight how the funding structure of housing investment impacts user costs and the PR-ratio in line with Borgersen (2020).

More importantly, allowing for variations in funding structure as argued by Gilbukh et al. (2017), user cost might be affected by both risk pricing in mortgage markets and potential lending gains from mortgage financed housing as argued by Borgersen and Greibrokk (2012). The lending gain is related to both the funding structure and the excess return on mortgage financed housing (i.e., house price growth vs the mortgage rate). Both the risk pricing in mortgage markets and the lending gains impact the user cost of mortgage financed owner-occupied housing, and the net-effect of the two might increase or reduce user costs compared to equity financed owner-occupation.

We briefly elaborate on this latter aspect. The purchase price of a house, \(P\), is typically financed by equity (\(E\)) and mortgage (\(M\)): \(P = E + M\). Assuming rational agents with perfect foresight, households know the rate of house price appreciation \(p\), where \(p \geq 0\). Households require a return on housing investment that at least compensates for the mortgage rate (\(r^M\)), but may also receive a positive return on home equity (\(e\)), as expressed by \(p = e\frac{E}{P} + r^M\frac{M}{P}\). Rewriting this expression, the return on home equity is given by the sum of price growth \((p)\) and the leverage gain \(\frac{M}{E} (p - r^M)\), i.e., \(e = p + \frac{M}{E} (p - r^M)\). Hence, the

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2 Seeing housing equity as part of a household portfolio (see e.g., Yao and Zhang 2005), and acknowledging housing as a long-term investment object, the response to incentives for changing tenure is more complicated. During periods of house price growth, or binding wealth to income constraints, the housing share may differ from what is indicated by optimal portfolios. In addition, as home ownership is hump shaped over the lifecycle (Chambers et al. 2009), a young household entering homeownership may have a larger portfolio share for housing equity than an established homeowner that has repaid its mortgage debt. Naturally, uncertainty might also impact housing demand (Han 2010).
leverage gain depends on the ratio of mortgage to equity and the difference between the change in prices and the mortgage rate. Assuming that \((p - r^M) > 0\), then a higher mortgage to equity results in higher leverage, \textit{cet. par.} Let the LTV ratio be defined as \(\theta \equiv \frac{M}{P}\). Then it follows that \((1 - \theta) \equiv \frac{E}{P}\). The return on housing equity can then be rewritten as: 

\[ e = p + \frac{\theta}{1-\theta} (p - r^M). \]

When including the return to housing equity instead of only the capital gain produced by house price growth in the user cost, the per-dollar user cost of owner-occupied housing may then be expressed as:

\[ u = (1 - \theta)r^{RF} + \theta r^M + \gamma + \delta - p - \frac{\theta}{1-\theta} (p - r^M) \]

The risk-free interest rate is given by \(r^{RF}\), \(\gamma\) is the risk premium associated with owning versus renting and \(\delta\) is the sum of maintenance, repair and depreciation measured as a fraction of \(P\).

As follows from Eq. (3), while including conventional user cost components related to maintenance and repair (i.e., depreciation costs) and forgone interest on own equity, we abstract away from inflation and all types of taxes (on property, capital gains, imputed housing services, or tax-deductible mortgage interest payments). Moreover, while empirically there are transaction costs associated with change of tenure (Corradin et al. 2014), we ignore them. We simplify even more by assuming that the rate of depreciation, \(\delta\), is identical for rental and owner-occupied housing.

Referring to Eq. (3), we see that user costs are reduced by a price gain, \(p\), and a leverage gain, \(\frac{\theta}{1-\theta} (p - r^M)\), where the leverage gain is positively dependent on the LTV ratio and the difference between price growth and the mortgage rate. While the effect of price growth is well known, the lending gain effect on

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3The cost depreciation is included in the user cost expression for owner-occupied housing in quite a conventional way. Empirically, depreciation of rental housing is found to exceed that of owner-occupied housing (see for e.g., Gatzlaff et al. (1988) and Harding and Simans (2000)). Differences in the rate of depreciation between rental and owner-occupied housing might impact the housing market equilibrium, as expressed by the PR-ratio. A higher rate of depreciation of rental housing might be argued to influence the rental price set by a landlord and, hence, the choice of tenure of a household. As the PR-ratio is based on a demand side approach which equates the cost of different forms of tenure, the higher depreciation cost associated with rental housing would be reflected in the rental price paid by a tenant in a competitive market. While depreciation of owner-occupied housing is explicit in the user cost, the depreciation of rental housing is implicitly included in the rental price paid by a tenant, thus allowing for differences in depreciation rate across form of tenure to impact the housing market equilibrium.
the user cost is less familiar. Combining the PR-ratio (2) and the user cost (3) we find the LTV-adjusted PR-ratio as follows:

\[
\frac{P}{R} = \frac{1}{(1 - \theta) r_{RF} + \theta r^M + y + \delta - p - \frac{\theta}{1-\theta} (p - r^M)}
\]

