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Tenant Portfolio Selection for Managing a Shopping Center

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In managing shopping centers, Kariya et al. (2005) formulate an analytical framework to optimize the net present value (NPV) of profit from each individual tenant with two control measures of fixed and variable rent linked to tenant sales or tenant replacement rules. Here, the framework is extended to a two-tenant case, where the sales growth rates of the new and old tenants are correlated in replacement. Hence, like the portfolio theory in finance, the combined NPV of the correlated cashflows of two tenants with the two control measures is optimized. To select a new tenant for a specific space, the sales correlation data of leaving and remaining tenants in proximity to the space can be used for optimization. Our simulation results show that it is better to select a new tenant whose predicted sales growth rate is high and has a largely negative correlation with that of the local tenants.

Keywords

Shopping center, Tenant management, Return-risk ratio, Percentage rent, Tenant replacement rule, Correlation of tenants' sales

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1. Introduction

In line with Miceli and Sirmans (1995), Kariya et al. (2005) (hereinafter KKUS) formulate a solution to the problem of optimizing the net present value (NPV) of future cash flows (CFs) in a shopping center (SC) by managing the two fundamental uncertainty factors of market lease rate and sales of each individual tenant in view of a net profit return and lower semi-standard deviation (LSD) risk of income, where the basic methods to enhance return risks are to use a rent-mix (percentage-fixed rent) ratio and tenant replacement (lease termination) agreement specified in the lease contract. Furthermore, optimal solutions for the percentage rate and the termination agreement are derived through simulation, where the models used are geometric discrete-time diffusion (GDD) models. They use the models to treat the case of a single representative tenant.

We follow their framework but expand the framework to the case of two representative tenants, where two tenants are assumed to be located close to each other in an SC, so that their sales competitively, complementarily or neutrally influence each other in variation. In other words, we bring the missing factor of the effect of tenant portfolio into the control problem where the correlation of the sales of two tenants is taken into account and after conducting a simulation, some recommendations are offered for SC management, where specific representative cases of sales correlations are explicitly considered. The revenue of an SC fluctuates with market rent variations, the sales of tenants, and management processes to select tenants in view of sales correlation.

Shopping center (SC) business is now becoming recognized as a real business platform, the success of which depends, among others, on the tenants who work together with management given that locational and economic environments are provided and management informs the tenants of the business situation as transparently as possible. Management will be better off only through the CFs of tenants by sharing a common incentive when variable rent and tenant replacement factors are adopted in managing tenants.
To describe a practical situation, suppose that a tenant will leave a specific space in an SC in six months' time and the SC manager needs to find another tenant who can provide a better performance for the SC in terms of CF through percentage rent. In such a case, we can use the correlated information with the monthly sales growth data of a few local tenants who are located in proximity to the space that will be available. Our simulation results show that it is ideal to find a new tenant whose predicted sales growth rates are high and largely negatively correlated with those of the tenant who has the highest sales growth rate among the local tenants. In fact, we derive such practically important implications from our various simulation cases by deriving the distributions of the NPVs and return-risk ratios under the GDD models of market lease rate, percentage rent and tenant replacement rule with parameters that include correlation.

In the literature related to the effect of percentage rents on SC management, Colwell and Munneke (1998) examine the value-enhancing aspects of percentage leases and explore the mechanisms of tenant mix, risk sharing and rent discrimination through which value is created. Differentiating between fixed and percentage leases, Chun et al. (1999) treat base and percentage rents as functions of sales, which are based on models developed in Benjamin et al. (1990, 1993) and Miceli and Sirmans (1995). Meanwhile, Wheaton (2000) argues that percentage rents, with sales externalities, discourage SC managers from acting opportunistically and to always have the interest of the existing tenants in mind when expanding, modifying, or reletting space in an SC. Gould et al. (2005) demonstrate that SC contracts are set to internalize externalities through both the efficient allocation and pricing of space, and the efficient allocation of incentives across the SC. They show that since some tenants generate externalities by attracting customers for other tenants, the success of each tenant depends on the presence and efforts of the other tenants and the SC to attract customers. More recently, Monden et al. (2021) provide a game-theoretic model where an SC offers different contracts for large and small tenants. In their analysis, they show that an SC offers a lower percentage rent and a higher fixed rent for the large tenant unless the small retailer faces a largely uncertain demand, and that the SC should attract homogeneous tenants since the expected profit of the SC increases with uncertainty of tenant demand.

For tenant replacement, Miceli and Sirmans (1995) discuss the problems of leasing arrangements between an SC and individual tenants in the presence of externalities between tenants. They show that the key elements are to create leases that achieve the goals of internalization of externalities and adequate marketing efforts are made by the SC in the leases. To realize these goals, SCs need to have the power to cancel the leases of tenants who do not meet the targeted sales level. However, no study has demonstrated the effect with respect to tenant replacement rules.

2. Method

In this section, we describe some of the basic concepts of the analytical structure of our problem. The optimality of our framework is considered with respect to the parameter α of percentage rent rate, parameter c of tenant replacement and correlation ρ of the sales of two tenants.

Here, α is the mix of percentage rent and monthly market lease rates which we call fixed rent:

$$\text{mix-rent} = (1 - \alpha)\ [\text{fixed rent}] + \alpha\ [\text{C} - \text{sales}] \qquad (1)$$

where fixed rate is the market lease rate at the time of the contract and fixed for the lease period, which is 3 years with possible renewal in this paper. Monthly C-sales is the sales defined in the contract that adjusts the initial amount of sales revenue so that it is equal to the initial fixed rent when the tenant moves into the SC (see Section 4 for definition). Mix-rent in Equation (1) shows that if the amount of C-sales is less than the fixed rate, the SC incurs a loss of the amount (fixed rent minus C-sales) compared to the case of completely fixed rent in that month, thus implying a risk-sharing scheme between the tenant and SC.

The percentage rent in Equation (1) is different from that commonly adopted in the US markets, where percentage sum is the sum of a flat base rent for a long lease period plus a variable rent (overage rent), which is calculated as a proportion of the sales revenue above the "breakpoint".

In Section 3, the mix rent scheme in Equation (1) is viewed as an investment allocation scheme where in each month, a portion of $100\alpha\%$ of the fully obtainable fixed rent (when $\alpha = 0$) is invested into the business sales of the tenant, which may be viewed as a stock investment and the remaining $100(1-\alpha)$ % is invested into the fixed rent that may be viewed as a bond investment.

On the other hand, the parameter c for optimization is a cut-off point parameter in a contract continuity condition (CCC) which is given as $c(k)$ in the contract continuity rule:

$$F\left(S_{i36(k-1)+1}, \cdots, S_{i36(k-1)+36}\right) \geq c(k) \qquad (k = 1, \cdots, K). \qquad (2)$$

where $S_{i36(k-1)+m}$ $(m = 1, \cdots ,36)$ is the sales of the i-th tenant in the m-th month of the k-th lease term and F is a specific function of the monthly sales revenue that defines the CCC, which is specified as a function of the sales growth rates in Section 4. Only if the sales of the i-th tenant meets the CCC (2) in the k-th lease term, then the lease contract can be renewed for another 3-year term. Otherwise, the tenant is replaced in the (k+1)-th lease term by a different tenant.

In our analysis, we assume GDD processes (Kariya and Liu (2003)) for the market lease rate and sales, and in the simulation, we maximize the mean NPV of rent CFs over 30 years to address the risk of LSD with respect to (α, c). The total time horizon for analysis is 30 years (360 months), which is divided into 10 lease contracts with 3 years for each contract. In each lease contract, the tenant is required to pay the mix rent in Equation (1) on a monthly basis for each lease period. Here, the market lease rate fluctuates monthly and is typically expressed by using a market index, thus implying that the fixed rate changes after 3 years to the next fixed rate when the contract is renewed to the next contract term. On the other hand, the percentage rate is a monthly variable rent that is proportionally linked to the sales of the tenant.

In this paper, SC management has the right to require a tenant to vacate the rented space after giving 6 months of advance notice if the tenant fails to meet the conditions agreed in the provisions of the CCC based on sales performance.

3. Reformulation and Tenant Portfolio Selection

In this section, the formula to find an optimal percentage rate and CCC cut-off is extended to the formula that partially enables an SC manager to select a more profitable tenant one by one when a rental space is vacated one by one and then form a better tenant portfolio by taking into account the correlation of the business sales of the two tenants, although the process will never allow a final most optimal portfolio.

