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Housing Market Dynamics in Kazakhstan: An Estimated DSGE Model

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Appendix

A1. Log-linearized Equations

$$\begin{aligned}\hat{p}_{D/C,t} &= \Omega^b(\hat{\tilde{c}}_t^b - \hat{d}_t^b) + \beta_b(1-\delta)[(1-\sigma)\gamma E_t(\Delta\hat{d}_{t+1}^b) - \Gamma E_t(\Delta\hat{\tilde{c}}_{t+1}^b)] \\ &+\beta_b(1-\delta)E_t(\hat{p}_{D/C,t+1}) + \psi(1-\chi)(\hat{\psi}_t + \hat{p}_{D/C,t} + \hat{\pi}_{D,t+1} + \epsilon_t^{Pension}) + \Lambda^b\epsilon_t^{d,b}\end{aligned} \tag{A1}$$

$$\begin{aligned}\hat{p}_{D/C,t} &= \Omega^s(\hat{\tilde{c}}_t^s - \hat{d}_t^s) + \beta_s(1-\delta)[(1-\sigma)\gamma E_t(\Delta\hat{d}_{t+1}^s) - \Gamma E_t(\Delta\hat{\tilde{c}}_{t+1}^s)] \\ &+\beta_s(1-\delta)E_t(\hat{p}_{D/C,t+1}) + \Lambda^s\epsilon_t^{d,s}\end{aligned} \tag{A2}$$

$$\hat{\psi}_t = \frac{\beta_b}{\beta_s - \beta_b}\begin{bmatrix} \Gamma E_t(\Delta\hat{\tilde{c}}_{t+1}^b) - (1-\sigma)\gamma E_t(\Delta\hat{d}_{t+1}^b) \\ -(\hat{r}_t - \hat{\pi}_{C,H,t+1} - \alpha_C E_t(\Delta\hat{s}_{C,t+1})) + (1-\sigma)\gamma[lnD^b - ln\tilde{C}^b](1-\rho_{d,b})\epsilon_t^{d,b}\end{bmatrix} - \hat{r}_t \tag{A3}$$

$$\begin{aligned}\Gamma\hat{\tilde{C}}_t^s &= \Gamma E_t\hat{\tilde{C}}_{t+1}^s - (1-\sigma)\gamma E_t(\Delta\hat{d}_{t+1}^s) - (\hat{r}_t - \hat{\pi}_{C,H,t+1}) + \alpha_C\Delta\hat{s}_{C,t+1} \\ &+(1-\sigma)\gamma(lnD^s - ln(\tilde{C}^s))(1-\rho_{d,s})\epsilon_t^{d,s}\end{aligned} \tag{A4}$$

$$\begin{aligned}\hat{c}_t^b &= \frac{B_H^b}{\tilde{C}^b}[\hat{b}_{H,t}^b - \beta_s^{-1}(\hat{r}_{t-1} + \hat{b}_{H,t-1}^b - \hat{\pi}_{C,H,t} - \alpha_c\Delta\hat{s}_{C,t})] + M_C(\widehat{\tilde{w}}_{C,t} + \hat{n}_{C,t}^b) \\ &+P_{D/C}M_D(\widehat{\tilde{w}}_{D,t} + \hat{n}_{D,t}^b) - P_{D/C}\frac{D^b}{\tilde{C}^b}(\delta\hat{p}_{D/C,t} + \hat{d}_t^b - (1-\delta)\hat{d}_{t-1}^b)\end{aligned} \tag{A5}$$

$$\hat{b}_{H,t}^b = \hat{p}_{D/C,t+1} + \hat{d}_t^b - (\hat{r}_t - E_t(\hat{\pi}_{C,Ht+1} + \alpha_c\Delta s_{C,t+1})) + \epsilon_t^{Pension} \tag{A6}$$