(4)

which shows that the price to rent ratio equals the inverse of the user cost of housing.

It follows from the discussion that if the LTV-adjusted PR-ratio is larger or smaller than the reciprocal of per dollar user cost, then the housing market is not in equilibrium. So how will an increase in the LTV-ratio alter the PR-ratio? Comparative statistics show that the effect on the PR-ratio of a larger LTV-ratio is ambiguous:

\[
\frac{\delta P}{\delta \theta} = \frac{1}{u^2} \left[ (p - r^M) \frac{1}{(1 - \theta)^2} - (r^M - r_{RF}) \right] \geq 0
\]

(5)

As shown in Eq. (5) the effect of a larger LTV-ratio on the PR-ratio is derived from the relation between the excess return to mortgage-financed housing \((p - r^M)\) and the mortgage market risk premium \((r^M - r_{RF})\). A larger LTV-ratio produces a larger PR-ratio when the excess return to mortgage financed housing exceeds the mortgage market risk premium, i.e.:

\[
(p - r^M) \frac{1}{(1 - \theta)^2} > (r^M - r_{RF})
\]

(6)

and a smaller PR-ratio when the relation between the two is reversed:

\[
(p - r^M) \frac{1}{(1 - \theta)^2} < (r^M - r_{RF})
\]

(7)

House price growth \(p\) represents a capital gain and \(p > r_{RF} > 0\) produces excess return on housing equity. In addition, if housing purchase is financed by debt, homeowners may earn an excess return on the mortgage financed part of the housing investment when \(p > r^M\), i.e., there is a lending gain (in the literature, this is also referred to as leverage gain), related both to the funding structure and to the excess return of mortgage financed housing. The return on housing equity thus equals \(p + \frac{\theta}{1-\theta} (p - r^M)\). If the housing investment is fully financed by equity, i.e., \(\theta=0\), there is no leverage gain, and housing equity gain equals house price growth. If the housing investment is mortgage financed, \(\theta>0\), and house price growth exceeds the mortgage rate \(p > r^M\), a lending gain adds to the return to housing equity and reduces the user cost of owner-occupied housing compared to when equity financed. The lending gain is positively related to the LTV ratio, cet. par. When on the other hand the mortgage rate exceeds house price growth, risk pricing reduces the return to housing equity of mortgage financed housing investments.. That is, if \(p < r^M\) a higher LTV-ratio increases the user cost of owner-occupied housing compared to equity financed housing investments.
The LTV-ratio thus has an ambiguous effect on the user cost. Correspondingly, its effect on the PR-ratio is contingent on the size of excess return to mortgage financed housing relative to the mortgage market risk premium.

Simply put, in a mortgage market where risk pricing is aggressive, i.e., \((r^M - r^{RF})\) is high, then a higher leverage \((\Delta \theta > 0)\) implies increased funding cost, and correspondingly, higher user cost and thus a lower PR-ratio. Owner-occupied housing thus becomes relatively more costly than rental housing, which puts a downward pressure on \(P\). If, on the other hand, the credit risk policy is loose, i.e., \((r^M - r^{RF})\) is low, and excess return to mortgage financed housing is the dominant effect of a larger LTV-ratio on user cost, then the PR-ratio increases. Lower user cost of owner-occupied housing will put an upward pressure on \(P\) for an equilibrium to be established. Likewise, when monetary policy is “leaning against the wind” and the interest rate mirrors house price growth closely (i.e., \(r^{RF} \approx p\)), a larger LTV-ratio might increase user costs and thus reduce the PR-ratio. The opposite might come about in the case of strict inflation targeting and where asset inflation is not a real concern for the central bank.\(^5\)

Knowing the ambiguous effect of a larger LTV-ratio on the PR-ratio, a relevant question is then to ask how changes in the LTV-ratio influence the housing market. To answer this, we present an modified version of the DPW model including the LTV-adjusted PR-ratio. In the following, we consider three regimes. Our benchmark Regime A is a situation where housing is financed exclusively by equity, i.e., \((\theta = 0)\). The two other regimes are based on mortgage financed housing. Regime B is one in which a larger LTV-ratio increases the user cost compared to our benchmark, while Regime C is one in which a larger LTV-ratio reduces the user cost compared to our benchmark.\(^6\)

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\(^5\)Borio and Lowe (2002) provide a seminal discussion on whether monetary policy should target asset inflation.