In the case of rents in Equation (1), there are two uncertainty (variation) factors that affect the future CFs of the SC owner over the 30 years, one of which is linked to the market lease rate of the first month in each 3-year contract, and another which is linked to the monthly C-sales of each tenant. Interestingly, the mix rent structure in Equation (1) indicates that the α portion of each monthly fixed rent is invested in the business sales of the i-th tenant. Since the mix rent is equal to the fixed rent in the first month of the new contract as will be discussed, the performance of this investment will appear from the second month on as the α portion of the C-sales or equivalently as the α percentage rent. As long as the tenant successively meets the CCC, this will continue with the amount of C-sales continuing, even though the fixed rent is reset to a new market rate at each lease renewal. Hence from an investment viewpoint, the fixed rent may be regarded as fixed income redemption (bond that pays equal monthly fixed coupon) for 3 years while the monthly variable rent may be regarded as a monthly stock dividend from the i-th tenant. In this analogy, the tenant issues bond and stock for a space in the SC and the SC manager who receives them is an investor who is creating a bond and stock portfolio: $(1-\alpha)$ bond $+\alpha$ C-sales for each tenant business. Since the fixed rent is constant for each lease period, the C-sales is independent for each lease period and hence independent of the fixed rent over the ten 3-year lease periods. This is taken into account by modeling the market lease rate and processes of percentage-rent linked to sales. This implies that not only the mean of discounted cash flows (DCFs) but also the risk of DCFs in terms of LSD is the sum of those from the fixed rent and the percentage rent linked to sales in the 30 years. This fact is common to the relation between each tenant and the SC owner. Note that the fixed rate itself is changing in each lease period along with changes in the market lease rate.

On the other hand, the sales of the tenants are in general correlated and hence the percentage rents of 2 tenants are correlated over a common lease period. However, since the lease contract does not allow the SC to replace tenants freely, it is not possible to apply an argument of stock portfolio optimization in return and risk to a Set A of businesses of the general tenants in which the SC manager might try to make an optimal tenant portfolio, where anchor tenants, and other indispensable tenants are excluded from A. Hence, if possible, it would take a long time to adjust the tenant portfolio toward the desired direction.

Besides, an optimal portfolio will change over time. In fact, our society evolves slowly but steadily with changes in technology, preferences and social value systems, and correspondingly as a property manager, the SC is required to change the structure of buildings with new technology, floor structure and business model with new tenants and allocations in the SC. In short, it is important to respond to the needs of society and the demographics, and increase the attractiveness and competitiveness of SCs from the viewpoint of the tenants together with enhancement of their sales, their externalities and the correlations of their sales. Consequently, no formal application of the modern portfolio theory (MPT) works for SC management even for the Set A of general tenants.

According to our simulation results in Section 6, a new tenant with a high predicted rate of sales growth and a sales growth that has a largely negative correlation with that of the tenant with the highest sales growth rate among the local tenants is recommended.

A Critical View on the Correlation Structure of Sales Growth of Tenants

In the above, we have expressed our view that the MPT is not directly applicable to the entire set of general tenants (A), and that it is important to consider a pairwise effect of the tenant portfolio based on the NPV through the correlation in view of the tenant recruitment and allocation problem stated in Section 2. In terms of the portfolio correlation structure that the SC cash follows, we add three important remarks on how sales growth correlations are formed.

First, it is interesting how the total sales correlation of two tenants is formed. Each tenant has its own portfolio of goods and services associated with some business segments and brands and some other characteristics including pricing (managed by the tenants themselves), time of sales (seasonality, business cycles, etc.), and location of each tenant in the zoning of the SC along with its building structure and size. According to the composition of the portfolio, each competes with and supports other tenants via competitiveness and externality. However, even though the sales data of individual goods and services in the composition are made available to the SC management via point of sale (POS) systems and other management tools, the SC management will not be able to change the individual correlation structure of goods and services and/or segmented sales model between two tenants in a local neighborhood. Hence, if we are to model the correlation structure, it would be expressed as a complex model with a large number of parameters and limited data even for two tenants, and the estimated model would not be practically useful because of the complications and instability.

Second, when a zoning system is adopted in an SC, the location of each tenant together with the building structure will be relatively important in the formation of the correlation structure because it determines the stream of customers. This will in fact differentiate the rent base for each area or zone and probably the sales growth rate in the segmented zoning. As such, we recommend setting a different growth rate and correlation for each zone and/or business in the modelling process. In this paper, we set the volatility of the growth rate equal to 20 as a case of comparatively volatile businesses, which seem to enhance the SC revenue in view of the high-risk and high return where the risk can be controlled through the diversification effect. In addition, a differentiated case with different volatilities for zoning and/or businesses can be treated in our analytical framework, although we do not pursue the problem and leave it for future research work.

In terms of the area-wise setting of the rent structure, even though the mean base in our framework is regarded as the market rent that applies to the general tenants, the base rent for essential shops such as those in food court areas and the proximity of anchor shops will be discounted. Therefore, it is difficult to consider the total sales correlation structure of all of the tenants at the same time from the outset of analysis. This is related to the inapplicability of the MPT even to the entire Set A under the current sales level. However, it is also true that SC management undoubtedly holds a portfolio of sales of the tenants as a whole, thus implying the need for further research. Without looking into the structurally heterogenous zoning and business relations for this problem, we do not think that it is appropriate to uniformly treat the sales growth rates of all of the tenants at the same time, which will cause a misleading portfolio analysis with a specious correlation structure. Consequently, it will be better to consider a pairwise effect of the tenant portfolio.

Third, let us briefly discuss the difficulty of developing an analytical framework that includes anchor tenants. We speculate that the anchor tenants will have completely different and varying lease contracts according to their (game-theoretic) market power. From the viewpoint of SC management, the prestigious anchor tenants in an SC have an important value and give the SC a brand image that signals the implied status of the SC to customers. Hence, the SC will indirectly pay for brand loyalty which will be reflected in the lease contracts including the contract termination conditions. The anchor stores have spillover value for the general tenants who will then adjust their strategies when forming their business portfolio in the SC. In particular, a new tenant that fills an available spot near an anchor will consider both the positive and negative effects of the presence of the anchor shop on its business.

4. Analytical Framework and Concepts for Analysis

The timeframe of our monthly analysis is 30 years, which is broken down into ten 3-year contract periods labeled as $k = 1,2,\cdots,10$, where each of 360 months of the 30 years is denoted by $n = 1,2,\cdots,360$ in a time series order. Thus the months in the first contract period are denoted as $n = 1,2,\cdots,36$, and those in the second period are denoted as $n = 37, 38,\cdots,72$, and so on. Let n=0 represent the time origin 0 of the analysis. Thus the n-th month corresponds to the time interval nh years with $h = 1/12$ from n=0.

The SC has *I* spaces for lease, indexed by $i = 1,2,\cdots,I$. Each space is occupied by a tenant with a specific type of business. For simplicity, it is assumed that a tenant is found upon vacancy.

For rent income, the rent of the nth month per 3.3 m^2 (a tsubo or informal unit of area in Japan) for the ith space (a specific type of business) can be expressed as the mix rent:

$$X_{i,n}^{M}(k) \equiv X_i^M(k, m(k)), \tag{3}$$

where n belongs to the k-th term and $m(k)$ is uniquely defined by:

$$n = 36(k-1) + m(k) \text{ for } 1 \leq m(k) \leq 36. \tag{4}$$

The mix rent in our terminology can be further expressed as:

$$X_{i,n}^{Mix}(k) = (1-\alpha_i)\bar{X}^f(k) + \alpha_i \tilde{S}_{i,n}(k) \tag{5}$$

where $\bar{X}^f(k)$ is the fixed rent of the k-th contract term that is the market lease rate in the [36(k-1)]-th month, and α_i represents the proportion of the variable rent linked to the C-sales $\tilde{S}_{i,n}(k) \equiv \tilde{S}_i(k, m(k))$. In what follows, we use $\tilde{S}_{i,n}$ interchangeably for $\tilde{S}_{i,n}(k)$.