$$\hat{y}_{C,t} = (1-\alpha_C)\hat{c}_t + \alpha_C\hat{c}_t^* + \alpha_C\upsilon_C\hat{s}_{C,t} + g_t \tag{A7}$$

$$\hat{y}_{D,t} = (1-\alpha_D)\hat{\imath}_{D,t} + \alpha_D\hat{\imath}^*_{D,t} + \alpha_D\upsilon_D\hat{s}_{D,t} + g_t \tag{A8}$$

$$\hat{c}_t = w\frac{C^b}{C}\hat{c}_t^b + (1-w)\frac{C^s}{C}\hat{c}_t^s \tag{A9}$$

$$\hat{d}_t = w\frac{D^b}{D}\hat{d}_t^b + (1-w)\frac{D^s}{D}\hat{d}_t^s \tag{A10}$$

$$\hat{\pi}_{j,H,t} = \beta_s\theta_j\phi_j E_t\hat{\pi}_{j,H,t+1} + \tau_j\phi_j\hat{\pi}_{j,H,t-1} + \kappa_j\widehat{mc}_{j,t} + \epsilon_t^{\mu_j}, j = C, D \tag{A11}$$

$$\widehat{mc}_{C,t} = \Gamma\hat{\tilde{c}}_t - (1-\sigma)\gamma\hat{d}_t + \phi_N\hat{n}_{C,t} + \alpha_C\hat{s}_{C,t} - a_{C,t} - \sum\epsilon_t^d \tag{A12}$$

$$\widehat{mc}_{C,t} = \Gamma\hat{\tilde{c}}_t - (1-\sigma)\gamma\hat{d}_t + \phi_N\hat{n}_{C,t} + \alpha_C\hat{s}_{C,t} - a_{C,t} - \sum\epsilon_t^d \tag{A13}$$

$$\hat{s}_{C,t} = \frac{1}{1-\alpha_C}[\Gamma\hat{\tilde{c}}_t^s - (1-\sigma)\gamma\hat{d}_t^s - \hat{\tilde{c}}_t^* + (1-\sigma)\gamma\Gamma^{-1}\hat{d}_t^*] \tag{A14}$$

$$\hat{y}_{j,t} = \hat{\alpha}_{j,t} + \hat{n}_{j,t}, j = C, D \tag{A15}$$

$$\hat{\widetilde{w}}_{j,t} = \Gamma\hat{\tilde{c}}_t - (1-\sigma)\gamma\hat{d}_t + \phi\hat{n}_{j,t} - \left[(1-\sigma)\gamma(lnD - ln\tilde{C})\frac{\gamma}{1-\gamma}\right]\epsilon_t^D \tag{A16}$$

$$\hat{\widetilde{w}}_{j,t} = \Gamma\hat{\tilde{c}}_t^i - (1-\sigma)\gamma\hat{d}_t^i + \phi\hat{n}_{j,t}^i - \left[(1-\sigma)\gamma(lnD^i - ln\tilde{C}^i)\frac{\gamma}{1-\gamma}\right]\epsilon_t^D \tag{A17}$$

$$\hat{p}_{D/C,t} = \hat{p}_{D/C,t-1} + \hat{\pi}_{D,H,t} - \hat{\pi}_{C,H,t} + \alpha_D\Delta\hat{s}_{D,t} - \alpha_C\Delta\hat{s}_{C,t} \tag{A18}$$

$$\Delta\hat{s}_{C,t} = \hat{\pi}_{C,F,t} - \hat{\pi}_{C,H,t} \tag{A19}$$

$$(1-\alpha_c)\hat{s}_{C,t} - (1-\alpha_D)\hat{s}_{D,t} = \hat{p}_{D,C,t} - \hat{p}^*_{D,C,t} \tag{A20}$$