\(^6\)Highlighting the links between the different parts of the housing market system, risk is not our main focus. Risk is also not the main focus in the original DPW model. Still, as the shocks incorporated in our DPW model is related to a change in funding structure, risk is indirectly accounted for. However, rather than focusing on how housing market risk is important for how households adapt to housing market, the model considers how risk impacts housing markets. Related to the LTV-ratio being an indicator of risk exposure, and how higher LTV-ratios represent increasing risk exposure in housing markets, a substantial body of literature analyses the increase in housing market risk across the western hemisphere in the years before the US Sub-prime crisis. It is beyond the scope of this paper to survey this literature, and instead we offer some reflections on the effects of risk our model implicitly carries with it. For instance, Han (2010) analyses the impact of risk on housing demand and finds effects from increased price risk, separating between a negative financial risk effect and a positive hedging effect. In our benchmark regime, where we consider equity funding, these two effects would both be included as we constrain financial risk to price risk. A funding risk effect, which regimes B and C introduce by allowing for mortgage financed housing investments, would impact mortgage financed housing investments. While both regimes imply higher
Hence, the three regimes are as follows:

A. Equity funding: \( \theta = 0 \) and \( u^A = \bar{u} \)

B. Mortgage funding and an aggressive risk pricing regime: \( \theta > 0 \) and \( u^B > \bar{u} \)

C. Mortgage funding and high lending gain: \( \theta > 0 \) and \( u^C < \bar{u} \)

Referring to the four quadrants in the graphical illustration of the DPW model, we introduce these three regimes into the DPW model by using the second quadrant (north-west diagram). Assuming a given level of rent on rental housing, \( R^0 \), we can illustrate how the market valuation of owner-occupied housing varies with user cost. Each of the three regimes are represented by a separate PR-ratio line that relates the house price to the rental price for different levels of user cost (\( u^B > u^A > u^C \)) as given by Figure 1.

At a given level of rent \( R^0 \), we can derive the effect of the different LTV-ratios on the price of owner-occupied housing in the different regimes. Equity funding, i.e., Regime A, is the benchmark case. In Regime B, risk pricing is aggressive, and a larger LTV-ratio increases the funding cost and therefore also the user cost of owner-occupied housing, thus leading to lower equilibrium house prices than those found in Regime A: \( P^B < P^A \). This is illustrated in Figure 1 by a leftward rotation of the PR-line relative to our benchmark. In Regime C, a larger LTV-ratio will reduce the user cost, thus implying that the PR-line will be less steep. The price of owner-occupied housing will be higher compared to our benchmark case: \( P^C > P^A \).

We now turn to the other components of the housing market. By means of the DPW model, we study how the different parts of the housing market interact in the three different user cost regimes, i.e., Regimes A, B and C.

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housing market risk compared to our benchmark, the two would have different implications for short-run housing demand. In a regime where risk perception and risk pricing are high, user cost increase and housing demand is reduced compared to the benchmark scenario. In this regime, house prices will be lower due to the higher risk households face with this funding structure. In a situation where risk pricing is less tough, and the lending gain dominates the effect of a higher LTV-ratio, the user cost is reduced. A lower user cost will increase housing demand and house prices. This latter regime represents a situation with higher house prices despite higher risk (compared to the benchmark situation), as the market lacks focus on the underlying risk development. This increased risk taking would represents a boom, which is our illustration of the period before the US sub-prime crisis.
3. A modified DiPasquale-Wheaton model

The four-quadrant model is developed by DiPasquale and Wheaton (1992), while some important model extensions are given by Colwell (2002). This section only presents a simplified version of the model. In our modified version, henceforth referred to as the modified DPW model, we consider both the rental and the owner-occupied housing markets illustrated in the first and second quadrants respectively. Thus, we depart from the original model by explicitly addressing these two types of tenures directly in the illustration. The non-arbitrage condition implies that in equilibrium, the price of rental housing \( R \) should equal the user cost of owner-occupied housing \( uP \). Figure 2 gives a graphic presentation of the model.