To define our C-sales $\tilde{S}_{i,n}(k)$, note that the fixed rent of the k-th contract term is $\bar{X}^f(k) = \tilde{X}_{36(k-1)}$, where $\tilde{X}_n \equiv \tilde{X}(k, m(k))$ is the market lease rate in the nth month with $n = 36(k-1) + m(k)$ and independent of the space index i. Hence when $\alpha_i = 0$, then

$$X_{i,n}^{M}(k) = \bar{X}^f(k) = \tilde{X}_{36(k-1)}, \tag{6}$$

which is the case of a lease agreement with fixed rent. Then the mix rent at $n = 1$ is

$$\begin{cases} X_{i,1}^{M}(1) = (1-\alpha_i)\bar{X}^f(k) + \alpha_i \tilde{S}_{i,1}(1) \\ \bar{X}^f(1) = \tilde{X}_0 \end{cases} \tag{7}$$

$\tilde{X}_0$ is the market lease rate at time 0. Now let $S_{i,n}$ be the actual sales of the i-th tenant (space) in the n–th month and let

$$r_{i,n} = \frac{1}{h}\log\left(S_{i,n}/S_{i,n-1}\right), \tag{8}$$

be the actual sales growth rate from the (n-1)-th month to the n-th month. Then the actual level of sales can be expressed as:

$$S_{i,n} = S_{i,n-1}\exp\left(r_{i,n}h\right). \tag{9}$$

When $n=1$, $S_{i,1}$ is observable, but $S_{i,0}$ is not, and neither is $r_{i,1}$. However, when $n \geq 2$ then $r_{i,n}$ is observable. Hence defining

$$\tilde{S}_{i,1} = \bar{X}^f(1) = \tilde{X}_0, \tag{10}$$

the initial mix rent is $X_{i,1}^M(1) = \tilde{X}_0$, which is nothing but the initial fixed rent that the i-th tenant is required to pay at the beginning of the first month, whatever the percentage rate α_i may be.

Our contract sales at time $n = 2$ with an observable $r_{i,2}$ is defined as:

$$\tilde{S}_{i,2} = \tilde{S}_{i,1}\exp\left(r_{i,2}h\right) = \tilde{X}_0\exp\left(r_{i,2}h\right). \tag{11}$$

The mix rent in this case is $X_{i,2}^M(1) = (1-\alpha_i)\bar{X}^f(1) + \alpha_i\tilde{S}_{i,2}(1)$. Similarly, contract sales with observable $r_{i,n}$ is defined as:

$$\tilde{S}_{i,n}(k) = \tilde{S}_{i,n-1}\exp\left(r_{i,n}h\right) = \tilde{X}_0\exp\left(\textstyle\sum_{j=2}^{n} r_{i,j}\, h\right), \tag{12}$$

and the mix rent in month n with $36 \geq n \geq 3$ is given by $X_{i,n}^M(1) = (1-\alpha_i)\bar{X}^f(1) + \alpha_i\tilde{S}_{i,n}(1)$. This is merely Equation (5) with $k = 1$.

For the kth contract period, the mix rent for the tenant who continues to lease space *i* in the *k*-th contract period is given by Equation (5). However, in the case of a new tenant that takes space *i* at the end of $n = 36(k-1)$, the fixed rent part is the market lease rate of month $36(k-1)$, i.e., $\bar{X}^f(k) = \tilde{X}_{36(k-1)}$ and the variable (percentage) rent in month $n = 36(k-1)+1$ is the contract sales; $\tilde{S}_{i,36(k-1)+1}(k) = \bar{X}^f(k)$ and for $n \geq 36(k-1)+2$

$$\tilde{S}_{i,n}(k) = \tilde{S}_{i,n-1}(k)\exp\left(r_{i,n}h\right). \tag{13}$$

From time $n \geq 36(k-1)+2$ onward, the mix rent is defined in Equation (5).

4.1 Sales Level Tenant Replacement Rule

In creating value through the SC management, whether or not percentage rent is introduced, it is still crucial to replace tenants with lower sales performances by those with higher sales performances. In fact, the latter tenants will have greater ability to attract customers and keep them longer in the SC once they come so that the tenants benefit mutually from the external economies, enhance the brand value, and render the SC competitive.

A process of selecting or keeping good tenants is now formulated as a CCC in Equation (2). As a function F in Equation (2), we adopt the following C-Sales Level Tenant Replacement (SLTR) rule. For a tenant to renew its contract, its C-sales six months prior to the end of the contract period must meet the CCC:

$$\tilde{S}_{j,36k-6}(k) = \tilde{S}_{i,36(k-1)+1}(k)\exp\left(\sum_{j=36(k-1)+2}^{36k-6} r_{i,j}h\right) \geq c(k). \qquad (14)$$

Here, $c(k)$ is assumed to have a constant c.

4.2 NPV Distribution, Mean as Return and Lower Semi-Standard Deviation as Risk

Given the above lease agreement structure and tenant-replacement rules, the dynamic discounted cash flow (DDCF) value of future CFs is expressed as:

$$V_i = \sum_{k=1}^{K} V_i(k), \qquad (15)$$

where $V_i(k)$ is the DDCF value of future CFs from the i-th tenant during the k-th contract period. An expression of $V_i(k)$ that distinguishes the i-tenant between a new tenant in the k-th term and a continuing tenant from the (k-1)-th term is given by:

$$V_i(k) = \sum_{n=n_{k-1}+1}^{n_k} \left[(1-\alpha_i)\bar{X}^f(k) + \alpha_i \widetilde{U}_{i,n}(k)\right] \cdot A_i \cdot D(n). \qquad (16)$$

Here, $D(n) = (1+d)^{-nh}$ which denotes the discount rate for CFs of month n where the cap rate d (a constant) reflects the complex risks associated with the uncertain profitability of real estate investments and A_i is the area of space i in terms of 3.3 m^2. In the following, we set $d = 0.02$.

The term $\widetilde{U}_{i,n}(k)$ denotes a notation of the sales that discriminates the sales of tenant leaving in the k-th term and the sales of tenants staying in the k-th term. In the case of the first contract, it is $\widetilde{U}_{i,n}(1) \equiv \tilde{S}_{i,n}(1)$. To distinguish a tenant who is staying in the k-th term and one who is leaving in the k-th term, we use the following:

$$L_i(k) = \begin{cases} 1 & if \;\; F_k\left(\tilde{S}_{i,36(k-1)+1}, \cdots, \tilde{S}_{i,36(k-1)+36}\right) \geq c \\ 0 & otherwise \end{cases}. \qquad (17)$$

Then the contract sales for the tenant in the k-th contract period is

$$\widetilde{U}_{in}(k) = \widetilde{U}_{in}(k-1)L_i(k-1) + \tilde{S}_{in}(k)[1 - L_i(k-1)]. \qquad (18)$$

When $L_i(k-1) = 1$, the tenant in the (k-1)-th contract term continues to stay in the kth term and this equation expresses the contract sales of a tenant who continues to stay from the $(k-1)$-th contract period. If $L_i(k-1) = 0$, which is the case of tenant replacement, $\widetilde{U}_{in}(k) = \tilde{S}_{in}(k)$ is the contract sales for a tenant newly taking the space with $\tilde{S}_{i,36(k-1)+1}(k) = \bar{X}^f(k) = \tilde{X}_{36(k-1)}$ being the rent of the first month of the k-th term.

The sales of each tenant follows its process along with the described procedure even though the sales realization of one tenant is correlated with that of another tenant. What we are interested in is the total NPV distribution of the sum $V = V_1 + V_2$ of two tenants with basic characteristic statistics of the mean, LSD and so on.

5. Model Formulation

In order to treat the tenant portfolio management problem stated in Sections 1, 2 and 3, the analytical framework is extended to the case where the sales of the tenants are correlated. For this purpose, the market lease rate process and the sales processes of two tenants with correlation are specified respectively in Sections 5.1 and 5.2. Although the models are individually expressed, we derive the total NPV distribution of $V = V_1 + V_2$ via simulation, where the sales of two tenants are generated correlatedly from the GDD models.

5.1 Market Lease Rate Model (per 3.3 m^2)

The following GDD process is assumed as our market lease rate model:

$$\tilde{X}_n = \tilde{X}_{n-1}\exp\left(\mu_{X_{n-1}}h + \gamma_{X_{n-1}}\sqrt{h}\tilde{\varepsilon}_{X_n}\right), \qquad \tilde{\varepsilon}_{X_n} \sim iid\ N(0,1), \qquad (19)$$

where the notation $\tilde{\varepsilon}_{X_n} \sim iid\ N(0,1)$ means that the disturbance factor $\tilde{\varepsilon}_{X_n}$ independently and identically follows a standard normal distribution. In general, the drift $\mu_{X_{n-1}}$ and volatility $\gamma_{X_{n-1}}$ may depend on the past values of $\tilde{X}_n$. However, while the volatility $\gamma_{X_{n-1}}$ is assumed to be constant γ_{X_i} here, i.e., independent of past values, the drift $\mu_{X_{n-1}}$ of market rents is assumed to follow the following exponentially smoothing model;

$$\mu_{X_{n-1}} = \phi_X \log\left[\frac{\tilde{X}_{n-1}}{\tilde{X}_{n-2}}\right] + (1 - \phi_X)\mu_{X_{n-2}}, \qquad (20)$$

which is non-Markovian, as is expected for sales movements. In fact, sales variations in SCs in general tend to carry seasonal movements.