$$\hat{\pi}_{C,t} = \hat{\pi}_{C,H,t} + \alpha_C \Delta\hat{s}_{C,t} \tag{A21}$$

$$\hat{y}_t = \frac{P_{D/C}^{-\xi} C}{Y}\hat{y}_{C,t} + \frac{\delta P_{D/C}^{1-\xi} D}{Y}\hat{y}_{D,t} + \Xi\hat{p}_{D/C,H,t} - \xi ln P_{D/C}(\epsilon_t^D + \epsilon_t^{D,*}) \tag{A22}$$

$$\hat{p}_{D/C,H,t} = \hat{p}_{D/C,t} - \alpha_D\hat{s}_{D,t} + \alpha_C\hat{s}_{C,t} \tag{A23}$$

$$\hat{n}_t = \frac{N_C}{N}\hat{n}_{C,t} + \frac{N_D}{N}\hat{n}_{D,t} \tag{A24}$$

$$\phi_j \equiv \frac{1}{\theta_j + \tau_j(1-\theta_j(1-\beta_s))} \tag{A25}$$

$$\kappa_j \equiv \frac{(1-\tau_j)(1-\theta_j(1-\beta_s\theta_j))}{\theta_j + \tau_j(1-\theta_j)(1-\beta_s)} \tag{A26}$$

$$\Gamma \equiv \frac{\sigma + (1-\sigma)\gamma}{1-h_c} \tag{A27}$$

$$\lambda^j \equiv \left[\frac{\Omega^j}{1-\gamma} - (1-\rho_{d,j})\beta_j(1-\delta)\left\{(1-\sigma)\gamma(lnD^j - ln\tilde{C}) - \frac{\gamma}{1-\gamma}\right\}\right] \tag{A28}$$

$$\Omega^j \equiv \frac{\gamma}{1-\gamma}\frac{\widetilde{C^j}}{D^j P_{D/c}} \tag{A29}$$

$$M_j \equiv \frac{1}{1+\mu} \frac{N_j}{\tilde{C}^b} \tag{A30}$$

$$\Sigma \equiv \left[(1-\sigma)\gamma(lnD - ln\tilde{C}) - \frac{\gamma}{1-\gamma}\right] \tag{A31}$$

A2. Steady State

$$R = \beta_s^{-1} \tag{A32}$$

$$\psi = \beta_s - \beta_b \tag{A33}$$

$$MC_j = \frac{1}{1+\mu_j} \tag{A34}$$

$$P_{D/C} = \frac{1+\mu_D}{1+\mu_C} \tag{A35}$$

$$\frac{C^b}{D^b} = \frac{1-\gamma}{\gamma} \frac{1-\beta_b(1-\delta)-(1-\chi)(1-\delta)(\beta_s-\beta_b)}{(1-h_c)} P_{D/C} \tag{A36}$$

$$\frac{C^s}{D^s} = \frac{1-\gamma}{\gamma} \frac{1-\beta_s(1-\delta)}{1-h_c} P_{D/C} \tag{A37}$$

$$\frac{B_H^b}{D^b} = \beta_s(1-\chi)(1-\delta)P_{D/C} \tag{A38}$$

$$D^b = \frac{\frac{1}{1+\mu_c} N}{\frac{C^b}{D^b} + (\delta - (1-\chi)(1-\delta)(\beta_s - 1))P_{D/C}} \tag{A39}$$

$$C^b = \frac{C^b}{D^b} D^b \tag{A40}$$

$$N = N_C + N_D \tag{A41}$$

$$N_D = \frac{\omega\delta \frac{C^s}{D^s}}{\frac{C^s}{D^s} + \delta} D^b + \frac{\delta}{\frac{C^s}{D^s} + \delta}(N - \omega C^b) \tag{A42}$$

$$D^s = \frac{1}{(1-\omega)\delta}(N_D - \omega\delta D^b) \tag{A43}$$

$$C^s = \frac{1}{1-\omega}(N_c - \omega C^b) \tag{A44}$$

$$C = N_C = Y_C \tag{A45}$$

$$\delta D = N_D = Y_D \tag{A46}$$