In the four-quadrant model, the rent on rental housing \( R \) is determined in the first quadrant (north-east diagram) by the intersection of a downward sloping demand curve \( D \) and a given housing supply \( S_0 \). The demand for rental housing is (conventionally) related to a set of macroeconomic variables like household income \( \alpha \) and the level of rent: \( D = \alpha - \beta R \).

\[ \Delta u > 0 \]
\[ \Delta u < 0 \]
As the model is recursive, the rental market equilibrium is given directly by the rental price:

$$R_0 = \frac{\alpha - S_0}{\beta} \quad (8)$$

where household income and housing supply have conventional effects on the rental price ($\frac{\partial R}{\partial \alpha} > 0$, $\frac{\partial R}{\partial S} < 0$). Having determined the rental price in the first quadrant, and given our assumption that rental and owner-occupied housing are perfect substitutes, the price of owner-occupied housing $P$ follows directly in the second quadrant (north-west diagram) by:

$$P_0 = \frac{R_0}{u_A} \quad (9)$$

Substituting for the equilibrium level of rent from Eq. (8), the equilibrium house price equals $P_0 = \frac{\alpha - S_0}{\beta u_A}$, where a higher level of household income increases house prices, while a housing supply increase reduces house prices, *cet. par.* Also, a higher user cost means lower equilibrium house price.

The third (south-west) quadrant describes the housing construction industry, where the number of housing starts is related to house prices. The housing construction industry carries both fixed and variable costs, and by assumption, the sector has no excess return. The number of housing starts is given as $C = c_0 + c_1 P$. Due to the recursive model structure, the equilibrium house price...
level is already determined, which makes the gross flow of housing in equilibrium equal to:

$$C_0 = c_0 + c_1 P_0$$  \hspace{1cm} (10)

In the fourth (south-east diagram) quadrant, we turn to the housing stock. The change in housing stock is determined by the interaction between the construction of new housing and the rate of depreciation $\delta$ (assumed proportional to the level of stock $S$): $\Delta S = C - \delta S$. Assuming that the reduction in stock due to depreciation equals the number of housing-starts, i.e., $\Delta S = 0$, then the equilibrium housing stock equals:

$$S_0 = \frac{C_0}{\delta}$$  \hspace{1cm} (11)

Colwell (2002) extends the model by introducing a long-run supply (LRS) curve for housing, which is graphically positioned in the first quadrant and defines the long run equilibrium for rent, owner-price, housing supply and level of stock. From an approach similar to that of Colwell (2002) positioned in our context, we find the LRS-curve by using $\Delta S = C - \delta S$ and $\Delta S = 0$. Substituting for housing starts from the relation $C_0 = c_0 + c_1 P_0$, the LRS-curve may be expressed as $S = \frac{c_0}{\delta} + \frac{c_1}{\delta} P$.

Finally, when substituting the equilibrium house price expression $P_0 = \frac{R_0}{u_A}$, the LRS-curve equals

$$S_0 = \frac{c_0}{\delta} + \frac{c_1 R_0}{\delta u_A}$$  \hspace{1cm} (12)

The positive relation between $R_0$ and $S_0$ gives the LRS-curve a positive slope in the first quadrant. The position of the LRS-curve is in our modelling framework affected by rent, the cost components of the construction industry, and the user cost of owner-occupied housing. From Eq. (12) we see that higher user cost, cet. par., rotates the LRS-curve upwards in the first quadrant: higher user costs entail lower prices in the housing market, reduced supply of new housing, lower stock of space and higher rent in the rental market, cet. par.

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As discussed earlier, we assume that the rate of depreciation is identical between rental and owner-occupied housing. Empirically, this rate is shown to differ. Assuming a competitive market, the rental price would include the higher cost of depreciation. The rate of depreciation for owner-occupied housing that closes the model by assuming a constant housing stock, may thus be seen as a lower bound for depreciation. Not allowing for different quality type of houses, the housing stock in the economy is constant as the number of housing starts is equal to the rate of depreciation.
4. Housing market dynamics and LTV-effects in the housing market

The LTV-adjusted PR-ratio allows us to study the short- and long run effects on the housing market system that emanate from changes in the funding structure of owner-occupied housing. The question regarding implications of a larger LTV-ratio on the housing market is interesting and highly relevant also from an empirical point of view. In the aftermath of the US sub-prime crisis, there has been a substantial focus on the LTV-ratio, both in terms of how larger LTV-ratios increased mortgage market risk prior to the crisis (Goodhart 2008, Scanlon et al. 2011, Taylor 2009), as well as how LTV-caps constrained risk in the crisis aftermath (see for e.g., Gelati and Moessner 2011; Claessen et al. 2013).