The smoothing parameter ϕ_X indicates the portion for which new information on rate of rent changes $x_{n-1} \equiv \log[\tilde{X}_{n-1}/\tilde{X}_{n-2}]$ is discounted with ϕ_X and reflected in the market rent level $\tilde{X}_n$. A smaller smoothing parameter ϕ_X means that the drift moves more slowly and smoothly. The rent system is usually adjusted to the dependence on the type of business although we omit the argument here.

5.2 Sales Processes of Two Tenants with Correlation

By formulating the rate of sales return $r_{i,n}$ of the i-th tenant as

$$r_{i,n} = \mu_{i,n-1}h + \gamma_i\sqrt{h}\tilde{\varepsilon}_{i,n},\ \tilde{\varepsilon}_{i,n} \sim iid\ N(0,1),$$

the correlated contract sales processes of the ith and jth spaces are given as follows.

$$\begin{cases} \tilde{S}_{i,n}(k) = \tilde{S}_{i,n-1}(k)\exp\left(\mu_{i,n-1}h + \gamma_i\sqrt{h}\tilde{\varepsilon}_{i,n}\right) \\ \tilde{S}_{j,n}(k) = \tilde{S}_{j,n-1}(k)\exp\left(\mu_{j,n-1}h + \gamma_j\sqrt{h}\tilde{\varepsilon}_{j,n}\right) \\ \qquad Corr\left(\tilde{\varepsilon}_{i,n}, \tilde{\varepsilon}_{j,n}\right) = \rho(i \neq j) \end{cases} \tag{21}$$

Here, the noise terms $\tilde{\varepsilon}_{i,n}$ and $\tilde{\varepsilon}_{j,n}$ are correlated which denotes that the contract sales of different tenants are mutually related to each other. This assumption is crucial for investigating the portfolio effect.

In addition, as in the case of the rent variation, drift $\mu_{i,n-1}$ and volatility γ_i depend on the past values of $\tilde{S}_{i,n}$. More precisely, we assume an exponential smoothing model:

$$\begin{aligned} \mu_{i,n-1} &= \phi_i \log\left[\frac{\tilde{s}_{i,n-1}(k)}{\tilde{s}_{i,n-2}(k)}\right] + (1-\phi_i)\mu_{i,n-2} \\ &= \phi_i \gamma_{i,n-1}h + (1-\phi_i)\mu_{i,n-2} \end{aligned} \tag{22}$$

On the other hand, the volatility γ_i of the sales rate is assumed to be a constant. The volatility of sales is therefore set to be higher than that of market rents.

5.3 Total NPV Distribution of $V_1 + V_2$ via Simulation

By generating 100,000 DDCF paths (scenarios) of $\left\{\left(\tilde{X}_n, \tilde{S}_{i,n}\right)(n = 1,2,\cdots,360)\ (i = 1,2)\right\}$ under the GDD type models with relevant parameters including correlation (see next section), the total NPV distribution of $V = V_1 + V_2$ is derived based on 100,000 values of $\left\{V^{(l)} = V_1^{(l)} + V_2^{(l)}\right\}$ generated by using a Monte Carlo simulation. From the distribution, the mean (expected) value is given by:

$$\mu_{i,n-1}M = \sum_{l=1}^{L} V^{(l)}/L \quad (L = 100{,}000) \tag{23}$$

and some other statistics including standard deviation and quantile points are obtained. Among others, the LSD is adopted to evaluate the downside uncertainty of the total NPV and as a risk measure of the total NPV, since the distribution is not symmetric. The LSD is defined as:

$$LSD \equiv R = R(\alpha, c\!:\!\rho) = \left[\sum_{l=1}^{L} \left\{\min\left(V^{(l)} - M\,, 0\right)\right\}^2 / L\right]^{1/2}, \tag{24}$$

and we use this risk measure to evaluate the NPV performance of two tenants with a return-risk ratio of M/R; the mean NPV income per unit risk. Let us call the case of the fixed rate with $\alpha = 0$ throughout the 30 years the "base case", whatever the sales performance of the tenants may be.

6. Valuation Using Monte Carlo Simulation

In this section, we consider our problem with the correlated sales of two tenants in the models in Equation (21) and tenant replacement rule in Equation (14) as described in Sections 4 and 5.

The simulation results of the return-risk performance in our two-tenant portfolio are presented, where the following cases are considered:

[1] Fixed rent only ($\alpha = 0$) and no replacement of tenants;
[2] Percentage rent ($\alpha > 0$), but no replacement of tenants; and
[3] Percentage rent ($\alpha > 0$), and SLTR rule applied.

Here, the tenant replacement rule is given by Equation (14), which is called the SLTR rule, where the C-sales level is measured two years and six months after the contract starts.

The following parameters are fixed:

1. Initial drifts of market rent and sales processes $\mu_{X_0} = 0,\ \mu_{i,0} = 0$ for $i = 1,2$
2. Volatilities of market rent and sales processes: $\gamma_X = 0.05,\ \gamma_i = 0.2$ for $i = 1,2$
3. Smoothing rates of market rent and sales processes: $\phi_X = 0.2$, $\phi_i = 0.2$ for $i = 1,2$
4. Continuously compound interest rate $r^* = 0.02$
5. Mixing rate $\alpha = 0.35$

However, the correlation of ρ in Equation (21) and volatility parameters γ_X and $\gamma_i{}'\ s$ are control parameters in the simulation to determine the changes in the characteristics of the NPV distribution of the two tenants. Therefore, they are changed in the simulation.

6.1 Case [1] Fixed Rent Only ($\alpha = 0$) and No Replacement of Tenants

In the base case with fixed rent and no replacement in Equation (19), the total NPV distribution of $V = V_1 + V_2$ has the statistical values of the mean (*M*), LSD, minimum (Min), and lower 5% (L5%) and 10% (L10%) percentile points:

M= 718.49, LSD= 75.02, Min=383.91, L5%=497.89, and L10%=580.10.

Then in this base case, the return-risk ratio is

$$M/LSD = 718.49/75.02 = 9.58. \tag{25}$$

Hence in the case of mix rent, the mean of the NPV return should be higher than 718.49 and the LSD as its risk is less than 75.02. Consequently, the return-risk ratio should be larger than that of the base case, i.e., 9.58.

6.1.1 Efficient Region in the total NPV relative to its Risk (LSD)

This return-risk ratio of the base case is a reference value to evaluate the performance of other cases with mix rent that have a value at n as the sum $X_{1,n}^M(k) + X_{2,n}^M(k)$ with $X_{i,n}^M(k) = (1-\alpha)\bar{X}^f(k) + \alpha\tilde{S}_{i,n}(k)\ (i = 1,2)$. The total NPV V of the two tenants in Equation (16) depends on α and c. Hence, the *M* and LSD of the NPV distribution depends on α and c. Let them be denoted with $M = M(\alpha, c)$ and $LSD = LSD(\alpha, c)$. Comparing these to those in the base case, let the return-risk ratio be:

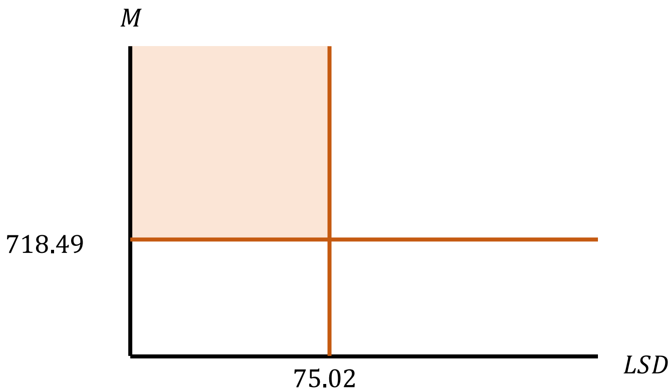
$$G(\alpha, c) = M(\alpha, c)/LSD(\alpha, c). \tag{26}$$

Then, we aim to make this ratio as large as possible with respect to (α, c) in the following efficient region:

$$EFF = \{(\alpha, c): M(\alpha, c) > 718.49, LSD(\alpha, c) < 75.02\} \tag{27}$$

If (α, c) belongs to this efficient region, the *M* of the total NPV is higher than that of the base case and the risk as LSD is smaller than the base case, thus implying that so long as (α, c) belongs to this region, the return-risk performance of mix rent in G is better than that of the base case. Therefore, mix rent and the tenant replacement rule are recommended for use.

Figure 1 Efficient Region in Total NPV Relative to its Risk



6.1.2 Detailed Results of Case [1]

Figure 2 shows the NPV distributions of the total value $V = V_1 + V_2$, when market rent volatility γ_X changes, where the horizontal axis is measured in units of 1,000 yen (about USD 7 (June 2023)). Since tenants are not replaced in Case [1], the NPV distribution tends to have a longer-tailed distribution toward the right direction as γ_X increases.