In the years prior to the US sub-prime crisis, a situation with low mortgage rates, strong house price growth and larger LTV-ratios emerged. As house price growth outpaced income growth, larger LTV-ratios were necessary for some households that were entering owner-occupied housing (see for instance, Barba and Pivetti (2009) for an interesting analysis on household debt and debt-to-income developments). For others, a house price growth that exceeded the mortgage rate provided incentives to increase leverage. Analysing the period before the US sub-prime crisis, Goodhart and Hoffman (2008) argue that mortgagees used larger LTV-ratios to fulfil nominal return targets in order to compensate for low mortgage rates. The lack of a focus on such increased risk taking before the crisis was reversed after the crisis.

So, with reference to our modified DPW model, we have shown that funding structure impacts user costs and hence the PR-ratio. We now wish to illustrate how a “shock” in the form of an increase in leverage will change the market outcome. We allow for three regimes: A, B and C as discussed in Section 2. These regimes differ with respect to the funding structure of housing investments. Our benchmark regime (Regime A) is that of equity funded housing investment, i.e., an LTV ratio ($\theta$) equal to zero. In Regimes B and C, housing purchase is partly financed by mortgages, that is, $\theta > 0$. Hence, Regimes B and C represent two possible scenarios when there is a change towards a funding structure with higher gearing: (i) a housing market bust and (ii) a housing market boom.

Indeed, the short-term housing market dynamics that may be explained by the link from the mortgage to the housing market, have been analysed by a number of papers. Some see the credit cycle approach of Kiyotaki and Moore (1997) and the financial accelerator of Bernanke et al. (1999) as seminal. However, different modelling frameworks might find different market dynamic structures. Both searching and matching (Wheaton 1990) and up-trading (Ortalo-Magne and Rady 2006) might influence short-run housing market
dynamics. In this paper, we abstract away from both searching and up-trading and allow the mortgage market momentum to impact the dynamics in rental housing markets, housing construction and the market for owner-occupied housing.

4.1 A housing market bust

The first of our two scenarios is that of a housing market bust. In this situation, a larger LTV-ratio increases the user cost, as the risk pricing effect of higher leverage dominates the lending gain effect on the user cost, i.e., Regime B. The effect of a larger LTV-ratio on house prices (assuming a given level of rent) equals:

\[
\frac{\delta P}{\delta \theta} = -R \frac{\delta u_B}{u_B} < 0 \quad \text{as} \quad \frac{\delta u_B}{\delta \theta} > 0
\]

This implies that the PR-curve illustrated in the second quadrant rotates inwards, thus reducing house prices at the given level of rent, as illustrated in Figure 3.

The initial housing market equilibrium is characterised by \((R_0, P_0, C_0, S_0)\). When a larger LTV-ratio increases user cost, house prices fall \(P_0 \rightarrow P_1\). The housing industry responds to lower house prices and reduced profitability by reducing the number of housing-starts \(C_0 \rightarrow C_1\). As the construction of new houses falls short of the depreciation of the existing housing stock necessary for \(\Delta S = 0\), there is a reduction in the housing stock \(S_0 \rightarrow S_1\). The reduced housing supply increases the rent level \(R_0 \rightarrow R_1\). In the new long-run equilibrium, the higher user cost implies that the LRS-curve rotates upwards, as illustrated by the new LRS curve denoted \(LRS'\). The equilibrium level of housing stock is now given by \(S_1 = S_2\) and the market clearing level of rent is given by \(R_2\). The new long-run equilibrium will of course depend on the rent-sensitivity of both the LRS and demand. Long-run equilibrium is ensured by the PR-ratio as the increase in rent feeds back into house prices and \(P_1 \rightarrow P_2\). In this three-period set-up (Periods 0, 1 and 2), the positive price effect in the third period (Period 2) has a positive impact on the activity in the housing industry \(C_1 \rightarrow C_2\). In our model set-up, the new housing market equilibrium is \((R_2, P_2, C_2, S_2)\).²