Figure 2 Case [1]: NPV Distributions for Changes in Market Rent Volatility γ_X

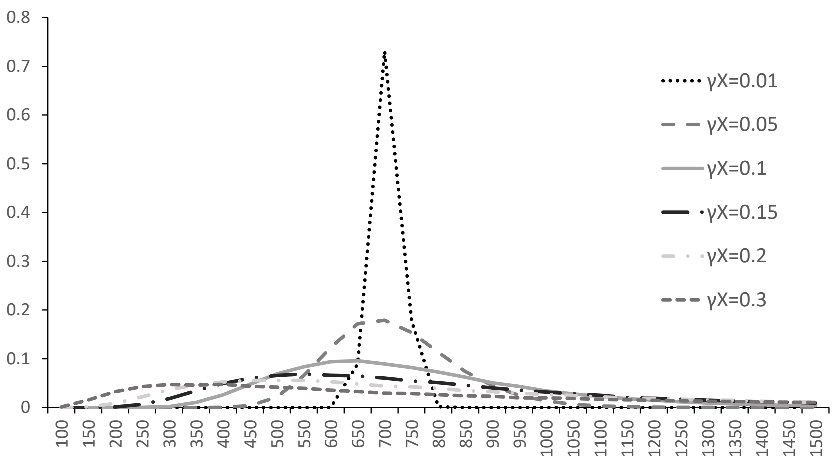


Table 1 shows the summary of the NPV distributions when γ_X changes, which includes the LSD, Min, quantiles of L1% (1%) through to L50% (50%), and average returns (M). A higher γ_X, smaller Min, and lower distribution quantiles result in longer right-side tails because once rents have an upward trend, the exponential smoothing scheme allows rents to maintain the trend at a certain rate, thus producing further increases in rent. The average returns M also increases as γ_X increases, but the increment of LSD (R) is much larger than that of M, thus resulting in lower M/Rs.

Table 1 Case [1]: Summary Statistics for Changes in Market Rent Volatility γ_X

	γx=0.01	γx=0.05	γx=0.1	γx=0.15	γx=0.2	γx=0.3
LSD	15.56	75.02	151.19	243.09	366.47	860.63
Min	622.81	383.91	239.11	164.84	129.82	93.67
1%	654.50	497.89	367.99	283.82	226.72	160.59
5%	668.81	549.63	440.22	362.45	303.34	226.54
10%	676.41	580.10	486.66	417.12	360.60	283.64
20%	685.85	620.68	553.63	499.25	456.09	390.80
50%	704.25	707.34	715.85	729.29	749.67	807.96
Mean	704.71	718.49	763.65	849.55	989.17	1605.38

Table 2 provides the return-risk ratio M/R. M/R represents a risk-return tradeoff as in the case of the Sharpe Measure in the portfolio theory, although we stated in Section 2 that it is difficult to apply the theory to our case. We adopt the case where $\gamma_X = 0.05$ as a practical reference for the annual volatility of a 3-year term fixed lease rate, in which M/R = 9.58 as in Equation (25), and use the value to derive the NPV distributions of the cases with mixed rent and tenant replacement (Cases [2] and [3]), where α and c change. Note that as γ_X increases, M/R decreases as provided in Table 2.

Table 2 Case [1]: M/R for Changes in Market Rent Volatility γ_X

	γ_X = 0.01	γ_X = 0.05	γ_X = 0.1	γ_X = 0.15	γ_X = 0.2	γ_X = 0.3
M/R	45.28	9.58	5.05	3.49	2.70	1.87

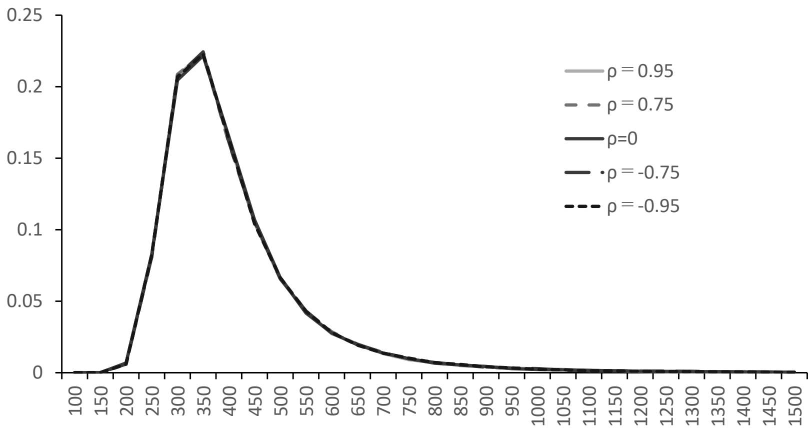
6.2 Case [2] Percentage Rent ($\alpha > 0$), but No Replacement of Tenants

In Case [2], it is assumed that the market fixed rent follows the same process in Case [1] with $\gamma_X = 0.05$ and the contract sales processes $\tilde{S}_{i,n}(k)$ follow the models in Equation (21), where each drift μ_{in-1} is determined by its history. The sales volatility γ_i is assumed to be 20% for both tenants, which is much higher than $\gamma_X = 0.05$.

In Case [2], we set $\alpha = 0.35$ as the percentage rate in Equation (1). In addition, the ρ between the sales growth rates of the two tenants in Equation (21) is our concern together with γ_i in this analysis to the return-risk enhancement effect via tenant sales, where ρ here varies over 0.95, 0.75, 0, -0.75, and -0.95.

Figure 3 shows that ρ does not have any significant effect on the shape of the NPV distributions when it varies. Table 3 shows that the *M*s of the distribution are completely the same as they are determined by the marginal distributions without any effect of ρ. Theoretically speaking, all the *M*s are the same. Interestingly, the *M*s in the tables are uniformly higher than the *M* of the fixed rent case (Case [1]) with volatility $\gamma_X = 0.05$, which means that an inclusion of a percentage rent may improve the profitability of SCs even with no tenant replacement.

Figure 3 Case [2]: NPV Distribution for Changes in ρ with No Replacement ($\alpha = 0.35$)



On the other hand, risk measures such as the Min and quantiles become smaller as ρ increases from -0.95 to 0.95. In other words, the risks are smaller when ρ becomes negatively larger since for e.g., the Min of $V = V_1 + V_2$ becomes larger. Hence in these measures, ρ should be smaller.
Meanwhile, the LSD risk measure increases from 31.06 to 84.16 when ρ increases from -0.95 to 0.95, thus implying that for the LSD, it will be better for ρ to be smaller. In fact, when $\rho = 0$, M/R allows us to compare Case [1] with $\gamma_X = 0.05$ and $\alpha = 0$ and Case [2] with $\gamma_X = 0.05$, $\gamma_i = 0.2$ and $\alpha = 0.35$. The LSD is 75.02 for Case 1 regardless of ρ and 63.74 for Case [2] at $\rho = 0$. Consequently, without introducing the tenant replacement rule, the return-risk ratio at $\rho = 0$ is enhanced from 9.58 to 12.95 by introducing a mix

rent ratio $\alpha = 0.35$ as in Table 4, which in fact shows that the M/Rs are all larger than that in Case [1]; see Table 2 ($M/R = 9.58$ at $\gamma_X = 0.05$). So long as $\alpha = 0.35$, the ratios are much more enhanced for negative ρs. In addition, $M/R = 9.78$ in Case [2] with $\rho = 0.95$ in Table 4 almost corresponds to $M/R = 9.58$ in Case [1], where the introduction of a percentage rent does not significantly have an effect on the ratio because the sales of two tenants move almost simultaneously together. Thus, in view of practices, this suggests that SC managers should choose a new tenant whose sales growth rates are negatively corelated to those of the other tenants in the neighborhood of a spot that will soon be available.

This suggests that mixed rents may have a large return-risk enhancement effect. However, from a practical viewpoint, it may be rather difficult to find a new tenant whose sales is negatively correlated with those of others in the neighborhood of a spot that will soon be available.

Table 3 Case [2]: Summary Statistics for Changes in ρ with No Replacement ($\alpha = 0.35$)

	ρ=0.95	ρ=0.75	ρ=0	ρ=−0.75	ρ=-0.95
LSD	84.18	79.83	63.74	40.93	31.06
Min	340.54	339.73	369.26	457.52	480.86
1%	462.99	469.82	500.42	559.00	591.89
5%	522.06	530.22	560.44	612.82	638.50
10%	558.67	566.60	597.98	645.79	665.38
20%	609.85	617.89	648.71	688.58	701.59
50%	741.90	749.14	772.61	786.36	785.92
Mean	823.18	825.12	825.18	824.20	825.20

Table 4 Case [2]: M/R Ratio for Changes in ρ with No Replacement ($\alpha = 0.35$)

	ρ=0.95	*ρ*=0.75	*ρ*=0	*ρ*=-0.75	*ρ*=-0.95
M/R	9.78	10.34	12.95	20.14	26.57

6.3 Case [3]: Percentage Rent ($\alpha > 0$), with SLTR

Now let us consider Case [3] where the SLTR rule in Equation (14) is introduced in the contract with a CCC, in addition to the inclusion of percentage rent ($\alpha > 0$). The CCC is a condition where the contract sales level at the end of two years and six months is required to be higher or equal to a cut-off point c. If a tenant fails to meet this CCC, it is replaced by a new tenant.

First, we consider in Case [3] that $\gamma_X = 0.05$, $\alpha = 0.35$ and $c = 0.8$ of the tenant replacement rule. If the sales level of the i-th tenant drops to less than

8,000 yen per m^2 in exactly two years and six months after the start of the contract period, then the tenant is replaced in the next lease term by a different tenant. Otherwise, the tenant can renew the lease contract for the next 3 years. Figure 4 shows that the probability distribution of the NPV in the SLTR rule when ρ of the contract sales between tenants is varied from 0.95, 0.75, 0, -0.75, to -0.95. A summary of the NPV statistics is provided in Table 5.

Figure 4 Case [3]: NPV Distribution for Changes in ρ ($\alpha = 0.35, c = 0.8$)

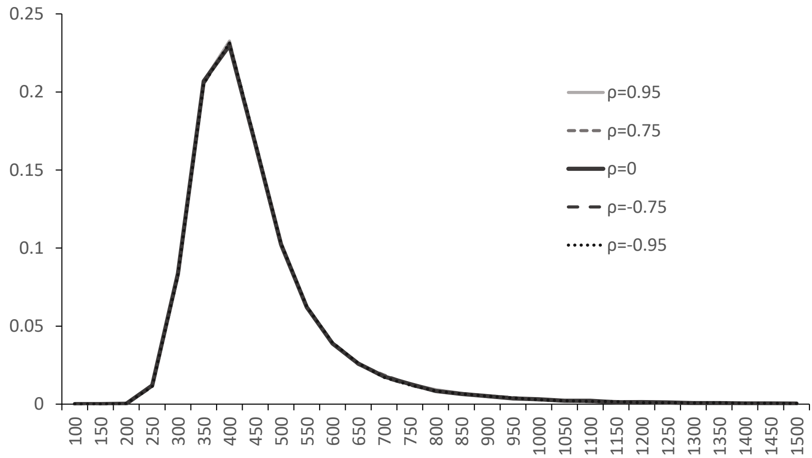


Table 5 Case [3]: Summary Statistics for Changes in ρ ($\alpha = 0.35, c = 0.8$)

	ρ=0.95	ρ=0.75	ρ=0	ρ=-0.75	ρ=-0.95
LSD	40.25	37.80	30.77	22.63	19.75
Min	393.11	404.85	405.56	396.74	410.48
1%	545.86	549.77	571.24	603.19	615.72
5%	614.91	621.67	643.91	671.57	680.65
10%	655.24	662.33	685.11	710.84	717.95
20%	708.48	715.55	738.70	761.73	766.18
50%	836.11	843.66	863.54	876.08	875.26
Mean	913.89	915.27	913.48	914.65	914.28

In all cases of the correlations, the Min and all quantile points are greater than those of the fixed rent ($\gamma_X = 0.05$). The *M*s are also higher than those of Cases [1] and [2]. This shows the value of such a tenant replacement as an SLTR in SC management with the aim to enhance profitability.

The *M*s are not affected by ρ, as in Case [2], so long as the other parameters are fixed. As ρ becomes negative, each quantile point increases, which implies that the downside bound increases or the risk becomes smaller, and that tenant replacement has a high impact on the enhancement of the NPV. In addition, a smaller ρ means a smaller LSD. Thus, as before, SC managers should select a new tenant whose contracted sales are negatively correlated with those of the tenants in the neighborhood of a spot that will soon be available.

Table 6 shows the M/R. In Case [3], M/R is large even when ρ is positive and large, since the SLTR significantly reduces the LSD. Furthermore, comparing Cases [2] and [3] (see Tables 4 and 5), it is found that the M/Rs in all cases are improved by the SLTR rule, and in particular, tenant diversification in terms of business sales works significantly. In fact, in the case of $\rho = -0.95$, the ratio is 46.53 as shown in Table 6, which is more than 4 times larger than that of Case [1], where $\alpha = 0.35, c = 0.8$. Table 6 shows the diversification effect in forming a tenant portfolio.

Table 6 Case [3] M/R for Changes in ρ ($\alpha = 0.35, c = 0.8$)

	ρ=0.95	ρ=0.75	ρ=0	ρ=-0.75	ρ=-0.95
M/R	22.71	24.21	29.69	40.42	46.30

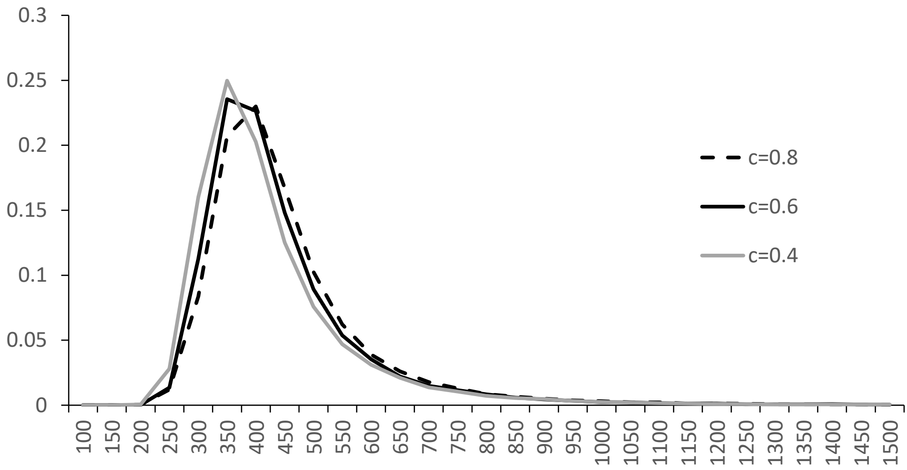
6.4 NPV Distributions with SLTR Cut-off Point c Changed

In this section, we consider the case where c takes values in the set of 80%, 60%, and 40% for (a) $\rho = 0$, (b) $\rho = 0.75$, and (c) $\rho = -0.75$, respectively.

a) $\rho = 0$

This case is treated in Figure 5, and it is observed that a larger c shifts the distribution of NPV more to the right, and so NPVs are more enhanced due to the increased frequency of tenant replacement. Table 7 shows that the Min, all quartile points and *M* become larger, while our risk measure LSD conversely becomes smaller as the threshold of c increases. As it stands, this is desirable, but we have to take into account the replacement cost, which can be calculated by replacing the *M* with a cost-adjusted *M* in M/R, although we do not carry this out here. Or if the M/R is evaluated to be large enough to cover the cost, actual observed ratios will help an SC manager to evaluate whether they are large enough in tenant management. In fact, since renovation costs may be covered by the deposit charged at the beginning of the tenancy, the expenses of an SC depend on the lease contract and its management strategy.

Figure 5 NPV Distribution for Changes in c with $\rho = 0$ ($\alpha = 0.35$)



In Table 8, the M/Rs are given for each c. Although the choices here are limited, an optimal c will lie in the interval (0.6, 0.8] when we look into the quantile values in Table 7. Compared to $M/R = 9.58$ in the base case (Case [1]) and $M/R = 12.95$ in Case [2] with $\rho = 0$, $M/R = 25.96$ in Case [3] with $c = 0.6$ and $\rho = 0$ is much larger, thus implying that tenant replacement can be a very important control scheme to significantly enhance the NPV.

Table 7 Summary of NPV Distribution for Changes in c with $\rho = 0$ ($\alpha = 0.35$)

	c=0.8	c=0.6	c=0.4
LSD	30.77	34.22	44.37
Min	405.56	391.31	401.18
1%	571.24	567.60	544.66
5%	643.91	630.31	603.39
10%	685.11	667.85	639.63
20%	738.70	717.83	688.71
50%	863.54	838.71	807.30
Mean	913.48	888.71	859.03

Table 8 M/R Ratio for Changes in c with $\rho = 0$ ($\alpha = 0.35$)

	c=0.8	c=0.6	c=0.4
M/R	29.69	25.97	19.36

b) $\boldsymbol{\rho = 0.75}$

The NPV distributions in this positively correlated case $\rho = 0.75$ are plotted in Figure 6. The plot is almost identical to that of Case [3] with $\rho = 0$ in Figure 5. As in Table 9, each quantile point increases as c increases, as in Case [3] with $\rho = 0$ in Table 7. However, it is found that all of the quantiles for each c in Table 9 are smaller than those of Case [3] with $\rho = 0$ (Table 7). This means that when the contract sales growth rates of two tenants are positively correlated, the quantiles when $\rho = 0.75$ are smaller than those when $\rho = 0$ for each case c, or equivalently the down-sided risks of the total value $V = V_1 + V_2$ in terms of quantiles that are larger than those of the uncorrelated case for each c. More importantly, the LSDs of this positively correlated case with $\rho = 0.75$ are larger than those of the uncorrelated case for each c, so that for each c, the M/Rs in the positively correlated case in Table 10 become smaller than those in the uncorrelated case in Table 8.