²As argued by Colwell (2002) and others, the equilibrium process has a cobweb structure in this modelling framework. For illustrative purposes and without loss of generality we position the third period as the long-run equilibrium in our illustrations. In Figure 3, the black-dotted line closes the model in the third period. (The same applies for Figure 5).
A change in the funding structure of housing investments and a larger LTV-ratio increases the user cost of owner-occupied housing when the mortgage market is characterised by aggressive risk pricing. This new funding structure creates a housing market bust. The reduction in house prices is transmitted through the housing market system and impacts the construction industry as well as the rental market. The long run effect on house prices is negative $P_0 > P_2$.

As illustrated in Figure 4, the market is characterised during the bust by overshooting in the process towards the new long-run equilibrium. Figure 4 pictures the overreaction in both house prices $P_0 \rightarrow P_1 \rightarrow P_2$ and housing construction $C_0 \rightarrow C_1 \rightarrow C_2$. 
4.2 A housing market boom

The other scenario is what we refer to as a housing market boom. In this situation, the lending gain dominates the risk pricing effect of a larger LTV-ratio, i.e., Regime C. Compared to the equity regime, leverage reduces the user cost of owner-occupied housing. From the housing market equilibrium condition, the effect of a larger LTV-ratio on house prices is given by:

\[
\delta P = \frac{-R \delta u_C}{\delta \theta} > 0 \quad \text{as} \quad \frac{\delta u_C}{\delta \theta} < 0
\]

This regime is illustrated in Figure 5 where the PR-ratio curve rotates downwards closer to the horizontal axis as a larger LTV-ratio reduces the user cost.

The initial housing market equilibrium is given by \((R_0, P_0, C_0, S_0)\). A change in the funding structure in favour of leverage now reduces the user cost in the beginning of Period 1 and house prices increase, \(P_0 \rightarrow P_1\). In response to the increased house prices the number of housing-starts increase \(C_0 \rightarrow C_1\). A shock to the flow of housing lifts the Period 1 housing stock \(S_0 \rightarrow S_1\), as new housing construction exceeds the depreciation of the existing housing stock. The increase in supply brings down equilibrium rent in period 1: \(R_0 \rightarrow R_1\). In the second period the reduced rent level pushes down house prices \(u_1 \rightarrow u_2\) for the housing market to be in equilibrium. The negative impact on house prices takes some steam off the housing industry, reducing the number of housing-starts to the new long-run equilibrium level \(C_1 \rightarrow C_2\). In our three-period set-up, \(S_1 = S_2\) and the Period 2 market clearing rent level, \(R_2\), ensures equilibrium in the market for owner-occupied housing at the price level \(P_2\). The reduction in user cost shifts the LRS-curve from \(LRS_0\) to \(LRS'\). The new housing market
equilibrium in our three-period model set-up is given by \((R_2, P_2, C_2, S_2)\). Naturally, the new long-run equilibrium will depend on the rent-sensitivity of both the LRS and the demand for space.

**Figure 5** Housing market effects of a larger LTV-ratio and a reduction in user cost

The reduction in user cost that follows from the change in funding structure stimulates the market for owner-occupied housing, but the process to the new equilibrium is again not smooth. While the long run effect on the house price is positive, the market is characterised by some price overshooting, as seen by the price process \(P_0 \rightarrow P_1 \rightarrow P_2\).

Figure 6 shows the overreactions in house prices and housing construction that come about during the housing market boom.

To summarise, we have illustrated how the mortgage market momentum contributes to booms and busts in the housing market. In Regime B, we find that a larger LTV-ratio leads to a housing market bust, while in Regime C, a larger LTV-ratio leads to a housing market boom. As follows from the discussion in Section 3, the two regimes are related to whether the risk premium or the lending gain dominates the effect of a larger LTV-ratio on the user cost of owner-occupied housing, and hence housing market equilibrium. Our boom-bust regimes may be related to either monetary policy or risk assessments in the
mortgage market. A bust may be seen as a situation where risk pricing is aggressive, and the user cost increases in response to a larger LTV-ratio, which causes house prices to fall relative to the equity funding regime. A boom can be related to a situation where a larger LTV-ratio produces a lending gain that reduces the user cost, either due to the absence of risk pricing, a monetary policy that is not “leaning against the wind” and allows for excess return to mortgage financed housing, or a combination of the two.