Figure 6 NPV Distribution for Changes in $\boldsymbol{c}$ with $\boldsymbol{\rho = 0.75}$ ($\boldsymbol{\alpha = 0.35}$)

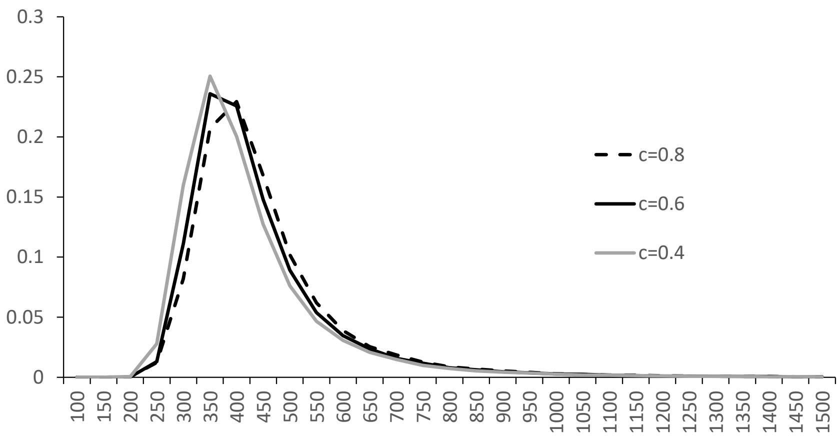


Table 9 Summary Statistics for Changes in $\boldsymbol{c}$ with $\boldsymbol{\rho = 0.75}$ ($\boldsymbol{\alpha = 0.35}$)

	c=0.8	c=0.6	c=0.4
LSD	37.80	42.23	53.95
Min	404.85	387.69	390.91
1%	549.77	547.40	526.11
5%	621.67	608.86	582.66
10%	662.33	645.55	617.01
20%	715.55	695.99	665.46
50%	843.66	817.79	785.97
Mean	915.27	889.26	859.15

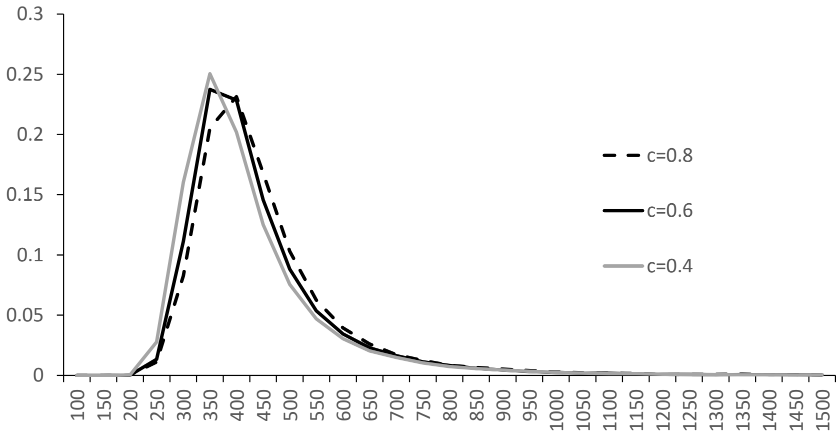
Table 10 M/R **Ratio for Changes in** c **with** $\rho = 0.75$ $(\alpha = 0.35)$

	c=0.8	c=0.6	c=0.4
***M*/R**	24.21	21.06	15.92

c) $\rho = -0.75$

In Figure 7, the NPV distributions of V are plotted with $\rho = 0.75$ and different cs which are similar to those when $\rho = 0$ and $\rho = 0.75$. Table 12 shows that the M/R when $\rho = -0.75$ increases as c increases. Also, the M/R ratio with $\rho =-0.75$ is significantly larger than those with $\rho = 0$ and $\rho = 0.75$ (Tables 8 and 10). It was found that the return-risk diversification effect increases as the correlation becomes more negative. In fact, as will be discussed in the next section, the risk reduction effect is the largest in the case of $\rho = -0.75$ for each c. In addition, the quantiles in this case are larger than those in the other cases, thus implying downside rigidity for the NPV.

Figure 7 **NPV Distribution for Changes in** c **with** $\rho = -0.75$ $(\alpha = 0.35)$



Summarizing the main point of the simulations from (a) to (c) above, tenant replacement is a crucial control tool to enhance the M/R in SC management and create a better tenant portfolio by using a sales correlation. In practice, tenant replacement involves the cost for finding a new tenant whose (contract) sales is negatively correlated with the sales of tenants in the neighborhood of a spot that will soon be available, and the cost for renovating the leased space. However, since renovation costs may be covered by the deposit charged at the beginning of tenancy, the expenses of the SC depend on the lease contract and its management strategy. KKUS discuss the impact of the inclusion of the rule for c on the income return and risk when the tenant replacement cost is proportional to the market fixed rent at the time of the contract. However, note that KKUS only analyzes the case with a single tenant whose activities are independent of the other tenants. It is necessary to consider how the rule for c should be set, when the replacement cost exists and contract sales of both tenants are correlated.

Table 11 Summary Statistics for Changes in c with $\rho = -0.75$ ($\alpha = 0.35$)

	c=0.8	c=0.6	c=0.4
LSD	22.63	25.45	32.65
Min	396.74	448.04	469.36
1%	603.19	596.20	577.79
5%	671.57	655.89	634.09
10%	710.84	691.32	667.00
20%	761.73	737.99	710.77
50%	876.08	847.38	816.82
Mean	914.65	887.94	858.86

Table 12 M/R Ratio for Changes in c with $\rho = -0.75$ ($\alpha = 0.35$)

	c=0.8	c=0.6	c=0.4
M/R	40.42	34.90	26.31

6.5 Overall Comparisons of Cases [2] and [3]: Effect of Tenant Replacement

Now let us more specifically compare the overall results in Cases [2] and [3].

Recall that the market lease (rent) rate is assumed to follow the discrete-time diffusion process in Equation (19) where the drift process follows an exponentially smoothing process (Equation (20)) with an initial smoothing parameter of 0.2 and the volatility process is held as a constant with a volatility of 0.05. Similarly, the sales processes of the two tenants follow the DD processes of the same type with the same parameters except for the fact that the sales volatilities are 0.2 and the sales growth rates are correlated with ρ as in Equation (21). In calculating the NPV, the interest rate is assumed to be 0.2. The percentage rent rate is $\alpha_i = 0.35$ for $i = 1,2$, and the tenant replacement rule is given by Equation (14). In Case [1] with no percentage rate linked to sales and no tenant replacement, the return-risk ratio is $M/LSD = 718.49/75.02 = 9.58$.

The simulations in Cases [2] and [3] with a percentage rate are respectively the cases without and with tenant replacement. It was observed that Case [3] performs much better than Case [2] in risk reduction as well as in return-risk ratio. The results on the ratios in Case [3] are summarized in pairs (ρ, c) of correlation and cut-off points for tenant replacement in Tables 8, 10 and 12.

Note that the case without tenant replacement is merely the case with $c = 0$, which is treated in Case [2].

Clearly from Table 13, a larger c and/or less correlation mean a larger ratio. The worst case is $\rho = 0.75$ and $c = 0.4$ with an NPV of 15.92. When $M/R = 15.92$ with $\rho = 0.75$ and $c = 0.4$ is compared to $M/R = 10.34$ in Case [2] with $\rho = 0.75$ and $c = 0$ (Table 4), it is shown that even in the case with $c = 0.4$ there are some tenant replacements so that the M/R is increased from 10.34 to 15.92.

In Table 14, the values of (M, LSD) for each c are listed to see the individual contributions to the ratios in Table 13. It is clear that the Ms of the NPV are almost same regardless of the ρ for each c, thus implying that ρ does not make a difference in the Ms. However, ρ significantly reduces the LSD, which contributes to the enhancement of the M/Rs. In the meantime, for each ρ, the Ms decrease as c becomes smaller, where the decrements are about 25 from $c = 0.8$ to $c = 0.6$ and about 30 from $c = 0.6$ to 0.4. This implies that tenant replacement contributes to the enhancement of the Ms as well as the reduction of LSD by increasing c.