**Figure 6**  
Response of house prices, rent and housing construction to a larger LTV-ratio when the leverage gain dominates the user cost effect

In our modelling framework, the short-run housing market dynamics are driven by the links between the markets for rental and owner-occupied housing and the construction industry. The dynamics depend on how closely the cost of different types of tenures is related to one another, as well as how the supply side responds to house price changes. To include a comprehensive survey of the literature on housing supply and the relationship between the different types of tenures is beyond the scope of this paper, but we highlight a few aspects related to these two. Empirically, one often finds the supply side of housing markets to be price inelastic (Caldera and Johansson 2013). As discussed earlier, the PR-ratio assumes perfect substitutability between rental and owner-occupied housing. As owner-occupied housing in addition to providing housing services also serves as an investment good and collateral for mortgages, the link between the two types of tenures might be different in real housing markets. Transaction cost, liquidity and risk aspects that impact the share of housing in a household portfolio as well as thin rental markets where rental alternatives to homeownership are lacking, might all impact the link between rental and owner-occupied housing. Hence, the degree of inelasticity of the housing supply, and the level of integration of the rental and owner-occupied housing markets are key aspects when analysing short-run housing market dynamics.
Analysing the Norwegian housing market and taking the changing funding structure of housing investments into account, Borgersen and Greibrokk (2012) calculate the return to housing equity and find a significant contribution from the lending gain as mortgage rates fall and house prices increase. Besides including the funding structure of housing investments in the return to housing equity, the estimated lending gain in the Norwegian market demonstrates the incentives for larger LTV-ratios emerging in a low interest rate environment with high house price growth. As the mortgage market momentum changes, and risk comes back to the agenda, increased risk taking is reflected in higher mortgage rates, thus changing the housing market regime.\footnote{Several papers argue that housing markets are characterised by regime switching structures (see for instance Chen and Leung 2008). Our boom- and bust scenarios represent two different regimes derived from the relation between the lending gain and risk pricing in mortgage markets. A similar situation including both a stable and an instable housing market regime is derived by Borgersen (2016) where the regimes are separated by a critical rate of appreciation. While the stable housing market regime is one where house prices are determined by market fundamentals, the instable regime is one where house price growth exceeds the critical rate and the prevailing lending gains allow “collateral policies” to produce an instable regime along the lines of Chen and Leung (2008).}

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A number of papers are concerned with housing market overreactions and prices deviating from long-run equilibrium (see Himmelberger et al. 2005). A significant body of literature analyses short-run housing market dynamics by using a dynamic stochastic general equilibrium (DSGE) model (see for e.g., Iacoviello and Neri (2010), Iacoviello (2010) and Tan et al. (2022)). Leung (2014) finds reduced form price dynamics consistent with error correction models proposed by Malpezzi (1999) and Capozza et al. (2004), linking equilibrium price dynamics to price-to-income ratios. Analysing US cities, Capozza et al. (2002) find house price overshooting especially in areas with high construction costs. Ortalo-Magne and Rady (2006) allow capital gains to amplify earning shocks which results in house price overshooting. André et al. (2017), analysing house prices in advanced and emerging economies, find asymmetric effects across booms and busts, as house prices undershoot less during busts than they overshoot during booms.

Zhou (2016) analyses overreactions in housing markets by addressing behavioural issues and different motives that impact housing demand. Based on a repeated sales index for Shanghai over the period of 2006-2015, the author finds evidence of overreactions in housing markets when differentiating between consumption and investment motives in housing demand. Fu and Qian (2014) also find that investment motive in housing demand contributes to house

\footnote{In the original DPW model, overreaction in the housing market may occur, since in the short run, the stock of space is fixed. Over time, the stock may adjust through construction or deterioration. The same mechanisms apply in our version of the model, when analysing a shock to the funding structure of housing investments.}
price overreactions in Shanghai. Xu et al. (2015) find overreaction as in Zhou (2016), when analysing the implications of housing policy.

The type of overshooting that we identify in this paper is as mentioned *not* related to individual behaviour, or shocks to household income, but to the *interaction* between different parts of the housing market system, as in the original DPW model. Due to the links between the segments, overshooting may also occur in other market segments than the segment directly hit by the shock. The *direct* effect of a larger LTV-ratio on the market for owner-occupied housing has *indirect* effects on both the housing construction industry and rental market. While there is a negative long run effect on house prices of a higher user cost, the short-run effect exceeds the long-run effect due to the sluggishness that characterises the indirect effects across the housing market system.

5. Conclusions and discussion

The housing market includes the rental market, the market for owner-occupied housing as well as the housing construction industry. Understanding the links between the different parts of the housing market might be challenging for both professionals and students. Moreover, knowledge is important to policy makers and analysts about the short-run market dynamics that these links create. Our ambition is to offer a simple but comprehensive framework for analysing housing markets that in a pedagogical way, assists with the structuring of arguments regarding housing market effects. This framework allows for both direct- and indirect effects from different types of shocks, and the assessment of short-run housing market dynamics and long run outcomes.

The original DPW model links the different parts of the commercial real estate market in an elegant and pedagogical way. This paper offers a straightforward extension to the DPW model by using an LTV-adjusted PR-ratio. This extension takes the DPW model into a housing market context where the user cost links the markets for rental and owner-occupied housing. Allowing for different sources of funding (equity and mortgage funding) that entail different costs, we study how funding structure impacts user costs. In our model, user cost is related to the LTV-ratio, hence linking the different parts of the housing market system to the mortgage market. By allowing for both risk pricing and lending gains, the model illustrates how the LTV-ratio might contribute to both housing market booms and housing market busts. Furthermore, our model highlights the indirect effects from a larger LTV-ratio on both the housing construction industry and the market for rental housing.

We consider two scenarios; one that represents a housing market boom where a large(r) LTV-ratio reduces the user cost and contributes to higher house prices. During the boom, the construction industry responds to higher house
prices by increasing the number of housing-starts. As the housing supply increases, the rental price falls, and impacts house prices negatively through the PR-ratio. Hence, during a housing market boom, the model predicts some house price overshooting and contributes to the understanding of short-run housing market dynamics. The other scenario considered represents a bust, where a larger LTV-ratio increases user costs. Higher user cost reduces house prices and makes the housing industry contract. The reduction in housing supply increases rent, an increase which is fed back to house prices through the equilibrium condition for the housing market. House prices are also during a bust characterised by some overshooting in the process towards the new equilibrium.

A stylised model always comes with a cost in terms of abstracting away from some aspects in real housing markets. In our model-based approach, the markets for owner-occupied and rental housing are linked by using a well-established equilibrium condition for housing markets, the PR-ratio, which equates the cost of living across different forms of tenures. The PR approach assumes perfect substitutability between rental and owner-occupied housing. As owner-occupied housing may serve multiple purposes (e.g., supply housing services, as collateral for mortgages and act as an investment object), while rental housing only serves the first of these three objectives, the perfect substitutability assumption comes with a cost. In addition, it is not cost free for households to move between rental and owner-occupied housing and transaction costs might dampen the argument in relation to our equilibrium condition for the housing market. Likewise, differences in the rate of depreciation between rental and owner-occupied housing may impact housing market equilibrium. Of course, uncertainty might matter for the choice of tenure. When considering the return to housing equity, our approach is partial, and does not take taxes into consideration. While a tax on housing equity might dampen the return to housing equity and hence, increases the user cost, tax deductible mortgage rates might have the opposite effect. An additional limitation in our modelling is the exogenous treatment of LTV and mortgage rates which ignores the link between them. Hence, our stylised approach abstracts away from aspects important both for the housing market as such, and for the link between the housing market and the mortgage market. Future modelling extensions should approach these abstractions.

The main aim of our paper is to provide a pedagogical tool for teaching housing market analysis in a comprehensive and non-technical framework for introductory courses in economics and finance. We have consequently made some simplifications abstracting from some empirical facts, but without loss of relevance. From our point of view, our modified model provides a didactic tool for discussing how housing financing impacts the housing market equilibrium and short-run market dynamics. The paper also provides a framework for discussions of monetary and financial policies including how access to credit affects different parts of the housing market. A comprehensive framework for
analysing housing markets provides a useful pedagogical tool to structure arguments regarding housing market effects. As the link(s) between the different parts of the housing market might be both bidirectional as well as direct and indirect, they could be difficult to frame-in. Allowing for both direct- and indirect effects from different kind of shocks, both short-run housing market dynamics and long run outcomes may be assessed in our non-technical framework.

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