Table 13 Ratios in Combinations of (ρ, c)

M/R	$c = 0.8$	$c = 0.6$	$c = 0.4$
$\rho = 0.75$	24.21	21.06	15.92
$\rho = 0.00$	26.69	25.97	19.34
$\rho = -0.75$	40.42	34.94	26.31

Table 14 Values of (M, LSD) in Combinations of (ρ, c)

(M, R)	c =0.8	c=0.6	c=0.4
$\rho = 0.75$	(915.27, 37.80)	(889.26, 42.23)	(859.15, 53.95)
$\rho = 0.00$	(913.48, 30.77)	(888.71, 34.97)	(859.03, 34.22)
$\rho = -0.75$	(914.65, 22.63)	(887.94, 25.45)	(858.86, 32.65)

To visualize the enhancements due to the adoption of the SLTR rule linked to the business performance (sales) of the tenants in [3] in addition to the percentage rent linked to sales ($\alpha = 0.35$) in [2], the NPV distributions of the corresponding cases are plotted for $\rho = 0$, $\rho = 0.75$ and $\rho = -0.75$. Here, we assume $c = 0.7$. Each figure (Figures 8 – 10) clearly shows the efficiency of the SLTR relative to the NPT distribution in Case [2] although the figures do not clearly distinguish the 3 cases as in the facts.

Finally, Table 15 summarizes the overall features of the NPV distributions in Cases [2] and [3] when $\rho = 0$, $\rho = 0.75$ and $\rho = -0.75$, in addition to Case [1]. The NPV is larger in Case [3] than in Case [2] at all quantile points and for all the *M*s, thus indicating that the value of the SC is increased with the SLTR rule. At each quartile, Case [2]/[3] is in the range of 0.85 to 0.93 for all correlations, thus indicating that the SLTR rule contributes to reducing the downside rigidity of the NPV distributions. As for the LSD, Case [2]/[3] is 2.07, 2.11, and 1.81 when $\rho = 0, 0.75$, and -0.75, respectively, which implies that regardless of the correlation, the SLTR rule significantly reduces the LSD for each case.

Table 16 shows that the M/R increases since the mean of NPV goes up and the LSD declines when ρ decreases. In all of the correlation cases, M/R more than doubles with the inclusion of SLTR, which suggests that the diversification effect is improved.

In short, it is important to well incorporate a tenant replacement scheme in SC management for sustainability.

Figure 8 Return-risk Enhancement Due to Tenant Replacement Effect ($\rho = 0$)

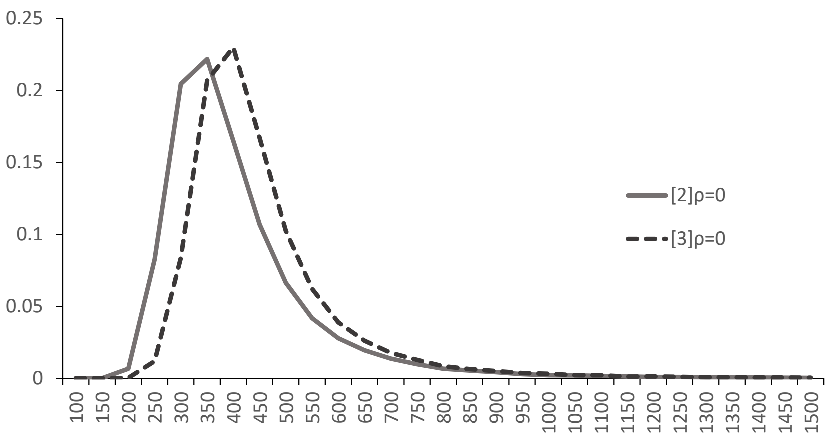


Figure 9 Return-risk Enhancement Due to Tenant Replacement Effect ($\rho = 0.75$)

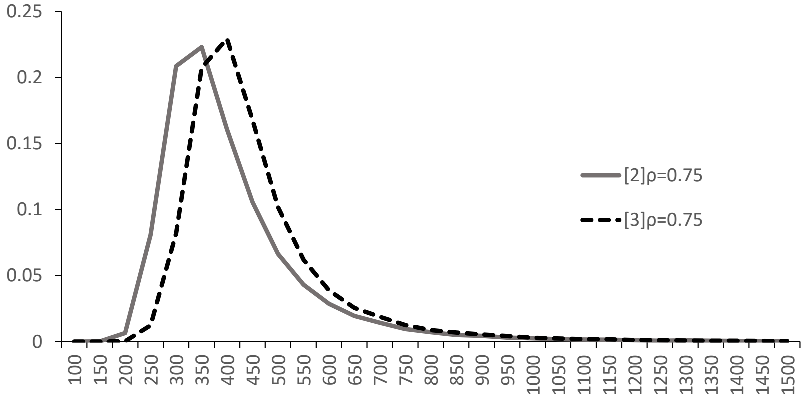


Figure 10 Return-risk Enhancement Due to Tenant Replacement Effect ($\rho = -0.75$)

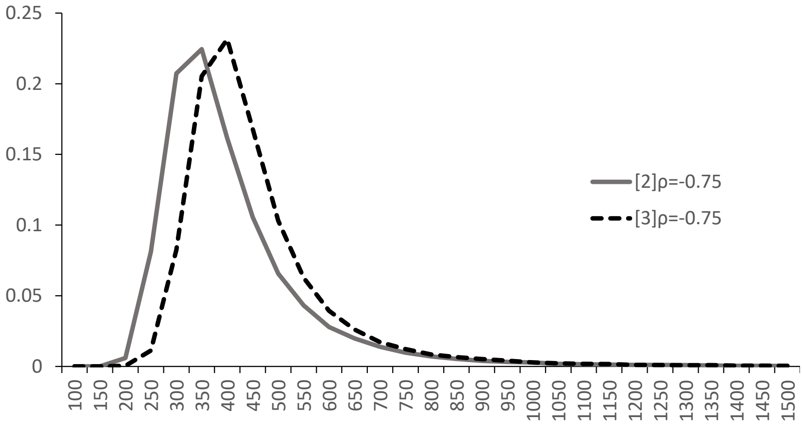


Table 15 Return-risk Enhancement: Summary

	[1]	**[2] ρ=0 (wo. SLTR)**	**[3] ρ=0 (w. SLTR)**	**[2] ρ=0.75 (wo. SLTR)**	**[3] ρ=0.75 (w. SLTR)**	**[2] ρ=-0.75 (wo. SLTR)**	**[3] ρ=-0.75 (w. SLTR)**
LSD	75.02	63.74	30.77	79.83	37.80	40.93	22.63
Min	383.91	369.26	405.56	339.73	404.85	457.52	396.74
1%	497.89	500.42	571.24	469.82	549.77	559.00	603.19
5%	549.63	560.44	643.91	530.22	621.67	612.82	671.57
10%	580.10	597.98	685.11	566.60	662.33	645.79	710.84
20%	620.68	648.71	738.70	617.89	715.55	688.58	761.73
50%	707.34	772.61	863.54	749.14	843.66	786.36	876.08
Mean	718.49	825.18	913.48	825.12	915.27	824.20	914.65

Table 16 Return-risk Enhancement in M/R Ratio

	[1]	[2] ρ =0 (wo. SLTR)	[3] ρ =0 (w. SLTR)	[2] ρ =0.75 (wo. SLTR)	[3] ρ =0.75 (w. SLTR)	[2] ρ =-0.75 (wo. SLTR)	[3] ρ =-0.75 (w. SLTR)
M/R	9.58	12.95	29.69	10.34	24.21	20.14	40.42

7. Concluding Remarks

In this paper, we have extended the analytical framework in KKUS to a framework that incorporates the correlation effect of the sales growth rates of two tenants into a revenue analysis for SCs to form a better and possibly optimal tenant portfolio in their tenant management efforts. Then, by taking into account the correlations of sales growth rates, the probability distributions of the NPV are derived through tenant management with the percentage rent linked to the sales of each tenant and the tenant replacement rule linked to the business performance of each tenant. This framework will enable SC managers to enhance the value of their SC to attract customers and create an innovative lease contract structure for value creation, in particular with a replacement management condition. Using simulations, these scenarios are modelled and under our specific models, we demonstrate the return-risk enhancements in terms of M/R with percentage rent α and a specific tenant replacement rule, where the correlation of the sales growth rates of two tenants is taken into account to consider a future problem of an optimal tenant portfolio. An important point in this part is that each SC has data on the details of sales components or categories with which the correlation structure of the sales of tenants can be formed empirically, which will be very valuable to them.

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